

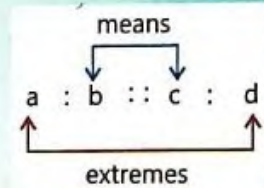
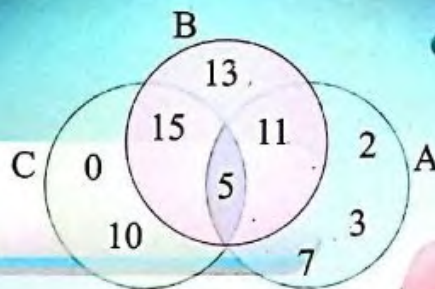
Mathematics

Test Edition

8

Free From Government
NOT FOR SALE

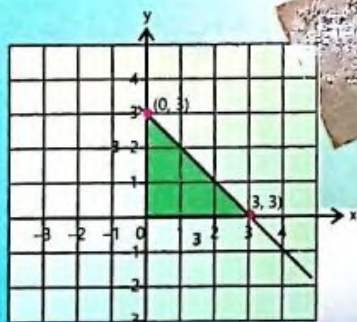
Sets



Financial

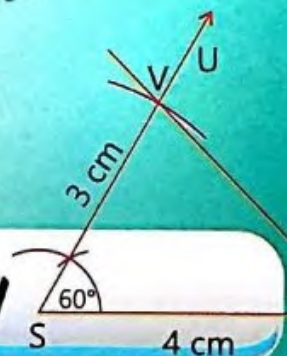
$$(a+b)(a-b) = a^2 - b^2$$

Gradient of a straight line = $\frac{\text{Vertical change}}{\text{Horizontal change}}$



Linear
Equations

Geometry



**KHYBER PAKHTUNKHWA TEXTBOOK BOARD,
PESHAWAR**

Download PDF MCQs, Books and Notes for any subject from the Largest website for free

awazeinqilab.com

آواز انقلاب ڈاٹ کام



and also Available in Play Store



Primary (5th) Level

Middle (8th) Level

Matric (10th) Level

F. Sc (12th) Level

BS (16th) Level

Urdu Grammar

Curriculum (Govt Books)

Past Papers

English Grammar

Readymade CV

NTS, FTS, CSS, PMS,
ETEA, KPPSC, PPSC,
SPSC, AJKPSC, BPSC,
KPTA ETC

Subject کے Notes اور MCQS فری
ڈاؤن لوڈ کریں۔ نیز CV کے MS word تیار

Templates ڈاؤن لوڈ کر کے اپنے حساب



سے Modify کریں۔

awazeinqilab.com

WhatsApp : 03025247378

Available Subjects

- Agriculture
- Computer Science
- Economics
- English
- Biology
- Chemistry
- Geology
- Management Sciences
- Microbiology
- PCRS/ Pak Study
- Pharmacy
- Sociology
- Zoology
- Chemistry
- Statistics
- Physics
- Mathematics
- Law
- Environmental Sciences
- Botany
- Biotechnology
- Library & Information Sciences
- Journalism & Mass Communication
- Political Science
- Geography
- General Science
- Education
- Tourism & Hotel Management
- Urdu
- Pashto
- Islamic & Arabic Studies
- Electrical Engineering
- Civil Engineering
- Mechanical Engineering
- Chemical Engineering
- Software Engineering
- Election Officer
- General Knowledge
- Pedagogy
- Current Affairs
- Medical Subjects
- Judiciary & Law
- other



Model Textbook

Mathematics

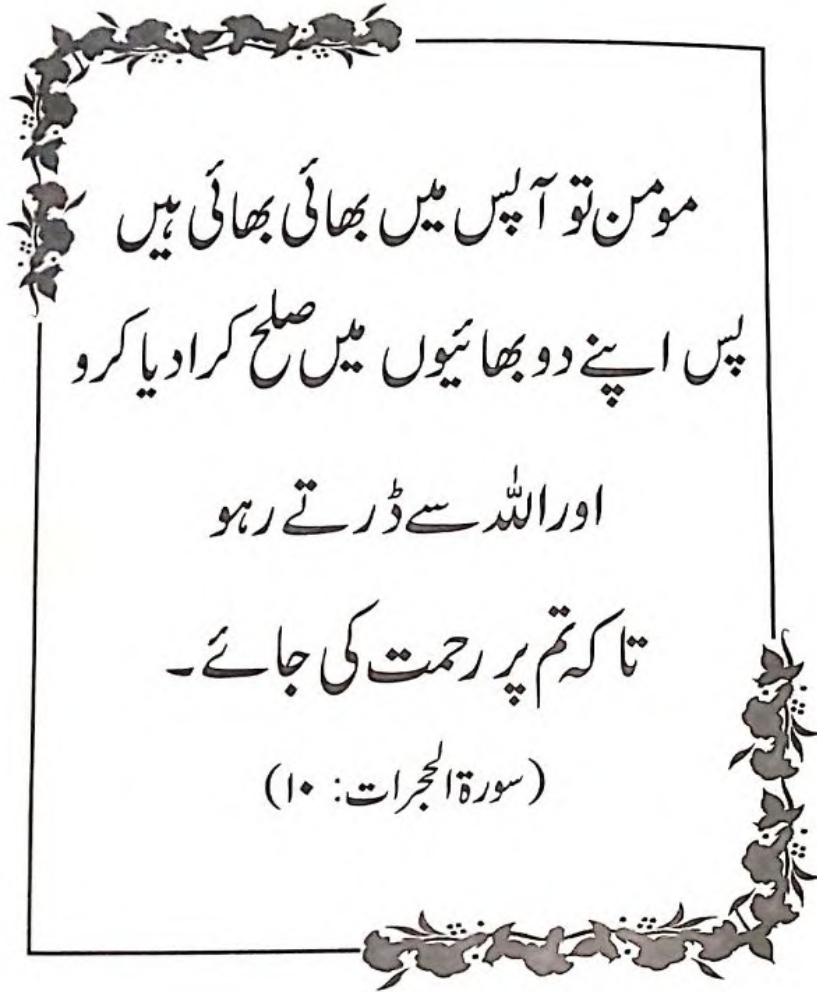
Grade 8

Based on Single National Curriculum 2022

One Nation, One Curriculum



**KHYBER PAKHTUNKHWA TEXTBOOK BOARD,
PESHAWAR**



All rights reserved with Association For Academic Quality (AFAQ), Lahore

Approved by the Directorate of Curriculum and Teachers Education (DCTE), Khyber Pakhtunkhwa Abbottabad vide NOC No. 2438-42/SNC-2022/Maths Dated 12-05-2022.

Authors:

- Ms. Madeeha Nuzhat Varaich
- Ms. Sadia Manzoor

Review Supervision:

Mr. Jaffar Mansoor Abbasi, Director, DCTE Khyber Pakhtunkhwa Abbottabad

Focal Person:

Syed Amjad Ali, Additional Director (CTR), DCTE Khyber Pakhtunkhwa Abbottabad

Reviewed by:

- Mr. Abdussalam, Principal GHSS Kokal Berseen Abbottabad
- Mr. Azhar Qayyum, Lecturer, GPGC Mansehra
- Mr. Amer Khan, SS, GHSS No. 2 Cantt Peshawar
- Mr. Abbas Khan, Desk Officer, DCTE Khyber Pakhtunkhwa Abbottabad
- Dr. Muhammad Shah, Khyber Pakhtunkhwa Textbook Board, Peshawar

Editor:

Mr. Nasir Kamal, Desk Officer (Maths)
Khyber Pakhtunkhwa Textbook Board, Peshawar

Printing Supervision:

Arshad Khan Afridi, Chairman
Abdul Wali Khan, Member (E&P)
Khyber Pakhtunkhwa Textbook Board, Peshawar

Academic Year: 2022-23

Email: memberbhb@yahoo.com

Website: www.kptbb.gov.pk

Phone: 091-9217714-15

بِسْمِ اللَّهِ الرَّحْمَنِ الرَّحِيمِ

Content

Sr.No	Unit	Page
1	Approximation and Estimation	1
2	Real Numbers	8
3	Financial Arithmetic	33
4	Square Roots and Cube Roots	60
5	Sets	86
6	Sequences and Algebra	104
7	Exponents and Laws of Exponents	147
8	Simultaneous Linear Equations	163
9	Surface, Area and Volume	208
10	Geometry	238
11	Data Management and Probability	275
	Answers	308



Unit 1

Approximation and Estimation

Student Learning Outcomes

After completing this unit, students will be able to:

- Round off numbers up to 5 significant figures.
- Analyze approximation error when numbers are rounded off.
- Solve real-world word problems involving approximation.



The capacity of an oil tanker is 24567.57 litres. What is this capacity to the nearest whole number?

NOT FOR SALE

Introduction

In many daily life situations, we are not able to calculate the exact amount or quantity of something. We need an answer that is close to the actual calculation. In such situations we need to estimate and get an approximate answer. While shopping, we need to estimate how much total bill we have to pay. This answer may not be the actual sum but it can be something around the actual bill. Similarly in mathematical calculations, we need to estimate the calculation before actually calculating and finding the exact result.

1.1 Estimation and Approximation

The most common and easiest type of estimation is rounding off. We know that rounding off helps us quickly estimate the number we need to use in various calculations. It helps us make a complicated number much simpler to work with. This include rounding off whole numbers to the nearest 10, 100, and so on.

Similarly rounding a decimal includes rounding it off to the nearest whole number, tenth, hundredths etc. This is where you make a long number simpler by 'rounding', or expressing in terms of the nearest unit, ten, hundred, tenth, or a certain number of decimal places.

Approximation Error

The difference between an exact value and the approximated value of something is known as **approximation error**.

For example, the mass of a sugar bag is 40.53 to be exact but the weighing scale allows to weigh only up to one decimal place or nearest 0.1. So, the approximate mass will be mentioned as 40.5 kilogram.

Similarly, the exact length of an electric wire is 25.82 metres. But when measured with measuring tape that only allows to measure to the nearest tenths. So, the approximate length will be mentioned as 25.8 metre.



1.1.1 Significant Figures

Significant figures are the number of figures or digits that indicate the accuracy of any calculation or measurement.



Ask students to analyze the rounding and approximation error by taking approximate measurements (lengths, masses, capacities) and then using tools to verify the exact ones.

NOT FOR SALE

Let's recall how to determine the number of significant figures in any number.

- All non-zero digits are significant figures. For example, 172.3 has 4 significant figures.
- Zeros at the start of a number are never significant. For example, 0.0789 has only 3 significant figures.
- Zeros that are within any two non-zero digits are significant. For example, 89.045 has 5 significant figures.
- Zeros at the end of a number after the decimal point are significant. For example, 9.500 has 4 significant figures.
- Zeros at the end of the whole numbers are not significant. For example, 9500 has 2 significant figures.

Note it down

The closeness of a measured value and a true(actual) value of something is known as measurement accuracy.

Example 1:

The actual or exact mass of a watermelon is 4.04 kilogram. When we round off it to the nearest whole number (kilogram), we get its mass equal to 4 kilogram and if we measure it to the nearest tenth, we get its mass equal to 4.0 kilogram. It means: The mass of watermelon is 4 kilograms rounded to one significant figure and the mass of watermelon is 4.0 kilogram rounded to two significant figures.



1.1.2 Rules to round to the specified number of significant figures

When rounding to the specified number of significant digits, round down if the next digit is less than 5 and round up when the next digit is greater than or equal to 5.

Example 1:

There are 657238 trees in forest. Round this off to 4 significant figures.



Solution:

Here we consider the four leftmost digits which are 6572. The digit next to the digit 2 is 3 which is smaller than 5. So, we will round down. So, keep 6572 as it is and replace next digits by zeros. So, 657238 rounded to 4 significant figures is 657200.

Example 2:

Round 872698 to 4 significant figures.

Solution:

Here we consider four leftmost digits which are 8726. The digit next to the digit 6 is 9 which is greater than 5. So, we will round up. So, we will add 1 to the digit 6 and replace next digits by zeros. So, 872698 rounded to 4 significant figures is 872700.



Ask the students to collect information about population and areas of different cities of Pakistan, and different countries of the world and then round the values to different numbers of significant figures.

Example 3:

Round 372345 to 5 significant figures.

Solution:

Here we consider five leftmost digits which are 37234. The digit next to the digit 4 is 5. So, we will round up. We will add 1 to the digit 4 and replace next digits by zeros.

So, 372345 rounded to 5 significant figures is 372350.

Example 4:

Round 1365831 to 5 significant figures.

Solution:

Here we consider the five leftmost digits which are 13658. The digit next to the digit 8 is 3 which is smaller than 5. So, we will round down. So, keep 13658 as it is and replace next digits by zeros.

So, 1365831 rounded to 5 significant figures is 1365800.

Example 5:

The exact length of a rope is 452.129 metre.

Round this length off to four significant figures.



Solution:

Here we consider 452.1. The next digit to 1 is 2 which is smaller than 5. So, we will round down. Keep 452.1 as it is and drop next digits.

So, 452.129 rounded to 4 significant figures is 452.1

Example 6:

Round 87.23178 to 5 significant figures.

Solution:

Here we consider 87.231. The next digit to 1 is 7 which is greater than 5. So, we will round up. Add 1 to the digit 1 and drop the next digits.

So, 87.23178 rounded to 5 significant figures is 87.232.

Quick Check

Round off 44.89216 to:
a) 3 significant figures
b) 4 significant figures
c) 5 significant figures

Exercise 1.1

1 Round the following whole numbers to 3 significant figures.

- a) 67245 b) 99244 c) 6378 d) 8230 e) 53772 f) 238796

- 2** Round the following whole numbers to 4 significant figures.
a) 83567 b) 178923 c) 873572 d) 93652 e) 83476 f) 45705
- 3** Round the following whole numbers to 5 significant figures.
a) 8723451 b) 5362852 c) 9024691 d) 8348347 e) 4528409 f) 5245891
- 4** Round the following decimal numbers to 3 significant figures.
a) 87.256 b) 58.2672 c) 9.24578 d) 2.6572 e) 4.8235
- 5** Round the following decimal numbers to 4 significant figures.
a) 72.5623 b) 89.0367 c) 56.8926 d) 6.09871 e) 23.23232
- 6** Round the following decimal numbers to 5 significant figures.
a) 4.10987 b) 65.23689 c) 872.12345
d) 0.234871 e) 65.72045
- 7** There are 87234 English language books and 63245 Art books, in a library. Round off these quantities to 4 significant figures and then add to find the approximate number of total books.
- 8** Zaeem want to buy a flat costing Rs 2,237,750. Round this amount off to 5 significant figures.
- 9** The mass of a bag of cement is 23.765 gram. Round this mass off to 4 significant figures.
- 10** The volume of water in a water tank is 6723.855 litres. Round this volume off to 5 significant figures.
- 11** The grocery bill has an exact amount of Rs 723.657. Round this off to 5 significant figures.



Think Higher

Write two decimals having 3 decimal places. Then multiply them and round the answer to 4 significant figures.

Summary

- The most common and easiest type of estimation is rounding off.
- The difference between an exact value and the approximated value of something is known as approximation error.
- Significant figures are the number of figures or digits that indicate the accuracy of any calculation or measurement.
- The closeness of a measured value and a true(actual) value of something is known as measurement accuracy.

Vocabulary

- Rounding off
- Estimation
- Approximation
- Rounding Error

Review Exercise

1 Encircle the correct option.

- a) How many significant figures are there in the number 3746000?
i) 4 ii) 5 iii) 6 iv) 7
- b) Which of these have 3 significant figures?
i) 23.78 ii) 67.07 iii) 0.350 iv) 680
- c) How many significant figures are there in the number 0.00589?
i) 3 ii) 4 iii) 5 iv) 6
- d) Which of these has 5 significant figures?
i) 0.0560 ii) 8.0560 iii) 81.0500 iv) 9.060
- e) How many significant figures are in 209110?
i) 4 ii) 5 iii) 6 iv) 7

Round the following to the given number of significant figures.

Numbers	Rounded off to 2 significant figures	Rounded off to 3 significant figures	Rounded off to 4 significant figures	Rounded off to 5 significant figures
a) 6729001				
b) 7609168				
c) 23.8712				
d) 0.672451				
e) 45.76217				

3 A library has 127462 monthly visitors. Round this value to:

- a) 2 significant figures b) 3 significant figures
c) 4 significant figures d) 5 significant figures

4 The population of Town A is 872345 and that of Town B is 624509. Round off these to 4 significant figures and then add to find the approximate population.

5 The distance between two landmarks is 452.9872 km. Round this off to
a) 3 significant figures. b) 4 significant figures. c) 5 significant figures.

Math Project

Material Required:

- Two types of calculators (four function and scientific) for each group
- Paper chits
- Baskets
- Recording sheet

Procedure:

- Work in groups.
- Teacher will put paper chits having various sums having non-terminating decimal results in a basket.
- Each pairs picks 5 chits randomly from the basket.
- Then they calculate the answers using both types of calculators.
- Then record the results from each calculator for the same question and analyze the rounding and approximating errors.



Unit 2

Real Numbers

Student Learning Outcomes

After completing this unit, students will be able to:

- Differentiate between rational and irrational numbers.
- Represent real numbers on a number line and recognise the absolute value of a real number.
- Demonstrate the ordering properties of real numbers.
- Demonstrate the following properties:
 - closure property -associative property-existence of identity element
 - existence of inverses -commutative property -distributive property
- Solve real-world word problems involving calculation with decimals and fractions.
- Identify and differentiate between decimal numbers as terminating (non-recurring) and non-terminating (recurring).



A certain road that is 812.5 kilometres long, has rest area after every $\frac{65}{2}$ kilometres. How many rest areas are on this road?

Introduction

We have learnt about rational numbers, their representation on a number line, comparison and also about their properties. Now we will learn about the real numbers (rational and irrational), their properties and representation on a number line but before it, let's learn about the types of decimal numbers which will help us understanding the concept of irrational numbers.

2.1 Types of Decimal Numbers

There are two types of decimal numbers.

- a) Terminating Decimal Numbers b) Non-Terminating Decimal Numbers

a) Terminating Decimal Numbers

When converting a fraction to a decimal by division method we mostly notice that the division is complete after certain steps and we get the remainder zero. In such cases, the quotient obtained as the decimal is called the terminating decimal.

So, the terminating decimal numbers are those decimal numbers that have a finite number of digits after the decimal point. Let's consider a few examples.

Example 1:

Express $\frac{43}{5}$ as a decimal number.

Solution:

- First divide the dividend 43 by the divisor 5. 5 goes into 43 eight times with 3 leftover. i.e. $5 \times 8 = 40$ r 3
- Now the remainder 3 is less than the divisor 5. So, put a decimal point after 8 in the quotient. Also put a decimal point after the dividend 43 and write an additional zero after it. So, 43 will be written as 43.0 (43 is same as 43.0). The decimal point in the quotient and dividend must be aligned.
- Bring down the 0. The remainder 3 will become 30.
- 5 goes into 30 six times, so the quotient is 8.6 with zero remainder now.
- The division process ends here.

$$\begin{array}{r} 8.6 \\ 5 \overline{) 43.0} \\ \underline{-40} \\ 30 \\ \underline{-30} \\ 0 \end{array}$$

So, $\frac{43}{5} = 8.6$ is a terminating decimal number.

Few more examples of terminating decimals include:

$$\frac{1}{2} = 0.5, \frac{1}{4} = 0.25, \frac{5}{2} = 2.5, \frac{27}{2} = 13.5$$

In the above examples, the division process is complete with the remainder zero. So, these are terminating decimals.

b) Non-Terminating Decimal Numbers

Many times when converting a fraction to a decimal by division method, the division process does not end and we do not get a remainder equal to zero. In such a case, the obtained decimal as quotient is called the non-terminating decimal.

Non-terminating decimal numbers are those decimal numbers that have an infinite number of digits after the decimal point.

Example 1:

Express $\frac{22}{7}$ as a decimal number.

Solution:

- First divide the dividend 22 by the divisor 7. 7 goes into 22 three times with 1 leftover. i.e. $7 \times 3 = 21$ r 1.
- Now the remainder 1 is less than the divisor 7. So, put a decimal point after 3 in the quotient. Also, put a decimal point after the dividend 22 and write an additional zero after it. So, 22 will be written as 22.0 (22 is same as 22.0). The decimal point in the quotient and dividend must be aligned.
- Bring down the 0. The remainder 1 will become 10.
- Carry on the division process. We can see that the division process continues infinitely and we cannot get the remainder of zero.

So, $\frac{22}{7} = 3.142...$ is a non-terminating decimal number.

Few more examples of non-terminating decimals include:

$$\frac{1}{6} = 0.1666..., \frac{5}{6} = 0.8333..., \frac{1}{3} = 0.3333..., \frac{7}{9} = 0.7777...$$

In the above examples, the digits are infinite after the decimal point. So, these are non-terminating decimals.

We can further classify the non-terminating decimals in 2 types:

- a) Repeating or Recurring Decimals b) Non-recurring Decimals

a) Repeating or Recurring Decimals

If a decimal number that has a single digit or a block of digits that repeats itself an infinite number of times after the decimal point is called a repeating or recurring decimal number. These repeated digits after the decimal numbers can be represented by putting a bar on the repeated part after the decimal point.

$$\begin{array}{r} 3.142 \\ 7 \overline{) 22.000} \\ \underline{-21} \\ 10 \\ \underline{-7} \\ 30 \\ \underline{-28} \\ 20 \\ \underline{-14} \\ 6 \end{array}$$

Example 1:

Express $\frac{4}{9}$ as a decimal fraction.

Solution:

- First divide the dividend 4 by the divisor 9. Here the dividend 4 is less than the divisor 9. So, $9 \times 0 = 0$ r 4.
- Now the remainder 4 is less than the divisor 9. So, put a decimal point in the quotient. Also put a decimal point after the dividend 4 and write an additional zero after it. So, 4 will be written as 4.0 (4 is same as 4.0). The decimal point in the quotient and dividend must be aligned.
- Bring down the 0. Now the remainder of 4 will become 40.
- Carry on the division process. We can see that the division process continues infinitely and we cannot get the remainder of zero.

$$\begin{array}{r} 0.444... \\ 9 \overline{) 4.000} \\ \underline{- 0} \\ 40 \\ \underline{- 36} \\ 40 \\ \underline{- 36} \\ 40 \\ \underline{- 36} \\ 4 \end{array}$$

Note it down

A small bar over the repeated digit(s) after the decimal point is used to represent the repeating or recurring decimals.

Quick check

Identify the recurring decimal.

- a) 0.3456754 b) 0.66666
c) 0.343434 d) 0.123123

$$\begin{array}{r} 0.666... \\ 3 \overline{) 2.000} \\ \underline{- 0} \\ 20 \\ \underline{- 18} \\ 20 \\ \underline{- 18} \\ 20 \\ \underline{- 18} \\ 20 \\ \underline{- 18} \\ 2 \end{array}$$

Example 2:

Express $\frac{2}{3}$ as a decimal fraction.

Solution:

By performing the division, we can see that:

$\frac{2}{3} = 0.666...$ is a repeating or recurring decimal number as the number 6 is infinitely repeated.

Note it down

The dot over the repeated digit(s) after the decimal point is also used to show that the digit or blocks of digits repeats itself an infinite number of times.

We can represent it as $0.666... = 0.\overline{6}$ or $0.\dot{6}$

Similarly, $\frac{7}{11} = 0.636363... = 0.\overline{63}$.

b) Non-Recurring Decimal Numbers

A non-terminating decimal number that continues endlessly, with no digit or a block of digits repeating, is called a non-recurring decimal number.

Example 1:

Express $\frac{32}{7}$ as a decimal fraction.

Solution:

By performing the division, we can see that the remainder is not zero. In the quotient, the digits after the decimal point are not repeating themselves.

So, $\frac{32}{7} = 4.571...$ is a non-recurring decimal number.

Few more examples of non-recurring decimals include:

$$\frac{8}{7} = 1.1428571..., \frac{20}{19} = 1.05263157..., \frac{5}{7} = 0.7142857...$$

In the above examples, the division process is not complete with a remainder of zero and in the quotient, the digits after the decimal point are not repeating themselves. So, these are non-recurring decimal numbers.

$$\begin{array}{r} 4.571... \\ 7 \overline{) 32.000} \\ \underline{- 28} \\ 40 \\ \underline{- 35} \\ 50 \\ \underline{- 49} \\ 10 \\ \underline{- 7} \\ 3 \end{array}$$

2.1.1 Standard form of a rational number

A rational number $\frac{p}{q}$ where $p, q \in \mathbb{Z}$ and $q \neq 0$ is said to be in the standard form if q is a positive integer, and p and q have no common factor other than 1.

For example, $\frac{2}{5}$, $\frac{-7}{3}$, $\frac{6}{11}$ and $\frac{17}{5}$ are in standard form.

The rational numbers $\frac{12}{10}$, $\frac{-24}{-16}$, $\frac{10}{-3}$, and $\frac{44}{-11}$ are not in standard form.

Example 1:

Convert the rational number $\frac{12}{-20}$ to standard form.

To convert it to standard form we need to make the denominator positive and bring the rational number to its simplest form.

$$\frac{12}{-20} = \frac{12 \times -1}{-20 \times -1} \text{ (multiplying numerator and denominator by } -1)$$



Explain to the students the difference between different types of decimal numbers. Also explain the process of dividing numbers when the divisor is greater than the dividend.

$$= \frac{-12}{20}$$

$$= \frac{-12 \div 4}{20 \div 4} \text{ (dividing numerator and denominator by 4)}$$

$$= \frac{-3}{5}$$

So, the standard form of $\frac{12}{-20}$ is $-\frac{3}{5}$.

2.1.2 Rules to find whether a given Rational Number is Terminating or Not

By division process we can find if the given rational numbers are terminating or non-terminating decimals. But there is a rule to check if a rational number is terminating or not without actually dividing. It states that, "If the denominator of a rational number in standard form has no prime factor other than 2, 5 or 2 and 5, then and only then the rational number is a terminating decimal".

Example 1:

Find if $\frac{28}{8}$ is a terminating decimal or not without dividing. Then verify your answer by division.

Solution:

Here, $\frac{28}{8}$ is not in standard form. First convert it to standard form.

$$\frac{28}{8} = \frac{28 \div 4}{8 \div 4} = \frac{7}{2}$$

The standard form of $\frac{28}{8}$ is $\frac{7}{2}$. The denominator 2 of $\frac{7}{2}$ has only one factor i.e. 2.

So, $\frac{28}{8}$ is a terminating decimal number.

Verification:

To verify our answer, let's divide.

$$\begin{array}{r} 3.5 \\ 8 \overline{) 28.0} \\ \underline{-24} \\ 40 \\ \underline{-40} \\ 0 \end{array}$$

As, $\frac{28}{8} = 3.5$ (a terminating decimal), so our answer is correct.



Explain to the students that by dividing rational numbers we can convert them into decimal numbers. Guide them by finding the factors of the denominator. We can guess if the rational number is terminating or not.

Example 2:

Find if $\frac{11}{10}$ is a terminating decimal or not without dividing. Then verify your answer by division.

Solution:

Here $\frac{11}{10}$ is already in standard form. $\frac{11}{10}$ has the denominator 10 whose factors are 2 and 5. So, it is a terminating decimal number.

Verification:

To verify our answer, let's divide.

$$\begin{array}{r} 1.1 \\ 10 \overline{) 11.0} \\ \underline{-10} \\ 10 \\ \underline{-10} \\ 0 \end{array}$$

As, $\frac{11}{10} = 1.1$ (a terminating decimal), so our answer is correct.

Example 3:

Find if $\frac{70}{15}$ is a terminating decimal or not without dividing. Then verify your answer by division.

Solution:

Here $\frac{70}{15}$ is not in standard form. The standard form of $\frac{70}{15}$ is $\frac{14}{3}$. The denominator 3 of $\frac{14}{3}$ has a factor 3. So, $\frac{70}{15}$ is not a terminating decimal number.

Verification:

To verify our answer, let's divide.

$$\begin{array}{r} 4.666... \\ 15 \overline{) 70.000} \\ \underline{-60} \\ 100 \\ \underline{-90} \\ 100 \\ \underline{-90} \\ 100 \\ \underline{-90} \\ 10 \end{array}$$

As, $\frac{70}{15} = 4.666...$ (a non-terminating decimal), so our answer is correct.

Note it down

Identify the terminating rational numbers.

- a) $\frac{10}{18}$ b) $\frac{11}{15}$ c) $\frac{33}{20}$

Example 4:

Find if $\frac{6}{7}$ is a terminating decimal or not without dividing. Then verify your answer by division.

Solution:

Here, $\frac{6}{7}$ is in standard form. It has a denominator whose factors is not 2 or 5. So, $\frac{6}{7}$ is a non-terminating decimal number.

Verification:

To verify our answer, let's divide.

$$\begin{array}{r} 0.857... \\ 7 \overline{) 6.000} \\ \underline{- 0} \\ 60 \\ \underline{- 56} \\ 40 \\ \underline{- 35} \\ 50 \\ \underline{- 49} \\ 1 \end{array}$$

As $\frac{6}{7} = 0.857...$ (a non-terminating decimal), so our answer is correct.

Note it down

Identify the non-terminating rational numbers.

- a) $\frac{3}{7}$ b) $\frac{7}{5}$ c) $\frac{65}{55}$

Exercise 2.1

1 Identify terminating and non-terminating decimal numbers.

- a) 0.45 b) $2.\overline{6}$ c) 0.12 d) $3.\overline{67}$ e) $8.\overline{3}$ f) 0.26
g) 1.2747 h) $2.\overline{63}$ i) $0.\overline{123}$ j) 3.12 k) 9.62 l) $8.\overline{71}$

2 Identify recurring and non-recurring decimal numbers.

- a) 0.561345... b) 0.4324567... c) 0.123123... d) 0.1289765...
e) 0.9999999... f) 4.231... g) 0.781... h) 8.1454545...

3 Convert the following to decimals and identify which type of decimal it is.

- a) $\frac{2}{3}$ b) $\frac{15}{4}$ c) $\frac{1}{7}$ d) $\frac{11}{8}$ e) $\frac{20}{21}$ f) $\frac{13}{8}$ g) $\frac{4}{7}$ h) $\frac{23}{15}$

4 Write the following in short form.

- a) 0.353535 b) 0.1272727 c) 7.44444 d) 8.123123123
e) 0.666666 f) 3.46464646 g) 12.33333 h) 9.333333

5 Identify the rational numbers which are not in standard form and given reason. Then convert them into standard form.

- a) $\frac{-12}{16}$ b) $\frac{41}{-61}$ c) $\frac{-21}{2}$ d) $\frac{3}{4}$ e) $\frac{5}{15}$ f) $\frac{17}{4}$ g) $\frac{44}{-88}$ h) $\frac{-75}{135}$

6 Find if the following are terminating decimals or not without dividing. Then verify your answer by division.

- a) $\frac{7}{9}$ b) $\frac{8}{13}$ c) $\frac{7}{-6}$ d) $\frac{10}{11}$ e) $\frac{-3}{12}$ f) $\frac{7}{5}$ g) $\frac{13}{10}$ h) $\frac{25}{15}$

2.2 Word Problems Involving Decimals and Fractions

A non-terminating decimal number that continues endlessly, with no digit or a block of digits repeating, is called a non-recurring decimal number.

Example 1:

A jug can hold $1\frac{1}{4}$ litres of water. How much water can 8 such jugs hold?

Solution:

$$1 \text{ jug holds } = 1\frac{1}{4} \text{ litres} = \frac{5}{4} \text{ litres}$$

$$8 \text{ such jugs hold } = \frac{5}{4} \times 8 = 10 \text{ litres}$$

So, 8 jugs can hold 10 litres of water.

Example 2:

A cook has $18\frac{3}{4}$ kg of minced beef. How many $\frac{1}{4}$ kg packets of beef can be made out of it?

Solution:

$$\text{Total weight of beef} = 18\frac{3}{4} \text{ kg}$$

$$\text{Packets of beef in } \frac{1}{4} \text{ kg} = 18\frac{3}{4} \div \frac{1}{4}$$

$$= \frac{75}{4} \times \frac{4}{1}$$

$$= \frac{300}{4}$$

$$= 75 \text{ packets}$$

Thus, 75 packets can be made out of this.



Ask the students to work in groups and create real-world word problems involving decimals and fractions. Then exchange their problems with other groups to solve.

Example 3:

One kg of dates costs Rs 443.75. Find the cost of 17 kg of dates.

Solution:

$$\begin{aligned}\text{Cost of 1 kg of dates} &= \text{Rs } 443.75 \\ \text{Cost of 17 kg of dates} &= \text{Rs } 443.75 \times 17 \\ &= \text{Rs } 7543.75\end{aligned}$$

Thus, 17 kg of dates cost Rs 7543.75.

Example 4:

For iftar, Sania prepared 6.4 litres of orange juice. If she poured it into 8 jugs equally, how much juice is poured in each jug?

Solution:

$$\begin{aligned}\text{Total litres of orange juice} &= 6.4 \text{ litres} \\ \text{Number of jugs} &= 8 \\ \text{Each jug can hold} &= 6.4 \div 8 \\ &= \frac{64}{10} \div 8 \\ &= \frac{64}{10} \times \frac{1}{8} \\ &= \frac{8}{10} \\ &= \frac{4}{5} \\ &= 0.8 \text{ litre}\end{aligned}$$

So, 0.8 litre of juice is poured in each jug.

Exercise 2.2

- 1 Saafi bought $3\frac{1}{3}$ litres of mango juice and $2\frac{1}{2}$ litres of orange juice. How much more mango juice did he buy than orange juice?
- 2 Adeela bought 6 packets of dates for Ramadan. If the mass of one packet is $3\frac{1}{2}$ kg, find the total mass of dates she bought.

NO FOR SALE

- 3 Mahad ran 3 rounds of a jogging track in the morning and covered a total distance of $4\frac{2}{3}$ km. What is the length of the jogging track?
- 4 Adeela and her family travel to another town at a distance of 60.27 km. On their way back, they choose another route which is 12.3 km shorter than the previous one.
 - a) How much distance did they cover on their way back?
 - b) Find the total distance covered by them.
- 5 The distance of Saafi's home to the masjid is 2.2 km. If Saafi offers all his 5 prayers regularly, how much distance does he cover every day from the masjid to home and home to the masjid? Estimate the answer to check its reasonableness.
- 6 Haadia bought 2.5 kg of honey. She used it equally to make 2 different desserts. How much honey did she use in each dessert.

2.3 Rational and Irrational Numbers.

We know that Natural numbers and 0 make the set of whole numbers. The set of whole numbers along with the negative integers make the set of integers. Integers along with fractions and decimals make the set of rational numbers.



A **rational number** is a number (positive or negative) that can be represented as a fraction or quotient of two integers. In general, we express rational numbers in the form of $\frac{p}{q}$ where p and q are integers and $q \neq 0$. For example, $\frac{1}{2}$, $\frac{2}{6}$, $\frac{3}{5}$, $\frac{12}{7}$, 0.25, 12, -15, etc. all are rational numbers as they can be represented in the form of $\frac{p}{q}$. The set of rational numbers include whole numbers, positive and negative integers, decimals and fractions. We represent the set of rational numbers with letter Q .

An **irrational number** is a number that cannot be represented as a fraction or quotient of two integers i.e. Irrational numbers cannot be presented in the form $\frac{p}{q}$ where p and q are integers and $q \neq 0$. We represent the set of irrational numbers with letter Q' .

Note it down

Every natural number, whole number, and integer is a rational number with a denominator of 1.



Put various number cards on the table having rational and irrational numbers. Show the cards one by one and ask the students to tell if it is rational or irrational. Then ask them to verbally tell the difference between rational and irrational numbers.

NO FOR SALE

For example, π (pi) is an irrational number as $\pi = 3.141592...$ as it is non-terminating and non-recurring. Similarly, $\sqrt{2}$, $\sqrt{5}$, $\sqrt{11}$, $4\sqrt{3}$, $-\sqrt{7}$ etc., are irrational numbers as they cannot be represented in the form $\frac{p}{q}$.

Number	Type and Explanation
$\sqrt{2}$	Irrational: 2 is not a perfect square.
$\sqrt{9}$	Rational: 9 is a perfect square, $\sqrt{9} = 3$.
0.01010101...	Rational: repeating decimal, it has a pattern
0.01001000100001...	Irrational: non-repeating, non-terminating decimal

If a decimal number has endless number of digits with no repeating pattern after the decimal, it is an irrational number, e.g. 2.151155515555111..., 1.10101100111101.... Also, the square root of any prime, the sum of a Rational and an Irrational number, and the product of a rational and an irrational number are always irrational

Quick Check

Which of the following are irrational?

- i) $\frac{5}{7}$ ii) $\sqrt{3}$ iii) $\frac{2}{3}$

Exercise 2.3

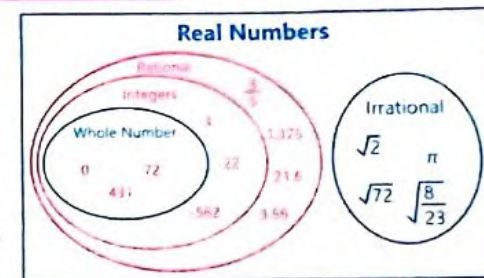
1 Which of the following are rational and which are irrational numbers? Justify your answer.

- a) 5 b) $\frac{1}{2}$ c) $-\frac{2}{7}$ d) $\frac{10}{0}$ e) $-\frac{8}{9}$ f) $\sqrt{2}$
 g) $2.\overline{25}$ h) 1.75 i) $\sqrt{11}$ j) $\sqrt{5}$ k) -23 l) $\frac{5}{2}$
 m) $\sqrt[3]{2}$ n) $\frac{1}{4}$ o) $6.\overline{33}$

2.4 Real Numbers

The sets of rational and irrational numbers together make up the set of real numbers. All these natural numbers, whole numbers, integers, decimals, fractions, irrational numbers etc. are subsets of the set of real numbers.

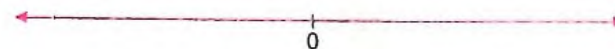
The set of real numbers is represented by R.
 $R = Q \cup Q'$



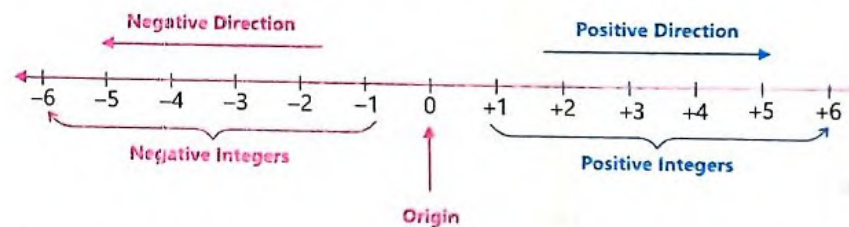
2.4.1 Number Line

We know that a number line helps us represent and display numbers by linking each number with a unique point on it.

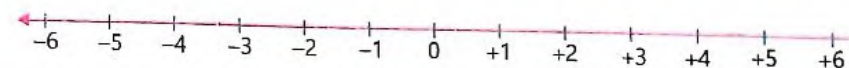
We can represent real number on numbers line by drawing a horizontal line with arrows on both ends. These arrows indicate that the number line keeps extending on both sides endlessly, as there are infinite positive and negative real numbers. As in the set of real numbers, we have positive, negative and zero, so we must choose a point to represent the number zero on it. For a real number line, this point zero acts as an origin.



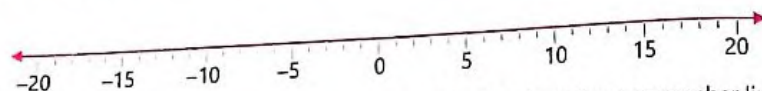
Mark the positive numbers to the right side and the negative numbers to the left of 0.



Next mark points on both sides of the origin with equal distances between them.



It is not necessary that the distance between two marks is always 1. It can be altered based upon the set of numbers we are representing on it. For example, for greater numbers, we can set the scale such that the distance between each mark on the number line is 5 units.



Sometimes when representing rational numbers and irrational numbers on number line, we need to first convert them to equivalent decimal numbers before plotting them on number line. As there can be non-terminating decimals, so many times it is not possible to mark a number on the exact point. In such cases, we use estimated value of numbers to plot on the number line.

Example 1:

Let's represent -4 , $\sqrt{5}$, $\frac{3}{4}$, $\sqrt{11}$ on number line.

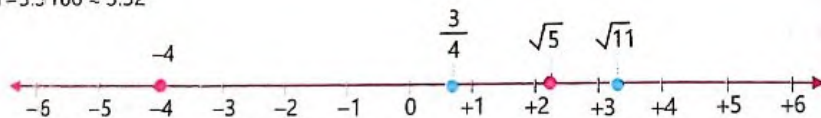
Solution:

First, we will convert $\sqrt{5}$, $\frac{3}{4}$ and $\sqrt{11}$ to decimal equivalents.

$$\sqrt{5} = 2.2360 \approx 2.24$$

$$\frac{3}{4} = 0.75$$

$$\sqrt{11} = 3.3166 \approx 3.32$$



Example 2:

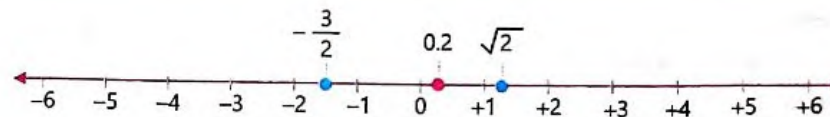
Let's represent $-\frac{3}{2}$, 0.2 and $\sqrt{2}$ on a number line.

Solution:

Here

$$-\frac{3}{2} = -1.5$$

$$\sqrt{2} = 1.4142 \approx 1.41$$



Quick Check

Represent -1.8 , 4.5 , $\sqrt{3}$, $\frac{5}{6}$, $1\frac{2}{3}$ on number line

2.4.2 Absolute value of a Real Number

The absolute value of a real number a is defined as the distance between zero (the origin) and that real number on the number line. Since it is a distance, it is always positive or zero.

The symbol $|x|$ means the absolute value of x

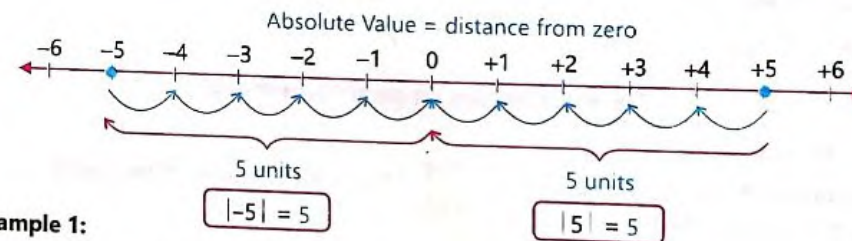
For example, the absolute value of -5 is 5 and also, absolute value of 5 is 5.

$$|-5| = 5 \text{ and } |5| = 5$$

Both 5 and -5 are "five" units away the origin 0.

Note it down

Absolute value of any numbers is simply its distance from 0 and it is always positive.



Example 1:

What is the absolute value of $-\sqrt{2}$ and $\sqrt{2}$?

Solution:

Both $-\sqrt{2}$ and $\sqrt{2}$ are same distance away from the origin on a number line. Therefore,

$$|-\sqrt{2}| = \sqrt{2}$$

$$|\sqrt{2}| = \sqrt{2}$$



Draw a number line on the board. Say a random real number and call students randomly to show it on the number line.

Example 2:

What is the absolute value of $-\frac{22}{7}$ and $\frac{22}{7}$?

Solution:

Both $-\frac{22}{7}$ and $\frac{22}{7}$ are same distance away from the origin on a number line. Therefore,

$$|-\frac{22}{7}| = \frac{22}{7}$$

$$|\frac{22}{7}| = \frac{22}{7}$$

Copyright © Awaz e Inqilab

Find is the absolute value of -1.5 and -1.5?

Exercise 2.4

1 Represent the following real numbers on a number line.

a) -1.7, 0.2, 2, -3.2, 4.

b) $\sqrt{2}$, $-\sqrt{5}$, $2/7$, $-1/2$

c) -2, $-1\frac{2}{3}$, $-1\frac{1}{3}$, -1, $-\frac{1}{3}$

d) $\frac{3}{2}$, 2^2 , 2.2, -2

e) 2, 4, 0, 1.5, $\frac{1}{2}$, $-\frac{3}{2}$

f) $1\frac{3}{7}$, 2.5, 3.5, -2.2, $-4\frac{3}{7}$, $-\frac{30}{6}$

2 Find the Absolute value of the following real numbers.

a) $\sqrt{11}$ and $-\sqrt{11}$

b) 3.18 and -3.18

c) $-\frac{3}{4}$ and $\frac{3}{4}$

d) -18.9 and 18.9

e) $\sqrt{7}$ and $-\sqrt{7}$

f) 1.44 and -1.44

2.5 Properties of real Numbers

2.5.1 Properties of inequality

Trichotomy Property of Inequality $\forall a, b \in \mathbb{R}$, one of the following can be true. Either $a = b$ or $a > b$ or $a < b$	Transitive Property of Inequality $\forall a, b \in \mathbb{R}$, i) $a > b \wedge b > c \Rightarrow a > c$ ii) $a < b \wedge b < c \Rightarrow a < c$
Addition Property of Inequality $\forall a, b, c \in \mathbb{R}$, a) i) $a > b \Rightarrow a + c > b + c$ ii) $a < b \Rightarrow a + c < b + c$ b) i) $a > b \wedge c > d \Rightarrow a + c > b + d$ ii) $a < b \wedge c < d \Rightarrow a + c < b + d$	Multiplication Property of Inequality a) $\forall a, b, c \in \mathbb{R}$ and $c > 0$, i) $a > b \Rightarrow ac > bc$ ii) $a < b \Rightarrow ac < bc$ b) $\forall a, b, c \in \mathbb{R}$, and $c < 0$. i) $a > b \Rightarrow ac < bc$ ii) $a < b \Rightarrow ac > bc$

a) Reflexive Property

This property states that $\forall a \in \mathbb{R}$: $a = a$ i.e. a number or value is equal to itself. This property states that for all real numbers.

For example,

$$3 = 3$$

$$\sqrt{2} = \sqrt{2}$$

$$-1.3 = -1.3$$

$$1.3 + \sqrt{7} = 1.3 + \sqrt{7} \text{ and so on.}$$

b) Symmetric Property

This property states that $\forall a \in \mathbb{R}$: if $a = b$ then $b = a$.

i.e. interchange the sides of an equation doesn't affect the result.

For example:

$$\text{If } a + b = c, \text{ then } c = a + b$$

$$\text{If } 2 + a = b, \text{ then } b = 2 + a$$

$$8 - x = c \text{ then } c = 8 - x \text{ and so on.}$$

c) Transitive Property

This property states that $\forall a, b, c \in \mathbb{R}$, if $a = b$ and $b = c$, then $a = c$. i.e. if a is related to b under a rule, and b is related to c under the same rule, then a is related to c under the same rule. For example,

$$3x + 5 = y \text{ and } y = x - 1, \text{ then } 3x + 5 = x - 1.$$

d) Addition Property

This property states that $\forall a, b, c \in \mathbb{R}$, if $a = b$, then $a + c = b + c$. i.e., adding the same number on both sides of equation, doesn't change the equation.

For example:

$$\text{If } x = 5, \text{ then } x + 1 = 5 + 1.$$

e) Subtraction Property

This property states that $\forall a, b, c \in \mathbb{R}$, if $a = b$, then $a - c = b - c$, i.e., subtracting the same number from both sides of equation, doesn't change the equation.

For example:

$$\text{If } x = 6, \text{ then } x - 2 = 6 - 2.$$

f) Multiplication Property

This property states that $\forall a, b, c \in \mathbb{R}$, if $a=b$, then $a \times c = b \times c$, i.e., multiplying by the same number on both sides of equation, doesn't change the equation.

For example:

$$\text{If } x = 8, \text{ then } x \times 3 = 8 \times 3.$$

g) Division Property

This property states that $\forall a, b, c \in \mathbb{R}$ and $c \neq 0$, if $a = b$, then $\frac{a}{c} = \frac{b}{c}$, i.e., dividing by the same number on both sides of equation, doesn't change the equation.

For example:

$$\text{If } x = 10, \text{ then } \frac{x}{3} = \frac{10}{3}$$

2.5.2 Properties of Addition**a) Closure Property of Addition**

This property states that $\forall a, b \in \mathbb{R}$, $a + b \in \mathbb{R}$ i.e. The sum of two real numbers is also a real number.

For Example:

$$3 + 4 = 7 \in \mathbb{R}$$

$$\frac{2}{5} + \frac{1}{5} = \frac{3}{5} \in \mathbb{R}$$

b) Commutative Property of Addition

This property states that $\forall a, b \in \mathbb{R}$, $a + b = b + a$ i.e. If we add two real numbers in any order, the sum will always be the same.

For Example:

$$\frac{1}{2} + \frac{3}{4} = \frac{3}{4} + \frac{1}{2}$$

$$\sqrt{5} + 8 = 8 + \sqrt{5}$$

$$4.5 + 3.9 = 3.9 + 4.5$$

c) Associative Property of Addition

This property states that $\forall a, b \in \mathbb{R}$, $(a + b) + c = a + (b + c)$ i.e. If we are adding three real numbers, the sum is always the same regardless of the grouping of numbers.

For Example:

$$(3 + 9) + 1 = 3 + (9 + 1)$$

$$(1.35 + 0.2) + 2.1 = 1.35 + (0.2 + 2.1)$$

$$\sqrt{17} + \frac{1}{2} + \frac{4}{5} = (\sqrt{17} + \frac{1}{2}) + \frac{4}{5}$$

NOT FOR SALE

d) Additive Identity Property

This property states that $\forall a \in \mathbb{R}$, $a + 0 = a$ i.e. If we add 0 to a real number, the sum will be the real number itself.

For Example:

$$10 + 0 = 10$$

$$-4 + 0 = -4$$

$$\frac{1}{5} + 0 = \frac{1}{5}$$

$$-3.4 + 0 = -3.4$$

e) Additive Inverse Property

This property states that $\forall a \in \mathbb{R}$, $a + (-a) = 0$ i.e. If we add a real number and its opposite, the result will always be 0 (the additive identity).

For Example:

$$10 + (-10) = 0$$

$$-4 + 4 = 0$$

$$\frac{2}{3} + (-\frac{2}{3}) = 0$$

$$\sqrt{2} + (-\sqrt{2}) = 0$$

2.5.3 Properties of Multiplication**a) Closure Property of Multiplication**

This property states that $\forall a, b \in \mathbb{R}$, $a \times b \in \mathbb{R}$ i.e. The product of two real numbers is also a real number.

For Example:

$$3 \times 4 = 12 \in \mathbb{R}$$

$$\frac{2}{5} \times \frac{1}{5} = \frac{2}{25} \in \mathbb{R}$$

b) Commutative Property of Multiplication

This property states that $\forall a, b \in \mathbb{R}$, $a \times b = b \times a$ i.e. If we multiply two real numbers in any order, the product will always be the same.

For Example:

$$\frac{1}{2} \times \frac{3}{4} = \frac{3}{4} \times \frac{1}{2}$$

$$\sqrt{5} \times 8 = 8 \times \sqrt{5}$$

$$4.5 \times 3.9 = 3.9 \times 4.5$$

NOT FOR SALE

c) Associative Property of Multiplication

This property states that $\forall a, b \in \mathbb{R}, (a \times b) \times c = a \times (b \times c)$ i.e. If we are multiplying three real numbers, the product is always the same regardless of the grouping of numbers.

For Example:

$$(3 \times 9) \times 1 = 3 \times (9 \times 1)$$

$$(1.35 \times 0.2) \times 2.1 = 1.35 \times (0.2 \times 2.1)$$

$$\sqrt{17} \times \left(\frac{1}{2} \times \frac{4}{5}\right) = (\sqrt{17} \times \frac{1}{2}) \times \frac{4}{5}$$

d) Multiplicative Identity Property

This property states that $\forall a \in \mathbb{R}, a \times 1 = a$ i.e. If we multiply 1 by a real number, the product will be the real number itself.

For Example:

$$10 \times 1 = 10$$

$$-4 \times 1 = -4$$

$$\frac{1}{5} \times 1 = \frac{1}{5}$$

$$-3.4 \times 1 = -3.4$$

e) Multiplicative Inverse Property

This property states that $\forall a \in \mathbb{R}, a \times \frac{1}{a} = 1$ i.e. If we multiply a real number by its inverse or reciprocal, the result will always be 1 (the multiplicative identity).

For Example:

$$10 \times \left(\frac{1}{10}\right) = 1$$

$$-4 \times \frac{1}{-4} = 1$$

$$\frac{2}{3} \times \left(\frac{3}{2}\right) = 1$$

$$\sqrt{2} \times \left(\frac{1}{\sqrt{2}}\right) = 1$$

f) Distributive Property of Multiplication over Addition

This property states that $\forall a, b, c \in \mathbb{R}, a(b + c) = ab + ac$ or $(a+b)c = ac + bc$.

For example:

$$2 \times (3+1) = 2 \times 3 + 2 \times 1$$

$$(4+5) \times 2 = 4 \times 2 + 5 \times 2$$

Exercise 2.5

1 Identify and the properties used in the following.

a) $17 + \sqrt{7} = \sqrt{7} + 17$

b) $12 \times 3.5 = 3.5 \times 12$

c) $7 \times \frac{1}{7} = 1$

d) $\frac{1}{3} + 0 = \frac{1}{3}$

e) $\frac{8}{11} \times 0 = 0$

f) $-y + 4 = 4 + (-y)$

g) $8.9 \times 1 = 8.9$

h) $\sqrt{13} \times 0 = 0$

i) $-\frac{1}{4} \times -4 = 1$

j) $(8+4) + 5 = 8 + (4+5)$

k) $(\sqrt{2} \times 2) \times 5 = \sqrt{2} \times (2 \times 5)$

2 Find the additive inverse of each number.

a) 9

b) -3.4

c) $\frac{3}{9}$

d) $\frac{1}{\sqrt{5}}$

e) $\frac{\sqrt{2}}{\sqrt{5}}$

f) -6

g) $\frac{-4}{9}$

h) 0

i) 1

3 Find the multiplicative inverse of each number.

a) -10

b) $-\frac{1}{2}$

c) 8.1

d) $\frac{\sqrt{5}}{4}$

e) $\sqrt{11}$

f) 14

g) $-\frac{9}{10}$

h) 0

i) 1

4 State and prove the following properties for any real numbers of your own choice:

a) The transitive property

b) The reflexive property

c) The symmetric property

d) Addition property

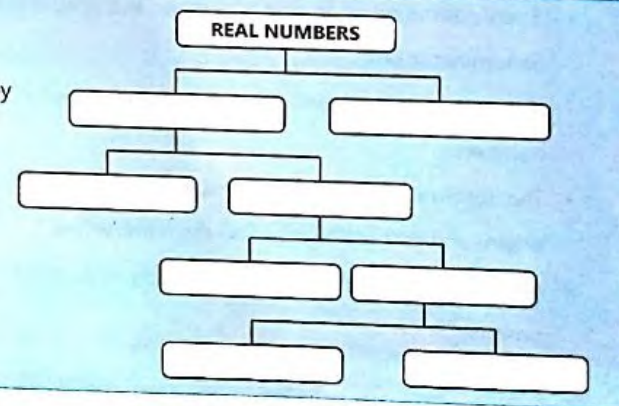
e) Subtraction property

f) Multiplication property

g) Division property

Think Higher

Complete the flow chart by filling in the boxes.



Summary

- The terminating decimal numbers are those decimal numbers that have a finite number of digits after the decimal point.
- Non-terminating decimal numbers are those decimal numbers that have an infinite number of digits after the decimal point.
- If a decimal number that has a single digit or a block of digits that repeats itself an infinite number of times after the decimal point is called a repeating or recurring decimal number.
- A non-terminating decimal number that continues endlessly, with no digit or a block of digits repeating, is called a non-recurring decimal number.
- A rational number is a number (positive or negative) that can be represented as a fraction or quotient of two integers. In general, we express rational numbers in the form of $\frac{p}{q}$ where p and q are integers and $q \neq 0$.
- An irrational number is a number that cannot be represented as a fraction or quotient of two integers i.e. Irrational numbers cannot be presented in the form $\frac{p}{q}$ where p and q are integers and $q \neq 0$.
- Every natural number, whole number, and integer is a rational number with a denominator of 1.
- The sets of rational and irrational numbers together make up the set of real numbers.
- The absolute value of a real number a is defined as the distance between zero (the origin) and that real number on the number line.
- Absolute value of any numbers is simply its distance from 0 and it is always positive.

Vocabulary

- Terminating Decimals
- Recurring decimals
- Repeating Decimals
- Rational Numbers
- Irrational Numbers
- Real Numbers
- Absolute Value

Review Exercise

1 Encircle the correct option.

- Terminating decimal numbers are those decimal numbers that have a _____ number of digits after the decimal point.
 - infinite
 - finite
 - even
 - Odd
- Decimal numbers that have an infinite number of digits after the decimal point.
 - non-terminating
 - terminating
 - like
 - unlike
- A decimal number that has a single digit or a block of digits repeat itself an infinite number of times after the decimal point is called a/an _____ decimal number.
 - Terminating
 - unlike
 - recurring
 - non-recurring
- Which of these is an irrational number?
 - 2.5
 - $\frac{1}{2}$
 - 0.23232323....
 - 1.01001110110111....
- $Q+Q'$ makes
 - The set off natural numbers
 - The set of real numbers
 - The set of rational numbers
 - The set of Integers
- The absolute value of -3 is:
 - 1
 - 3
 - 3
 - 6
- Reflexive property of real numbers states that $\forall a \in \mathbb{R}$:
 - $a+b=b+a$
 - $a=-a$
 - $a=a$
 - $a+3=a$
- Symmetric property is shown by:
 - If $a+2=10$, then $-10=a-2$
 - If $a+2=10$, then $10=a+2$
 - If $a+2=10$, then $10=2-a$
 - If $a+2=10$, then $-10=-a+2$

2 Define the following with examples.

- Terminating Decimals
- Non-Terminating Decimals
- Recurring Decimals
- Non-Recurring Decimals
- Standard form of rational numbers
- Rational Numbers
- Irrational Numbers
- Real Numbers
- Absolute value of a Real Numbers

3 Identify terminating and non-terminating decimal numbers.

- a) 1.23 b) 6.7777... c) 0.87676... d) 2.3456
e) 7.7 f) 7.89765... g) 0.25 h) 16.51...

4 Identify recurring and non-recurring decimal numbers

- a) 0.1111... b) 0.5555... c) 0.454545... d) 3.141645
e) 9.9999... f) 3.434343... g) 0.4535353... h) 7.6751053...

5 Which of the following are rational and which are irrational numbers? Justify your answer.

- $-3\frac{1}{2}$ $\sqrt{12}$ 5.7 0 -8π 9 $\sqrt{25}$ $\frac{\pi}{2}$
 $\frac{12}{5}$ $\frac{4}{7}$ $\sqrt{9}$ $\frac{15}{38}$ 5.2 $\sqrt{17}$ $-\sqrt{16}$

6 Represent the following real numbers on a number line.

- a) 2.4, -1.9, 3, $\frac{2}{3}$
b) $\sqrt{11}$, $-\sqrt{3}$, 3.3, $\frac{22}{7}$, $-\frac{1}{5}$
c) 6, 0, 0.5, $\frac{3}{4}$, $\frac{1}{2}$

7 Find the absolute value of the following real numbers.

- a) $\sqrt{17}$ and $-\sqrt{17}$
b) -0.06 and 0.06
c) $\frac{4}{5}$ and $\frac{4}{5}$

8 Identify and write the properties used in the following.

- a) $12 + \sqrt{2} = \sqrt{2} + 12$ b) $3 \times 5 = 5 \times 3$ c) $-2 \times -\frac{1}{2} = 1$
d) $2.5 \times 0 = 0$ e) $(1+6) + 2 = 1 + (6+2)$

9 Find the additive and multiplicative inverse of each number.

- a) 10 b) 2.5 c) $\frac{1}{9}$ d) $\frac{1}{\sqrt{2}}$
e) $\frac{\sqrt{2}}{3}$ f) -10 g) 0 h) 1

10 State and prove the following properties for any real numbers of your own choice:

- a) The transitive property b) The reflexive property
c) The symmetric property d) Addition property
e) Subtraction property f) Multiplication property
g) Division property

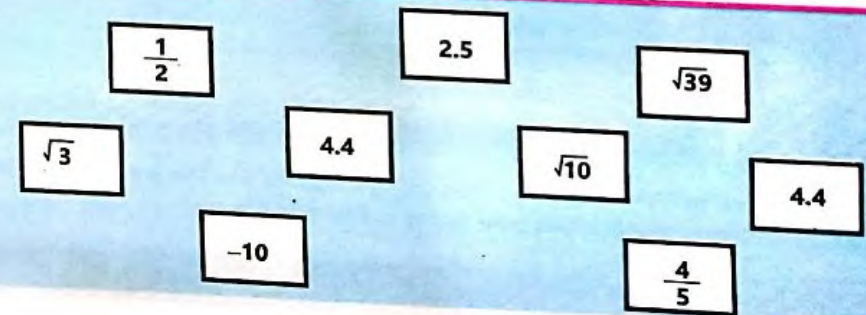
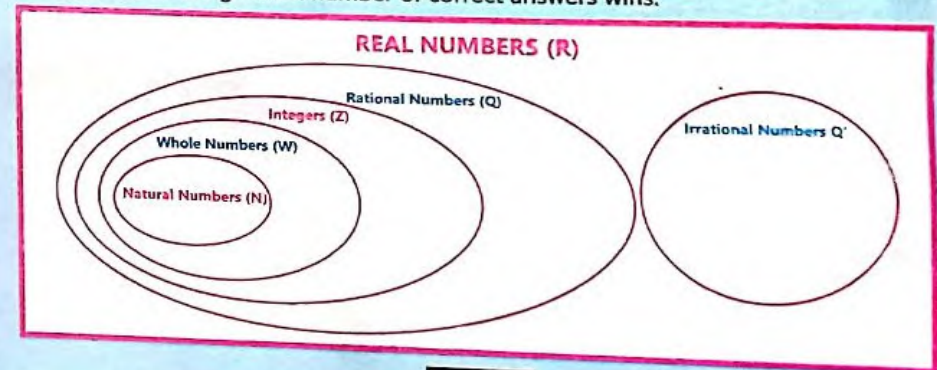
Math Project

Material Required:

- Real number charts for each pair (blank)
- Glu stick
- Real numbers cards (Multiple cards for each pair)
- Scoring sheet

Procedure:

- Work in pairs.
- Each member will get a real number Venn diagram sheet and a set of cards having variety of real numbers.
- Each pair will paste the correct real number in the correct circle of the Venn diagram.
- At the end, each pair will present their work and justify why have they chosen each number for each category.
- The correct and wrong answers will be recorded in the scoring sheet.
- The pair with the greatest number of correct answers wins.



Unit 3

Financial Arithmetic

Student Learning Outcomes

- Calculate direct and inverse and compound proportion and solve real-world word problems related to direct, inverse and compound proportion. (Using table, equation and graph)
- Explain and calculate profit percentage, loss percentage and discount.
- Convert Pakistani currency to well-known international currencies and vice versa.
- Explain and calculate profit/markup, principal amount and markup rate.
- Explain insurance, partnership and inheritance.
- Solve real world word problems involving profit %, loss %, discount, profit, markup, insurance, partnership and inheritance.



Ifthikhar ordered a wrist watch from an online shopping site. The price of watch is 28.50 USD. How much he will have to pay in Pakistani Rupees?

Introduction

In the previous class we have learned about profit, loss, discount and types of different taxes. Now we will learn about direct and inverse proportion, compound proportion using table, equation and graph, markup, markup rate, insurance, inheritance and partnership. We will also learn about conversion of Pakistani currency to well-known international currencies.

3.1 Propotion

Equality of two ratios is called proportion. We use the double colon :: to show equality of ratios.

For any four quantities or numbers a, b, c and d, if the ratio of $a : b = c : d$, then a : b is proportional to c : d.

$$a : b :: c : d$$

There are four terms in a proportion. a is the first term, b is the second term, c is the third term and d is the fourth term.

The first and the fourth terms of a proportion are called extremes and the second and the third terms are called the means.

3.1.1 Direct and Inverse Proportion

Two quantities will be called directly proportional to each other when:

- With the increase of one quantity, the other quantity also increases.
- With the decrease of one quantity, the other quantity also decreases.

Two quantities will be called inversely proportional to each other when:

- With the increase of one quantity, the other quantity decreases.
- With the decrease of one quantity, the other quantity increases.

Example 1:

A worker earns Rs 43000 in 25 days. How much does he earn in 4 days? (Consider the same amount of earning each day.)

Solution:

We can solve this problem using two methods.

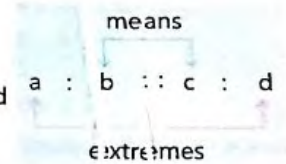
Unitary Method

$$\text{Earnings in 25 days} = \text{Rs } 43000$$

$$\text{Earnings in 1 day} = \text{Rs } \frac{43000}{25}$$



Explain to the students the term proportion, means and extremes with examples.



Note it down

- If the ratio $a : b$ is the same as the ratio of $c : d$, then:
 $a : b :: c : d$ or $a \times d = b \times c$
- If the ratio $a : b$ is the inverse of the ratio $c : d$, then:
 $a : b :: d : c$ or $a \times c = b \times d$

Note it down

Unitary method is a method in which we calculate the value of a number of things by finding the value of one thing.

Earnings in 4 days = Rs $\frac{43000 \times 4}{25}$ = Rs 6880
 So the worker will earn Rs 6880 in 4 days.

Proportion method

Here the days are directly proportional to the amount earned.

Rupees	:	Days
43000	:	25
x	:	4

$$\frac{x}{43000} = \frac{4}{25}$$

$$x \times 25 = 4 \times 43000$$

$$= \frac{4 \times 43000}{25}$$

$$x = 6880$$

Hence the worker will earn Rs 6880 in 4 days.

Example 2:

Grade VIII students are packing gifts for needy children. If they pack 128 gift boxes in 4 hours, how many gifts will they pack in 7 hours?

Solution:

As we know that time is directly proportional to the task (packing). So,

Gifts Packed	:	Time (In hours)
128	:	4
x	:	7

$$\frac{x}{128} = \frac{7}{4}$$

$$4x = 128 \times 7$$

$$x = \frac{896}{4}$$

$$x = 224$$

So, they will pack 224 gifts in 7 hours.



Explain to the students about the relationship between time, work and the number of workers by presenting different real life examples.

Quick Check

If the price of 25 books is Rs 1525, find the price of 18 books.

Quick Check

12 men water the field in 9 hours. How many men will be required to water the same field in 4 hours?

Example 3:

If 12 men complete the construction of a wall in 6 days, in how many days will 18 men construct the same wall?

Solution:

We know that number of men is inversely proportional to time. So,

Days	:	Men
6	:	12
x	:	18

$$\frac{x}{6} = \frac{12}{18}$$

$$18x = 12 \times 6$$

$$x = \frac{72}{18}$$

$$= 4$$

Hence, 18 men will construct the same wall in 4 days.

Example 4:

In a library, 11 persons arranged 319 books. How many persons are required to arrange 725 books in the same time duration?

Solution:

Here the number of persons is directly proportional to the number of books arranged.

Persons	:	Books arranged
11	:	319
x	:	725

$$\frac{x}{11} = \frac{725}{319}$$

$$x = \frac{725}{319} \times 11$$

$$= 25$$

So, 25 persons are required to arrange 725 books.

3.1.2 Graphical Representation of Direct Proportion

We can represent proportions through graphs. If the two quantities x and y are directly proportional to each other i.e. $y \propto x$, then:

$$y = kx \text{ (where k is a constant)}$$

Let's learn to graph proportions by the following examples.

Example 1:

Madiha is reading a story book. Daily she reads 2 pages of the book. Show this information through graph.

Solution:

Here, the number of pages is directly proportional to the number of days as the pages are increasing with number of days.

Note it down

Two quantities that are in direct proportion will always produce a straight-line graph that passes through the origin.

Note it down

We use symbol " \propto " for 'is proportional to'.

Let the number of days = x
The number of pages = y

Here $y \propto x$

So, we can write this relation as

$$y = kx$$

$y = 2x$ (where 2 is the constant of proportionality)

We will find a few pairs of numbers for x and y which satisfy the equation $y=2x$.

$$\text{Put } x = 0 \text{ in } y = 2x;$$

$$y = 2(0) = 0$$

$$\text{Put } x = 1 \text{ in } y = 2x;$$

$$y = 2(1) = 2$$

$$\text{Put } x = 2 \text{ in } y = 2x;$$

$$y = 2(2) = 4$$

$$\text{Put } x = 3 \text{ in } y = 2x;$$

$$y = 2(3) = 6$$

$$\text{Put } x = 4 \text{ in } y = 2x;$$

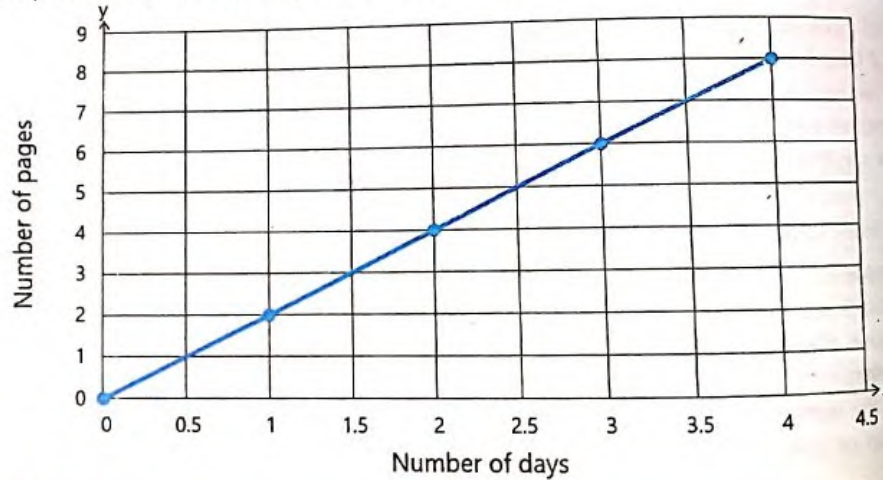
$$y = 2(4) = 8$$

and so on.

So, by showing this values on table, we get:

x (No. of days)	0	1	2	3	4
y (No. of pages)	0	2	4	6	8

If we plot these points on the graphs we get:



We can see that the graph is a straight line.

y increases as x increases shows direct proportion.

y decreases as x increases shows inverse proportion.

3.1.3 Graphical Representation of Inverse Proportion

If the two quantities x and y are inversely proportional to each other i.e. $y \propto \frac{1}{x}$, then:

$$y = \frac{k}{x} \text{ (where } k \text{ is a constant)}$$

Let's learn to proportion graph by the following examples.

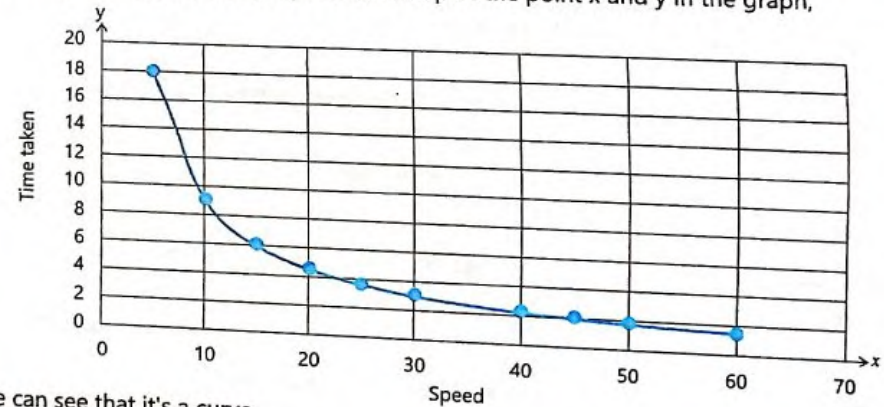
Example 1:

The following table shows the relationship between speed of vehicle and the time taken to cover a distance of 90 km. Plot this on graph and identify the proportion.

Solution:

Speed(km/hr) x	5	10	15	20	25	30	40	45	50	60
Time taken(hr) y	18	9	6	4.5	3.6	3	2.25	2	1.8	1.5

We can see that as the speed increased, the time taken to cover the distance decreased. So, this is representing inverse proportion. Let's plot the point x and y in the graph,



We can see that it's a curve.

Note it down

If the graph goes up as x increases (left to right) then the relationship is **direct**. If the graph goes down as x increases then the relationship is **inverse**.

Note it down

Two quantities that are in inverse proportion will always produce a curved graph.

Exercise 3.1

1 Find the value of y in the following.

a) $\frac{y}{32} = \frac{3}{8}$

b) $\frac{2}{y} = \frac{4}{6}$

c) $\frac{5}{15} = \frac{y}{30}$

d) $\frac{5}{20} = \frac{4}{y}$

e) $\frac{y}{24} = \frac{8}{16}$

f) $\frac{3}{21} = \frac{7}{y}$

g) $125 : 15 :: y : 60$

h) $70 : 14 :: 40 : y$

i) $y : 32 :: 8 : 16$

j) $25 : y :: 100 : 200$

2 In $21 : 28$, if we change the first term of the ratio to 70, what will be the second term?

3 What is the missing term in the proportional $8 : 11 :: 16 : x$?

4 5 men complete a task in 2 days. How many days are required to complete the same task by 1 man?

5 Ijaz reads 20 pages of the Holy Quran in 55 minutes. How many pages will he read in 120 minutes?

6 9 footballs cost Rs 3500. How much will 4 footballs cost?

7 The cost of 145 kg of rice is Rs 6743. How many kg of rice can be purchased for Rs 9876?

8 If 5 boxes occupy a space of 750 cm^3 , how much space will be required for 180 such boxes?

9 If 8 workers can do a job in 16 days, how many workers will it take to do the job in 32 days?

10 A man cleans 1450 clothes in 90 minutes. How many clothes will he wash in 18 more minutes?

11 Rabia irons 7 scarves in 12 minutes. How long will she take to iron 45 scarves?

12 14 tube wells can fill a water tank in 45 minutes. If 4 tube wells are out of order, how long will the remaining tube wells take to fill the tank?

13 Identify the proportion from the given table and then plot the graphs.

x	1	3	9	5
y	2	6	18	10

x	-35	20	55	-15
y	7	-4	-11	3

NOT FOR SALE

3.2 Profit

Previously we have learnt the concept of profit and loss. When a person runs a business or is involved in some activities of purchasing or selling goods or items we use the terms profit or loss.

The price of goods or items at which it is purchased by a shopkeeper or wholesaler from the manufacturer is known as cost price. We can represent cost price by CP.

The price of goods or items at which it is sold by a wholesaler or a shopkeeper to the customer is known as selling price. We represent selling price by SP.

3.2.1 Profit and Loss

If the selling price (SP) of the things or goods are more than its cost price (CP), we can say that there is a gain or a profit.

If the selling price of the things or goods is less than its cost price, we can say that there is a loss.

Therefore,

$$\text{Profit} = \text{SP} - \text{CP} \text{ (if } \text{SP} > \text{CP})$$

$$\text{Profit \%} = \frac{\text{Profit}}{\text{Cost price}} \times 100$$

$$\text{Loss} = \text{CP} - \text{SP} \text{ (if } \text{SP} < \text{CP})$$

$$\text{Loss \%} = \frac{\text{Loss}}{\text{Cost price}} \times 100$$

Note it down

Cost price and selling price tell us how much profit or loss a person gets when selling items.

Note it down

Selling price is always greater than the cost price when profit is earned.

3.2.2 Overheads

A wholesaler or a shopkeeper buys things or goods from the manufacturer for a certain amount. That price is the cost price. Then he adds extra expenses in the cost price. These extra expenses adding a surplus value to the cost of items or services are called overheads. It may include shipping charges, delivery charges, insurance, rent, taxes, labour, transportation, repairs, etc.

$$\text{SP} = \text{CP} + \text{Overheads}$$



Instruct the students to make real-life problems of profit and loss, solve them and share with their class-fellows.

NOT FOR SALE

Example 1:

A shopkeeper bought a book for Rs 560 and sold it for Rs 600. Find his profit percent.

Solution:

Cost price = Rs 560

Selling Price = Rs 600

$$\begin{aligned}\text{Profit} &= \text{SP} - \text{CP} \\ &= \text{Rs } 600 - \text{Rs } 560 \\ &= \text{Rs } 40\end{aligned}$$

$$\text{Profit \%} = \frac{\text{Profit}}{\text{Cost price}} \times 100$$

$$\begin{aligned}\therefore \text{Profit \%} &= \frac{40}{560} \times 100 \\ &= 7.14\%\end{aligned}$$

Quick Check

A shopkeeper bought a crate of cold drinks in Rs 950 and sold it for Rs 1100. What is the profit or loss percent?

Example 2:

Salman bought a deep freezer for Rs 52000. He spends Rs 950 on the transportation. Later he sold it for Rs 56250. Find his profit or loss. Also find the percentage of profit or loss.

Solution:

Cost price = Rs 52000

Overhead charges (transportation) = Rs 950

Total amount paid = Cost price + Overhead charges (transportation)
= Rs 52000 + Rs 950 = Rs 52950

Since the selling price is more than the cost price, so Salman earned a profit.

$$\begin{aligned}\text{Profit} &= \text{S.P} - \text{C.P} \\ &= 56250 - 52950 = 3300\end{aligned}$$

$$\text{Profit \%} = \frac{\text{Profit}}{\text{Cost price}} \times 100$$

$$\begin{aligned}\therefore \text{Profit \%} &= \frac{3300}{52950} \times 100 \\ &= 6.2\%\end{aligned}$$

Example 3:

A shopkeeper bought 14 packs of crayons at the rate of Rs 40 per pack. He spent Rs 70 on the transportation. He sold each pack at Rs 50. Calculate his profit or loss %.

Solution:

Cost of 14 packs = Rs 40 × 14 = Rs 560

Overhead charges (transportation) = Rs 70

Total cost = Cost price + Overhead charges

Total cost = Rs 560 + Rs 70 = Rs 630

Note it down

Faisal sells a television for Rs 52500 and he earns a profit of 8%. Find the cost price of the television.

Selling price = 14 × Rs 50 = Rs 700

Here, SP > CP, so there is a profit

Profit = SP - CP

$$= \text{Rs } 700 - \text{Rs } 630 = \text{Rs } 70$$

$$\text{Profit \%} = \frac{70}{630} \times 100 = 11.11\%$$

Quick Check

Fill in the blanks in the following:

C.P	S.P	Profit or Loss	%Profit or %Loss
Rs. 1500	Rs. 1200		
Rs. 4000	Rs. 4800		
		Profit = Rs. 112	
Rs. 2500	Rs. 672		%Loss = Rs. 16%

3.2.4 Discount

Usually, at the time of off-season and on the occasion of religious festivals like Eid, the shopkeepers announce reduction in the marked price of articles to increase their sale. Such a reduction in the marked price is called 'Discount'.

Actual selling price = Marked price - Discount

Percentage discount can be found by using the following formula,

$$\% \text{ Discount} = \frac{\text{Discount}}{\text{Marked price}} \times 100\%$$

Now we explain the use of the above formula with the help of the following examples.

Example 1:

The marked price of a shirt is Rs. 450. It is sold at 20% discount. Find the actual selling price of the shirt.

Solution:

Marked price of shirt = Rs. 450

Discount = 20% of Rs. 450

$$= \frac{20}{100} \times 450$$

Discount = Rs. 90

Actual selling price

$$= \text{Rs. } 450 - \text{Rs. } 90$$

$$= \text{Rs. } 360$$

Example 2:

A shopkeeper sold a wristwatch for Rs. 1485 after giving a discount of 10% to the customer. Find the marked price of the watch.

Solution:

Actual SP = Rs. 1485

Suppose marked price of the watch = Rs. x

10% discount means that if marked price is Rs. 100, then discount is Rs. 10 and therefore actual SP = Rs. 100 – Rs. 10 = Rs. 90

If actual SP is Rs. 90, then marked price = Rs. 100

If actual SP is Re. 1, then marked price = $\frac{100}{90}$

If actual SP is Rs. 1485, then marked price = $\frac{100}{90} \times 1485 = \text{Rs. } 1650$

Example:

The marked price of a coat is Rs. 2000. The shopkeeper offers his customer two successive discounts of 10% and 5%. Find the final selling price of the coat.

Solution:

Marked price of the coat = Rs. 2000

1st Discount = 10% of Rs. 2000

$$= \frac{10}{100} \times 2000$$

= Rs. 200

Selling price after 1st discount = Rs. 2000 – Rs. 200 = Rs. 1800

2nd discount = 5% of Rs. 1800

$$= \frac{5}{100} \times 1800$$

= Rs. 90

Final selling price = Rs. 1800 – Rs. 90 = Rs. 1710

Quick Check

The marked price of a suitcase is Rs. 1750. The shopkeeper offers a discount of 4%, but still earns a profit of 5%. Find the cost price of the suitcase.

Exercise 3.2

1 Find percentage profit or percentage loss if:

- | | |
|---------------------------|---------------------------|
| a) Cost price = Rs. 3900 | Selling price = Rs. 3800 |
| b) Cost price = Rs. 14500 | Selling price = Rs. 14000 |
| c) Cost price = Rs. 67800 | Selling price = Rs. 68900 |
| d) Cost price = Rs. 1500 | Selling price = Rs. 2550 |

- 2 Jamil purchased a house for Rs. 9250000. After sometime, he sold it at 8% profit. Find the selling price of the house.
- 3 An electric fan is sold for Rs. 19516 after giving a discount of 14%. Find the marked price of the fan.
- 4 The marked price of a photo-copier machine is Rs. 72000. The machine was sold at 5% discount. The shopkeeper still earned a profit of 14%. Find its cost price.
- 5 The marked price of a mobile phone is Rs. 24500. The shopkeeper offers his customer two successive discounts of 10% and 6%. Find the final S.P of the mobile phone.
- 6 The factory price of a T.V set is Rs. 42000. A wholesaler buys 20 TV sets at a discount of 5% and sells them at 10% profit on each TV set. Find the SP of each TV set.
- 7 Zain sold a can of oil for Rs 7780 at a loss of 15%. Calculate his cost price.
- 8 A merchant bought some goods for Rs 650,000. He spent Rs 12,000 on overhead charges including labor and transportation. He sold the goods for Rs 770,000. Find his profit or loss percentage.
- 9 Mr Qadeer sold a car for Rs 8,40,000 and suffered a loss of 18%. How much should he have sold it for to achieve a profit of 10.5%?
- 10 A shopkeeper sold a table for Rs 8400, he loses 6% of its cost price. Find the cost price of the table.
- 11 A buyer buys rice at Rs 120 per kg. What price per kg must he sell the rice to gain 20%?
- 12 A man gets a profit of Rs 370 by selling a cycle at Rs 10,500. Find its cost price and profit percentage.
- 13 A watch is bought for Rs 16780 and sold for Rs 17300. Find the profit and profit percentage.
- 14 A shopkeeper sold a television and makes a profit of 8%. The buyer sold it for Rs 52500 and suffered a loss of 10%. What was the original price of the television?

3.3 Markup

When someone borrows money from a bank, the borrower has to pay an extra amount when repaying the borrowed amount. Similarly, when services or goods are sold to customers by the seller, he must charge a price greater than the actual cost of goods or services in order to earn a profit. In simple words the price at which the merchant or seller buys a product/service

Note it down

An amount that is added to the cost price to calculate the sale price is called markup.

is the cost price, while the amount in which they increase the price by is called the markup or simple interest. This amount depends upon the amount borrowed and the time period for which it was borrowed.

The money borrowed by a person is known as the principal, and the additional amount which is repaid by the borrower to the lender is known as the markup or simple interest. Suppose "P" is the principal, "T" is the time period and "R" is the markup rate, then the Formula for calculating the amount of markup/simple interest will be:

$$\text{Markup/simple interest} = \frac{\text{Principal} \times \text{Rate of markup} \times \text{Time duration}}{100}$$

$$\text{Markup} = \frac{P \times R \times T}{100}$$

If A denotes the amount to be paid in the future, then

A = Principal Amount + Markup

A = P + Markup

Markup = A - P

3.3.1 Installments

Sometimes a person cannot buy an article or a property on cash payment at the time of purchase. For example, refrigerator, television, motorbike, car, plot, land, etc. To purchase these types of goods or things a scheme was introduced, known as installment scheme. In an installment scheme, the customer is not required to pay the full payment of the articles at the time of purchase, but is allowed to pay a part of it at that time and the rest of the money can be paid in easy installments. The installment must be paid with an additional amount as the purchasing amount. This amount is the **markup**.

To calculate the markup, we subtract the cash price from the total payment.

Markup = Total payment - Cash price

Note it down

A person who borrows the money is known as the debtor and the person who lends the money is called the creditor.

Note it down

Installments can be paid monthly, quarterly, half yearly or even yearly.

Note it down

The full payment that a customer has to pay to purchase an article is called the cash price.

Note it down

The amount a customer has to pay at the beginning of an installment plan is called the down payment.



Explain to the students how to find markup, rate, principal and time by using the formula. Write some examples on the board and ask them to solve them and share their results with their class-fellows.

NOT FOR SALE

Example 1:

Find the markup/interest on Rs 80000 for 5 years at the rate of 7% per annum.

Solution:

Principal = Rs 80000

Rate = 7%

Time = 5 years

Markup = ?

$$\begin{aligned} \text{Markup} &= \frac{P \times R \times T}{100} \\ &= \frac{80000 \times 7 \times 5}{100} = \text{Rs } 28000 \end{aligned}$$

Example 2:

The cash price of an air conditioner is Rs 65000. It is sold for 20% down payment and 10 equal installments of Rs 6000 each.

a) Find the cost of the air conditioner.

b) Find the difference between the cash price and the total amount paid through installments.

Solution:

Cash price of the air conditioner = Rs 65000

Down payment = 20% of 65000

$$= \frac{20}{100} \times 65000 = \text{Rs } 13000$$

Number of equal installments = 10

Amount of each installment = 6000

Total installments payment = $10 \times 6000 = \text{Rs } 60000$

Total payment a customer will have to pay = down payment + total amount of installments

Total payment a customer will have to pay = Rs 13000 + Rs 60000 = Rs 73000

Hence the cost of the air conditioner bought on installments is Rs 73000

Difference between the total payment and cash price

$$= \text{Rs } 73,000 - \text{Rs } 65,000 = \text{Rs } 8000$$

Example 3:

Calculate the principal amount, if Markup = Rs 4500, R = 5% and T = 3 years.

Markup = Rs 4500

Rate = 5%

Time = 3 years

Principal = ?

Example 4:

In what time will Rs 600 amount to Rs 840 at 10% per annum mark up ? :

P = Rs 600

Rate = 10%

Markup = A - P ($\because A = P + \text{Markup}$)

$$= \text{Rs } 840 - \text{Rs } 600 = \text{Rs } 240$$

Time = ?

NOT FOR SALE

$$\text{Markup} = \frac{P \times R \times T}{100}$$

$$4500 = \frac{P \times 5 \times 3}{100}$$

$$4500 = \frac{P \times 15}{100} = \frac{4500 \times 100}{15} = P$$

$$= \text{Rs } 30000 = P$$

$$\text{Markup} = \frac{P \times R \times T}{100}$$

$$240 = \frac{600 \times 10 \times T}{100}$$

$$\frac{240 \times 100}{6000} = T = 4 \text{ years} = T$$

So, the required time is 4 years.

Example 5:

Haris bought a laptop having a cash price of Rs 10000 on installments at the markup rate of 7% per annum. Find the price Haris paid if the time period is 2 years.

Solution:

Cash price (P) = Rs 10000

Markup rate = 7% per annum

Time period (T) = 2 years

Amount paid = ?

Using the formula,

$$\text{Markup} = \frac{10000 \times 7 \times 2}{100}$$

$$\text{Markup} = \frac{140000}{100} = \text{Rs } 1400$$

Amount paid = cash price + markup

$$= \text{Rs } 10000 + \text{Rs } 1400 = \text{Rs } 11400$$

Thus, Haris paid Rs 11,400 for the laptop.

Quick Check

Find the missing quantity.

a) If P = Rs 900, Markup = Rs 6000,
R = 3% and T = ?

b) If P = Rs 500, T = 3 years,
R = 12% and Markup = ?

Exercise 3.3

1 Find the missing quantities by using the formula.

- If P = Rs 400, T = 2 years, R = 8%, Markup = ?
- If Markup = Rs 1900, T = 3 years, R = 4%, P = ?
- If P = Rs 4000, R = 10%, Markup = Rs 12500, T = ?
- If P = Rs 700, T = 3.5 years, R = 12%, Markup = ?

2 The cash price of a washing machine is Rs 19500. Iqra buys it on installments by paying Rs 4500 as down payment and the rest by 9 monthly installments of Rs 2000 each. If she buys it using cash payment, how much money will she save?

3 A microwave oven is available for Rs 17000 in cash or Rs 2000 down payment followed by 7 equal monthly installments of Rs 2500 each. Compute the total payment.

NOT FOR SALE

- Mr Amir paid a markup of Rs 520 on an item that costs Rs 5800. If the time period is 2 years, calculate the rate of the markup.
- Find the markup on a principal amount of Rs 95,400 at the rate of 13% per annum for 5 years.
- The price of a mobile including markup is Rs 15610. If the markup rate is 7% per annum, find the amount of mark up for 2 years.
- Qasir bought a motorbike for Rs 125,000 and paid a markup of Rs 25,000 for 2 years. What markup rate did he pay?

3.4 Compound Proportion

The relationship between two or more proportions is known as compound proportion. In compound proportion two or more quantities are involved. Let "x" be the quantity required.

Quantity 1	Quantity 2	Quantity 3
a	b	c
d	e	x

If quantity 1 and quantity 3 are directly proportion and quantity 2 and quantity 3 are also directly proportion, then we will use this rule:

$$\frac{x}{c} = \frac{d}{a} \times \frac{e}{b}$$

Now if quantity 1 and quantity 3 are directly proportion, but quantity 2 and quantity 3 are inversely proportion then we use this rule:

$$\frac{x}{c} = \frac{d}{a} \times \frac{b}{e}$$

Once again if quantity 2 and quantity 3 are directly proportional but quantity 1 and quantity 3 are inversely proportion, then we use this rule:

$$\frac{c}{x} = \frac{d}{a} \times \frac{e}{b}$$

3.4.1 Partnership and Inheritance

Partnership is a business in which two or more than two individuals called the partners, establish and manage a business. All partners share the profit or loss of the business according to their shares.

Inheritance is the act of giving any property or share upon the death of an individual.

In our daily life we have to deal with numerous problems involving proportion, partnership and inheritance. The method of solving such question and problems is illustrated in the following examples.

NOT FOR SALE

Example 1:

If 30 persons can consume 20 Kg of flour in 25 days, how much of flour will 15 persons consume in 30 days?

Solution:

Let the flour consumed = x kg

Persons	Days	Flour (kg)
30	25	20
15	30	x

$$\frac{x}{20} = \frac{30}{25} \times \frac{15}{30}$$

$$x = 20 \times \frac{3}{5}$$

$$\Rightarrow x = 12 \text{ kg}$$

So, 15 persons will consume 12 kg flour in 30 days.

Example 2:

Zubair and Mumtaz started a business by investing Rs. 80,000 and Rs.60,000 respectively. At the end of the year they earned a profit of Rs.42,000. Find the share of Zubair and Mumtaz in the profit.

Solution:

To find their shares, we first have find the ratio of their investments.

Zubair's Share	Mumtaz's Share
80,000	60,000
80	60
4	3

Sum of the ratios = $4 + 3 = 7$

Total profit = 42,000

$$\text{Zubair's Share} = \frac{4}{7} \times 42000 = \text{Rs. } 24,000$$

$$\text{Mumtaz's Share} = \frac{3}{7} \times 42000 = \text{Rs. } 18,000$$

Example 3:

Imran passed away leaving a property of value Rs. 20,000,000. This amount is to be shared among his heirs including a widow, one daughter and one son, in such a way that the widow gets $\frac{1}{8}$ of the property and the daughter gets half of son's share.

Solution:

Total value of property = Rs 20,000,000

$$\text{Widow's share} = \frac{1}{8} \text{ of } 20,000,000 = \frac{1}{8} \times 20,000,000 = \text{Rs } 2,500,000$$

$$\text{Remaining value of property} = \text{Rs } 20,000,000 - 2,500,000 = \text{Rs } 17,500,000$$

According to the condition;

Daughter's share : Son's Share

$$1 : 2$$

Sum of ratios = $1 + 2 = 3$

$$\text{Daughter's share} = \frac{1}{3} \text{ of Rs. } 17,500,000 = \frac{1}{3} \times \text{Rs. } 17,500,000 = \text{Rs } 5,833,333 \text{ (approx).}$$

$$\text{Son's share} = \frac{2}{3} \text{ of Rs } 17,500,000$$

$$= \frac{2}{3} \times \text{Rs } 17,500,000 = \frac{2}{3} \times 17,500,000 = \text{Rs } 11,666,666 \text{ (approx)}$$

Exercise 3.4

- 1 For a family of 4 members, Rs. 40,000 is sufficient for 25 days. For how many days Rs. 40000 will be sufficient for a family of 6 members?
- 2 An inheritance of Rs. 500,000 is to be divided among two brothers and one sister such that ratio among their share is 2 : 2 : 1 respectively. Find their share.
- 3 Salman, Ahmad and Akram invested an amount in a business in the ratio 2:4:3 respectively. If the profit at the end of year is Rs. 45,000, find their shares in profit.
- 4 If 140 men working 10 hours a day can build a house in 16 days, find out how many men will build same kind of house in 12 days by working 13 hours a day?
- 5 Saleem left a property of value Rs. 150,000. If his widow gets $\frac{1}{8}$ of the total value and son gets twice than a daughter, find the share of his widow, a son and a daughter.
- 6 Aleem and Riaz started a business together. Aleem invests Rs.41,000 and Riaz invests Rs. 82,000. If they earned a profit of Rs. 21,000 after a year, find their shares?
- 7 If 10 men can construct a 75 km long road in 5 days. How many days will needed for 15 men to construct a 45 km long road.

3.5 Insurance

Insurance is designed to protect the financial well-being of an individual or a company in case of unexpected loss (accidents, deaths, fire etc.). Insurance policy creates a contract between the insured and the insurer. According to this contract, insurer is bound to pay an amount in the form of installments (monthly, quarterly, or yearly).

These installments are called premiums.

Premium is some percentage of the actual price of the object. The rate and time of a premium is already decided in the agreement at the time of insurance. Insurance is of different types. But here we discuss only two types.

i. Life Insurance

ii. Vehicle Insurance

3.5.1 Life Insurance

The life insurance is generally used to describe a form of insurance that pays for medical expenses. It is sometimes used more broadly to include insurance covering disability or long-term nursing.

In other words we can say that the life insurance covers the unexpected event like death, accidental death or sickness.

Now, we solve some examples.

Example 1:

Akram insured his life for Rs. 800,000. Find out the amount of premium at the rate of 3%.

Solution:

$$\text{Total amount of policy} = \text{Rs. } 800,000$$

$$\text{rate of premium} = 3\%$$

$$\text{amount of premium} = \frac{\text{rate}}{100} \times \text{total amount of policy}$$

$$= \text{Rs. } \frac{3}{100} \times 800,000$$

$$= \text{Rs. } 24,000$$

Example 2:

A person insures his life for Rs. 950,000 and he pays annual premium Rs. 30,000. Find the rate of the premium.

Solution:

$$\text{Total amount of insurance} = \text{Rs. } 950,000$$

$$\text{amount of premium} = \text{Rs. } 30,000$$

$$\text{rate of premium} = ?$$

$$\text{amount of premium} = \frac{\text{rate}}{100} \times \text{total amount of insurance}$$

Math History

The concept of life insurance was proposed in 1694 by Hugh, the Elder Chamberlen from the Peter Chamberlen family.

$$\text{Rs. } 30,000 = \frac{\text{rate}}{100} \times \text{Rs. } 950,000$$

$$\text{Rs. } 30,000 \times 100 = \text{rate} \times \text{Rs. } 950,000$$

$$\text{Rs. } 3,000,000 = \text{rate} \times \text{Rs. } 950,000$$

$$\text{rate} = \left(\frac{3,000,000}{950,000} \right)$$

$$\text{rate} = 3\% (\text{approx.})$$

Note it down

Depreciation is the decreasing rate of a vehicle or property.

Note it down

When depreciation rate is not given, then we consider it 10% yearly

3.5.2 Vehicle Insurance or Property Insurance

Insurance that covers the risk of fire, accidents, etc against vehicles or property is known as vehicle insurance or property insurance.

A person or a company gets insurance to secure their property or vehicles.

In vehicles or property insurance, the premium is some percentage of the actual amount of the object.

In this type of insurance, depreciation is allowed after every premium. Depreciation rate is also decided accordingly as rules of the company at different rates.

i. e.

We solve questions in the following steps.

$$1^{\text{st}} \text{ premium} = \frac{\text{rate of premium}}{100} \times \text{total price of policy}$$

Depreciated price

$$2^{\text{nd}} \text{ premium} = \frac{\text{rate of premium}}{100} \times \text{depreciate price after 1st premium}$$

Depreciated price

$$3^{\text{rd}} \text{ premium} = \frac{\text{rate of premium}}{100} \times \text{depreciate price after 2nd premium}$$

$$\text{Last premium} = 0$$

Note it down

The amount of the last premium is always equal to zero.

The above procedure is illustrated by the following examples.

Example 1:

Amir insured his car worth Rs. 685,000 at a rate of 3.5% for 4 years. Find the total amount paid as premium, if he did not claim any damages during the period. (If the depreciation rate is 10%).

Solution:

$$\text{Value of car} = \text{Rs. } 685,000$$

$$\text{Rate of premium} = 3.5\%$$

$$\text{Tenure} = 4 \text{ years}$$

$$1^{\text{st}} \text{ premium} = \text{Rs. } \frac{3.5}{100} \times 685,000 = \text{Rs. } 23,975$$

Depreciation rate = 10%

$$\begin{aligned} \text{Depreciation} &= \text{Rs. } \frac{10}{100} \times 685,000 \\ &= \text{Rs. } 68,500 \end{aligned}$$

$$\begin{aligned} \text{Depreciated price after 1 year} &= \text{Rs. } (685,000 - 68,500) \\ &= \text{Rs. } 616,500 \end{aligned}$$

$$\begin{aligned} 2^{\text{nd}} \text{ premium} &= \text{Rs. } \frac{3.5}{100} \times 616,500 \\ &= \text{Rs. } 21,577.5 \end{aligned}$$

$$\begin{aligned} \text{Depreciation} &= \text{Rs. } \frac{10}{100} \times 616,500 \\ &= \text{Rs. } 61,650 \end{aligned}$$

$$\begin{aligned} \text{Depreciated price after 2 years} &= \text{Rs. } (616,500 - 61,650) \\ &= \text{Rs. } 554,850 \end{aligned}$$

$$\begin{aligned} 3^{\text{rd}} \text{ premium} &= \text{Rs. } \frac{3.5}{100} \times 554,850 \\ &= \text{Rs. } 19,419.75 \end{aligned}$$

$$4^{\text{th}} \text{ premium} = 0$$

$$\begin{aligned} \text{The total amount paid as insurance} &= \text{Rs. } 23,975 + \text{Rs. } 21,577.5 + \text{Rs. } 19,419.75 \\ &= \text{Rs. } 64,972.25 \end{aligned}$$

Example 2:

A person insured his property at a rate of 2%. He paid an annual premium of Rs. 13,000. How much is the price of his property?

Solution:

$$\text{Let total price of property} = \text{Rs. } x$$

$$\text{Amount of premium} = \text{Rs. } 13,000$$

$$\text{Rate of premium} = 2\%$$

$$\text{Amount of premium} = \frac{\text{rate of premium}}{100} \times \text{total price}$$

$$\text{Rs. } 13,000 = \frac{2}{100} \times \text{Rs. } x$$

$$\text{Rs. } 13,000 \times 100 = \text{Rs. } 2x$$

$$\text{Rs. } 1,300,000 = \text{Rs. } 2x$$

$$\text{Rs. } x = \text{Rs. } \left(\frac{1,300,000}{2} \right)$$

$$\text{Rs. } x = \text{Rs. } 650,000$$

Which is the actual price of property.

Exercise 3.5

1. Najma insured her life for Rs. 650,000. Find the amount of premium at a rate of 5%.
2. Find the rate of premium if a person insured his life for Rs. 500,000 and he pays annual premium Rs. 25,000.
3. Anwar insured his rickshaw at a rate of 4.2%. He paid an annual premium Rs. 6500. Find the actual price of the rickshaw.
4. Hamid brought a car for Rs. 700,000. He insured it at a rate of 3.5% for 3 years. Find the total amount paid as premium, while the rate of depreciation is 5% yearly.
5. A person insured his vehicle worth Rs. 450,000 at a rate of 2.5% for 4 years. After 3 years he got a claim of damages worth Rs. 750,000. How much loss had he recovered through insurance.

3.6 Converting Currencies

When we want to know the value of a currency with respect to any other currency, we need to know the exchange rate. By knowing the exchange rate, we can easily convert any currency to other currencies.

PKR to USD (United States Dollar)

$$1 \text{ Pakistani Rupee} = 0.0055809287 \text{ United States Dollar (USD)}$$

$$1 \text{ United States Dollar} = 179.181648 \text{ Pakistani Rupee (Rs)}$$

Example 1:

convert 650 Pakistani Rupee to United States Dollar

Solution:

$$1 \text{ Pakistani Rupee} = 0.0055809287 \text{ USD}$$

$$\begin{aligned} 650 \text{ Pakistani Rupee} &= 650 \times 0.0055809287 \text{ USD} \\ &= 3.627 \text{ USD (approx.)} \end{aligned}$$

USD to PKR

Example 2:

Convert 50 United States Dollar to Pakistani Rupee

Solution:

$$1 \text{ United States Dollar} = \text{Rs. } 179.181648$$

$$\begin{aligned} 50 \text{ United States Dollar} &= \text{Rs. } 179.181648 \times 50 \\ &= \text{Rs. } 8959.08 \end{aligned}$$

PKR to GBP (British pound sterling)

$$1 \text{ PKR} = 0.0042 \text{ GBP}$$

$$1 \text{ GBP (British pound sterling)} = 239.66 \text{ PKR}$$

PKR to GBP**Example 3:**

Convert 200 Pakistani Rupee to British pound sterling

Solution:

$$1 \text{ Pakistani Rupee} = 0.0042 \text{ GBP}$$

$$\begin{aligned} 200 \text{ Pakistani Rupee} &= 200 \times 0.0042 \text{ GBP} \\ &= 0.83 \text{ GBP} \end{aligned}$$

GBP to PKR**Example 4:**

Convert 100 GBP to Pakistani Rupee

Solution:

$$1 \text{ GBP} = 239.6635 \text{ PKR}$$

$$\begin{aligned} 100 \text{ GBP} &= 239.6635 \text{ PKR} \times 100 \\ &= \text{Rs } 23966.35 \end{aligned}$$

PKR to Euro

$$1 \text{ PKR} = 0.0050 \text{ EUR}$$

$$1 \text{ EUR} = 199.64 \text{ PKR}$$

PKR to EUR**Example 5:**

Convert 100 Pakistani Rupee to EURO

Solution:

$$1 \text{ PKR} = 0.0050 \text{ EUR}$$

$$\begin{aligned} 100 \text{ Pakistani Rupee} &= 100 \times 0.0050 \text{ EUR} \\ &= 0.50 \text{ EUR} \end{aligned}$$

EUR to PKR**Example 6:**

Convert 50 EUR to Pakistani Rupee

Solution:

$$1 \text{ EUR} = 199.64 \text{ PKR}$$

$$\begin{aligned} 50 \text{ EUR} &= 50 \times 199.64 \\ &= \text{Rs } 9981.89 \end{aligned}$$

PKR to CAD (Canadian Dollar)

$$1 \text{ PKR} = 0.0069 \text{ CAD}$$

$$1 \text{ CAD} = 144.45 \text{ PKR}$$

PKR to CAD**Example:**

convert 100 Pakistani Rupee to CAD

Solution:

$$1 \text{ PKR} = 0.0069 \text{ CAD}$$

$$\begin{aligned} 100 \text{ Pakistani Rupee} &= 100 \times 0.0069 \text{ CAD} \\ &= 0.69 \text{ CAD} \end{aligned}$$

CAD to PKR**Example:**

Convert 50 CAD to Pakistani Rupee

Solution:

$$1 \text{ CAD} = 144.45 \text{ PKR}$$

$$\begin{aligned} 50 \text{ CAD} &= 50 \times 144.45 \text{ PKR} \\ &= 7222.71 \text{ PKR} \end{aligned}$$

Exercise 3.6

1 Convert the following currencies to PKR. (Use updated currency rates from the internet)

a) 340 USD

b) 120 CAD

c) 98 EUR

d) 250 GBP

e) 29 USD

f) 45 CAD

g) 444 EUR

h) 654 GBP

2 Convert the following amounts given in PKR to the mentioned currencies. (Use updated currency rates from the internet)

a) 600 PKR to USD

b) 35 PKR to CAD

c) 897 PKR to EUR

d) 600 PKR to GBP

e) 425 PKR to USD

f) 98 PKR to CAD

g) 326 PKR to EUR

h) 562 PKR to GBP

Summary

- If the ratio $a : b$ is the same as the ratio of $c : d$, then:
 $a : b :: c : d$ or $a \times d = b \times c$
- If the ratio $a : b$ is the inverse of the ratio $c : d$, then:
 $a : b :: d : c$ or $a \times c = b \times d$
- Unitary method is a method in which we calculate the value of a number of things by finding the value of one thing.
- Two quantities that are in direct proportion will always produce a straight-line graph that passes through the origin.
- We use symbol " \propto " for 'is proportional to'.
- If two quantities are directly proportional to each other, they will always result in a straight-line graph that passes through the origin.
- If the graph goes up as x increases (left to right) then the relationship is **direct**. If the graph shape goes down as x increases then the relationship is **inverse**.
- Two quantities that are in inverse proportion will always produce a curved graph.
- Cost price and selling price tells us how much profit or loss a person gets when selling items.
- Selling price is always greater than the cost price when profit is earned.
- An amount that is added to the cost price to calculate the sale price is called markup.
- A person who borrows the money is known as the debtor and the person who lends the money is called the creditor.
- Installments can be paid monthly, quarterly, half yearly or even yearly.
- The full payment that a customer has to pay to purchase an article is called the cash price.
- The amount a customer has to pay at the beginning of an installment plan is called the down payment.
- Depreciation is the decreasing rate of a vehicle or property.
- When depreciation rate is not given, then we consider it 10% yearly.
- The amount of the last premium is always equal to zero.

Vocabulary

- Direct
- Indirect
- Compound
- Inheritance
- Partnership
- Insurance
- Profit
- Loss
- Discount

Review Exercise

1 Encircle the correct option.

- a) If the ratio $a : b$ is the same as the ratio of $c : d$, then:
i. $a : c :: b : d$ ii. $a : b :: c : d$ iii. $a : d :: c : b$ iv. $a : b :: a : d$
- b) A method in which we calculate the value of a number of things by finding the value of one thing is called:
i. Direct Method ii. Inverse method iii. Unitary Method iv. Indirect Method
- c) If the two quantities x and y are directly proportional to each other i.e. $y \propto x$, then:
i. $y = k + x$ ii. $y = \frac{k}{x}$ iii. $y = x$ iv. $y = kx$
- d) If the selling price (SP) of a thing or goods is more than its cost price (CP), we can say that there is a:
i. Loss ii. Profit iii. Discount iv. Tax
- e) An amount that is added to the cost price to calculate the sale price is called:
i. Loss ii. Profit iii. Markup iv. Principal
- f) Markup =
i. Discount – Cash price ii. Total payment – Discount
iii. Cash price – Total payment iv. Total payment – Cash price

2 If 42 men working together can construct a road in 13 days, how long will it take 39 men to construct the same road?

3 A bus runs at an average speed of 35km per hour and completes a journey in 4 hours 12 minutes. How long will it take to cover the same distance at a speed of 45km per hour?

4 2.6 carpenters can make 8 chairs in 10 days, how many days will be required for 10 carpenters to make 16 chairs?

5 Find the profit or loss (percent) if Saima bought a house for Rs. 7,540,000 and sold it for Rs. 9,200,000.

6 Ahmad and Haroon started a business by sharing Rs. 400,000 and Rs. 300,000. Ahsan joined after 6 months with a capital of Rs. 390,000. They made a profit of Rs. 40,000 at the end of the year. Find the share of each in the profit.

- 7 Zubair passed away after leaving property worth Rs. 962,000. Distribute property among his heirs; a wife, a daughter and two sons.
- 8 A person bought 35 books for Rs. 7,000. He gained 10% profit by selling them. Find out the selling price.
- 9 Convert the following currencies to PKR. (Use updated currency rates from the internet)
- | | | | |
|------------|------------|-----------|------------|
| a. 23 USD | b. 556 CAD | c. 67 EUR | d. 85 GBP |
| e. 455 USD | f. 890 CAD | g. 23 EUR | h. 456 GBP |
- 10 Convert the following amounts given in PKR to the mentioned currencies. (Use updated currency rates from the internet)
- | | | | |
|-------------------|-------------------|-------------------|-------------------|
| a. 456 PKR to USD | b. 777 PKR to CAD | c. 89 PKR to EUR | d. 128 PKR to GBP |
| e. 778 PKR to USD | f. 765 PKR to CAD | g. 200 PKR to EUR | h. 989 PKR to GBP |

- 11 Identify the proportion from the given table and then plot the graphs.

x	3	4	5	6	x	-6	12	-8	-1
y	9	12	15	18	y	-4	2	-3	-24

Math Project

Material Required:

- Graph paper
- Paper sheets
- Scoring sheet

Procedure:

- Work in groups.
- Each group will draw one table and equation for direct proportion and one table and equation for inverse proportion on a paper.
- Then they will exchange their paper with other groups.
- The groups will then draw a graph based on the information and interpret it.

NOT FOR SALE

Unit 4

Square Roots and Cube Roots

Student Learning Outcomes

After completing this unit, students will be able to:

- Find the square root of natural numbers, common fractions and decimal numbers (up to 6-digit).
- Solve real-world word problems involving squares and square roots.
- Recognise perfect cubes and find:
 - cubes of up to 2-digit numbers
 - cube roots of up to 5-digit numbers which are perfect cubes
- Solve real-world word problems involving cubes and cube roots



A cube shaped ballot box has a capacity of 27000 centimeter cube. What is its height?

NOT FOR SALE

Introduction

We have already learnt about squares and square roots of numbers, fractions and Decimals (up to 3 digits). In this grade we are going to learn about finding the square of larger numbers. We will also learn about cubes and cube roots of different numbers.

4.1 Squares and Square Roots

We know that we can find the area of a square by multiplying its length with its length. We also know about exponents where repeated multiplication of numbers is written in the form of power to make calculation simpler. Now we will learn about the perfect square and square root of natural numbers.

1.1.1 Square of a Number

When we multiply a number by itself, we can write that number to the power of 2. For example, when we multiply 2 by itself i.e. $2 \times 2 = 2^2$ we read it as 2 raised to the power of 2 or 2 to the power 2 or square of 2.

When a number is multiplied by itself, the value (result) we get is called the square of that number.

Consider the following number sequence, 1, 4, 9, 16, 25, 36.....

Can you describe the rule of this sequence?

Observe that each term of the sequence is obtained by multiplying the respective natural number with itself.

We can write the pattern as:

$1 \times 1, 2 \times 2, 3 \times 3, 4 \times 4, 5 \times 5, 6 \times 6, \dots$

or

$(1)^2, (2)^2, (3)^2, (4)^2, (5)^2, (6)^2, \dots$

Note it down

The square of any two integers is always positive.

Quick Check

Find the square of these numbers;

- a) 18 b) 20
c) 55 d) 100

4.1.2 Perfect Squares

When a number is multiplied by itself, the product is called the square of that number. A number will be called a perfect square if it is a square of any number. A perfect square has a pair of the same factors.

In the above examples, 16, 25, 49, 81, etc., are perfect squares as they are squares of 4, 5, 7, 9, etc. respectively.

If we observe the pattern of perfect squares closely, we can note that:

1^2	1×1	1
2^2	2×2	4
3^2	3×3	9
4^2	4×4	16
5^2	5×5	25
6^2	6×6	36
7^2	7×7	49
8^2	8×8	64
9^2	9×9	81
10^2	10×10	100
11^2	11×11	121
12^2	12×12	144

- Numbers that have 2, 3, 7 or 8 in their ones place can never be perfect squares.
- Only the numbers ending with the digits 1, 4, 6, 9, 00 and 25 **can be** perfect squares. But not all numbers ending with the digits 1, 4, 6, 9, 00 and 25 are perfect squares.
- Numbers ending in an odd number of zeros can never be perfect squares.

We can use the above facts to quickly tell if a number can be a perfect square or not. We can also identify a perfect square by finding its prime factors. If all its factors can be written in the form of pairs, then the given number is a perfect square. In other words, if all the factors have even exponents (indices/powers), the number will be a perfect square.

Note it down

- A perfect square is also called a square number.
- There are an infinite number of perfect squares as they are infinite natural numbers.

Example 1:

Check whether the following numbers are a perfect square or not.

- a) 4356 b) 14175

Solution:

$$\begin{aligned} \text{a) The prime factors of } 4356 &= 2 \times 2 \times 3 \times 3 \times 11 \times 11 \\ &= 2^2 \times 3^2 \times 11^2 \end{aligned}$$

All factors of 4356 can be written in the form of pairs, so 4356 is a perfect square.

2	4356
2	2178
3	1089
3	363
11	121
	11



Make two groups of the students. Give one group even number cards and the other group odd number cards. Instruct them to find the square of the given numbers and write the answer on the blank sheet. Then share the answers with their teacher and class-fellows.

b) The prime factors of 14175 = $3 \times 3 \times 3 \times 3 \times 5 \times 5 \times 7$
 $= 3^4 \times 5^2 \times 7$

The prime factors 3, 5 can be written in the form of pairs. But 7 cannot be written in the form of a pair. So, 14175 is not a perfect square.

Note it down

All prime factors can be written in the form of pairs if a number is a perfect square.

3	14175
3	4725
3	1575
3	525
5	175
5	35
7	7
	1

Note it down

- The square of an even number is even.
- The square of an odd number is odd.
- The square of a proper fraction is less than itself.
- The square of a decimal less than 1 is smaller than the decimal.

Example 2:

Express the following numbers as perfect squares.

- i) 64 ii) 169 iii) 62 iv) 1024

Solution:

- i) $64 = 8 \times 8 = (8)^2$ ii) $169 = 13 \times 13 = (13)^2$
 iii) $625 = 25 \times 25 = (25)^2$ iv) $1024 = 32 \times 32 = (32)^2$

We can find various patterns in squares of natural numbers.

Adding of odd numbers:

$$1^2 = 1 = 1 \text{ (1st odd number)}$$

$$2^2 = 4 = 1 + 3 \text{ (1st two numbers)}$$

$$3^2 = 9 = 1 + 3 + 5 \text{ (1st three numbers)}$$

$$4^2 = 16 = 1 + 3 + 5 + 7 \text{ (1st four numbers)}$$

and so on.

NOT FOR SALE

We can match from above pattern that every square numbers can be written as the sum of consecutive odd numbers starting from 1. The number of odd numbers which are being added is same as the number being squared.

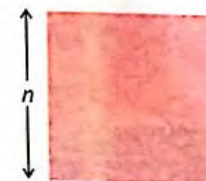
Quick Check

Complete the pattern for the given numbers.

Number	Square	Addition Pattern
1	1^2	1
4		$1 + 2 + 1$
	3^2	$1 + 2 + 3 + 2 + 1$
16	4^2	
25		
36	6^2	
	7^2	
64		
81		
	10^2	

4.1.3 Square Root

Observe the square in figure. It is formed of four equal sides each having the length n .



$$\begin{aligned} \text{Area of the square} &= n \times n \\ &= n^2 \end{aligned}$$

$\therefore n$ is the root of the square.

When we talk of square root of a number, we say that;

Square root of a number is a factor of the number that when multiplied by itself, gives the number.

Note it down

A square is a four sided closed figure with each side having the same length and each angle between the two sides, the right angle.

Note it down

$\sqrt{\quad}$ = Radical
 $\sqrt{\quad}$ = Radical sign
 \quad = Radicand

NOT FOR SALE

For example, 3 is the square root of 9 because $3 \times 3 = (3)^2 = 9$.

We write $\sqrt{9} = 3$.

Square Root by Prime Factorization Method

To find the square root of the numbers using factorization method, we follow these steps.

Step I: First find the prime factors of the given numbers and write them in pairs.

Step II: From each pair, take only one number and find their product. This product will be the square root of the given number.

Example 1:

Find the square root of the following numbers.

a) 81 b) 196 c) 2304

Solution:

$$\begin{aligned} \text{a) } \sqrt{81} &= \sqrt{3 \times 3 \times 3 \times 3} \\ &= \sqrt{3^2 \times 3^2} \\ &= 3 \times 3 = 9 \\ \therefore \sqrt{81} &= 9 \end{aligned}$$

3	81
3	27
3	9
3	3
3	1

$$\begin{aligned} \text{b) } \sqrt{196} &= \sqrt{2 \times 2 \times 7 \times 7} \\ &= \sqrt{2^2 \times 7^2} \\ &= 2 \times 7 = 14 \\ \therefore \sqrt{196} &= 14 \end{aligned}$$

2	196
2	98
7	49
7	7
	1

$$\begin{aligned} \text{c) } \sqrt{2304} &= \sqrt{2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 3 \times 3} \\ &= \sqrt{2^8 \times 3^2} \\ &= 2 \times 2 \times 2 \times 2 \times 3 \\ &= 48 \\ \therefore \sqrt{2304} &= 48 \end{aligned}$$

2	2304
2	1152
2	576
2	288
2	144
2	72
2	36
2	18
3	9
3	3
	1

Note it down

- Finding square root is the inverse process of squaring the number.
- The numbers that read same onward and backward are called palindromes.

Math History

Al-Nasawi (1010 – 1075AD), explained the extraction of square and cube roots almost in the modern manner.

Note it down

The factors of a number that are divisible by 1 and the number itself are called prime factors.

Note it down

If the prime factors of the given number cannot be written in pairs, the given number is not a perfect square.

As we know that square root of a fraction is obtained by finding the separate square roots of the numerator and denominator of the fraction. Let us follow some examples.

Example 2:

Find the square roots of the following fractions.

a) $\frac{49}{121}$ b) $\frac{64}{256}$ (By prime factorization method)

Solution:

$$\begin{aligned} \text{a) } \sqrt{\frac{49}{121}} &= \frac{\sqrt{49}}{\sqrt{121}} \\ &= \frac{\sqrt{7 \times 7}}{\sqrt{11 \times 11}} = \frac{\sqrt{7^2}}{\sqrt{11^2}} \\ &= \frac{7}{11} \end{aligned}$$

$49 = 7 \times 7$
 $121 = 11 \times 11$

$$\begin{aligned} \text{b) } \sqrt{\frac{64}{256}} &= \frac{\sqrt{64}}{\sqrt{256}} \\ &= \frac{\sqrt{2 \times 2 \times 2 \times 2 \times 2 \times 2}}{\sqrt{2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2}} \\ &= \frac{\sqrt{2^6}}{\sqrt{2^8}} \\ &= \frac{2 \times 2 \times 2}{2 \times 2 \times 2 \times 2} \\ &= \frac{8}{16} \end{aligned}$$

2	64
2	32
2	16
2	8
2	4
2	2
	1

2	256
2	128
2	64
2	32
2	16
2	8
2	4
2	2
	1

Example 3:

Find the square root of the following decimal fractions, using the prime factorization method.

- a) 1.21 b) 0.1089 c) 47.61

$$\begin{aligned} \text{a) } \sqrt{1.21} &= \sqrt{\frac{121}{100}} \\ &= \frac{\sqrt{121}}{\sqrt{100}} = \frac{11}{10} = 1.1 \end{aligned}$$

$$\therefore \sqrt{1.21} = 1.1$$

$$\begin{aligned} \text{b) } \sqrt{0.1089} &= \sqrt{\frac{1089}{10000}} \\ &= \frac{\sqrt{1089}}{\sqrt{10000}} = \frac{\sqrt{3 \times 3 \times 11 \times 11}}{\sqrt{2 \times 2 \times 2 \times 2 \times 5 \times 5 \times 5 \times 5}} \\ &= \frac{\sqrt{3^2 \times 11^2}}{\sqrt{2^2 \times 2^2 \times 5^2 \times 5^2}} \\ &= \frac{3 \times 11}{2 \times 2 \times 5 \times 5} \\ &= \frac{33}{100} \end{aligned}$$

$$\therefore \sqrt{0.1089} = 0.33$$

- c) 47.61

$$\begin{aligned} \sqrt{47.61} &= \sqrt{\frac{4761}{100}} \\ &= \frac{\sqrt{4761}}{\sqrt{100}} = \frac{\sqrt{3 \times 3 \times 23 \times 23}}{\sqrt{2 \times 2 \times 5 \times 5}} \\ &= \frac{\sqrt{3^2 \times 23^2}}{\sqrt{2^2 \times 5^2}} \\ &= \frac{3 \times 23}{2 \times 5} = \frac{69}{10} \end{aligned}$$

$$\therefore \sqrt{47.61} = 6.9$$

3	1089
3	363
11	121
11	11
	1

2	10000
2	5000
2	2500
2	1250
5	625
5	125
5	25
5	5
	1

3	4761
3	1587
23	23
23	1

Square Root by Division Method

The square root of the numbers can also be found by division method. Consider the following examples.

Example 1:

Find the square root of 236196 by division method.

Solution:

Step I: First make pairs of the digits of the number 236196 from the right to the left.

We have, $\overline{23} \overline{61} \overline{96}$.

Step II: Try to find the greatest number whose square is less than or equal to the number in the leftmost pair i.e. 23.

$2 \times 2 = 4$, $3 \times 3 = 6$, $4 \times 4 = 16 < 23$, $5 \times 5 = 25 > 23$

Here the number we need is 4.

Step III: Take this number 4 as the divisor and the quotient and write the product 16 below the number in the leftmost pair i.e. 23. We get the remainder 7.

Step IV: Take down the next pair 61. Add the divisor with the quotient (or double the quotient 4) $4 + 4 = 8$ and enter it with a blank to its right.

Step V: Find the greatest possible digit to fill the blank that will also be the next digit in the quotient, so when the divisor (made up of 8 and the new digit) is multiplied to the new digit in the quotient, the product is less than or equal to the dividend 761. Here $88 \times 8 = 704 < 761$, so 8 is the required digit here. The remainder will be 57.

Step VI: Take down the next pair 96. Add the divisor 88 with the new quotient 8 i.e. $88 + 8 = 96$ (or double the quotient 48) and enter it with a blank to its right.

Step VII: Find the greatest possible digit to fill the blank that will also be the next digit in the quotient, so when the divisor (made up of 96 and the new digit) is multiplied to the new digit in the quotient, the product is less than or equal to the

$$\begin{array}{r} 4 \\ 4 \overline{) 236196} \\ \underline{-16} \\ 7 \end{array}$$

$$\begin{array}{r} 4 \\ 4 \overline{) 236196} \\ \underline{-16} \\ 8 \overline{) 761} \\ \underline{-704} \\ 57 \end{array}$$

$$\begin{array}{r} 48 \\ 4 \overline{) 236196} \\ \underline{-16} \\ 88 \overline{) 761} \\ \underline{-704} \\ 57 \end{array}$$

$$\begin{array}{r} 486 \\ 4 \overline{) 236196} \\ \underline{-16} \\ 88 \overline{) 761} \\ \underline{-704} \\ 966 \overline{) 5796} \\ \underline{-5796} \\ 0 \end{array}$$

dividend 5796. Here $966 \times 6 = 5796$, So 6 is the required digit here. The remainder will be 0.

The quotient 486 is the square root of 236196.

So, $\sqrt{236196} = 486$

Verification:

$$486 \times 486 = 236196$$

Note it down

In division method, the quotient is the square root of the given number.

Example 2:

Find the square root of $\frac{3136}{4096}$ by division method.

Solution:

To find the square root of the given fraction by division method, we find the square root of the numerator and denominator separately.

$$3136 = \overline{31\ 36}$$

$$4096 = \overline{40\ 96}$$

Now follow the same steps as in the previous example and find the square root of 3136 and 4096

$$\begin{array}{r} 56 \\ 5 \overline{) 3136} \\ \underline{-25} \\ 636 \\ \underline{-636} \\ 0 \end{array}$$

$$\begin{array}{r} 64 \\ 6 \overline{) 4096} \\ \underline{-36} \\ 496 \\ \underline{-496} \\ 0 \end{array}$$

Quick Check

Find the square root of the following numbers by division method.

- a) 2209 b) 4624
c) 10816 d) 13225

The square root of $\frac{3136}{4096} = \frac{56}{64}$.

Verification:

$$56 \times 56 = 3136$$

$$64 \times 64 = 4096$$

Example 3:

Find the square root of the following decimal fractions using the division method.

- a) 1.0201 b) 18.4041 c) 904.2049

Solution:

Procedure:

- Make pairs of the whole part of the number from right to left and that of decimal part of the number from left to right.
- When a pair of digits after the decimal point is brought down place the decimal point in the quotient.
- Solve, like the whole numbers as explained before in division method for finding the square root of whole numbers.

$$\begin{array}{r} 1.01 \\ 1 \overline{) 1.0201} \\ \underline{-1} \\ 02 \\ \underline{-00} \\ 201 \\ \underline{-201} \\ 0 \end{array}$$

b) 18.4041

$$\begin{array}{r} 4.29 \\ 4 \overline{) 18.4041} \\ \underline{-16} \\ 240 \\ \underline{-164} \\ 7641 \\ \underline{-7641} \\ 0 \end{array}$$

$$\therefore \sqrt{18.4041} = 4.29$$

c) 904.2049

$$\begin{array}{r} 30.07 \\ 3 \overline{) 904.2049} \\ \underline{-9} \\ 04 \\ \underline{-00} \\ 420 \\ \underline{-000} \\ 42049 \\ \underline{-42049} \\ 0 \end{array}$$

$$\therefore \sqrt{904.2049} = 30.07$$

Note it down

In decimal fraction, if the number of digits after the decimal point is odd, we place, '0' to make complete pairs.

Quick Check

1. Find the square root of the following fractions by division method.

a) $\frac{25}{121}$ b) $\frac{1521}{2809}$

2. Find the square root of the following decimal numbers by division method.

a) 0.81 b) 0.02304

4.1.4 Number of Digits in the Square Root of a Perfect Square

While finding square roots of large numbers, it is useful to determine the number of digits in the square root.

This help us to avoid confusion, when bringing down two pairs together or solving decimal fractions.

We use the following rule to determine the number of digits in the square root of a perfect square.

Rule: Let n be the number of digits in $n!$

- $\frac{n}{2}$ digits if n is even and
- $\frac{n+1}{2}$ digits if n is odd

Example 1:
Find the number of digits in the square root of the following numbers in perfect square form.

- a) 157609 b) 1,014,049 c) 20.331081

Solution:
The number of digits in 157609 is 6. i. e, $n = 6$ (an even number)

a) $\therefore \sqrt{157609}$ contains $\frac{n}{2} = \frac{6}{2} = 3$ digits.

- b) The number of digits in 1,014,049 is 7. i.e $n = 7$ (odd)

Therefore, the number of digits in $\sqrt{1,014,049}$

$$= \frac{n+1}{2} = \frac{7+1}{2} = \frac{8}{2} = 4$$

- c) The number of digits in the decimal 20.331081 is 8.

Here $n = 8$ (even)

∴ Number of digits in $\sqrt{20.331081}$

$$\frac{n}{2} = \frac{8}{2} = 4$$

Let us find the square root of numbers or fractions which are not perfect square;

Find the square root of: a) 2 b) 3 c) 2.5 d) 4.225

a) 2

Using the division method,
in the second trial, the divisor is less than the dividend. So to continue the process, we place a decimal point after 2 along with three pairs of zeros as follows;

We observe that the process does not end, and if we continue the process, no digit or block of digits will repeat.

$$\begin{array}{r} 1 \\ 1 \overline{) 21} \\ \underline{2} \\ 1 \end{array}$$

This means $\sqrt{2}$ is equal to a non-terminating and non-recurring decimal. Such decimal fractions represent irrational numbers. If we continue the process, we get $\sqrt{2} = 1.4142135.....$

We round off the answer to three decimal places.

$$\therefore \sqrt{2} \approx 1.414$$

b) 3

1	1.7320...
	3.00000000
	-1
27	200
	-189
343	1100
	-1029
3462	7100
	-6924
24640	17600
	-
	17600

$\therefore \sqrt{3} \approx 1.732$

d) 4.225

	2.0554
2	4.225000
	-4
405	2250
	-2025
4105	22500
	-20525
41104	197500
	-164416
	33084

$\therefore \sqrt{4.225} \approx 2.055$

c) 2.5

1	1.5842
	2.500000
	-1
25	150
	-125
308	2500
	-2464
3164	13600
	-12656
31682	94400
	-63364
	31036

$\therefore \sqrt{2.5} \approx 1.58$

' \approx ' means
"approximately equal".

While finding the square root of a number other than perfect square, we round off the decimal to 3 decimal places.

1 Find the square root of the following numbers by factorization method.

- i) 36 ii) 100 iii) 2401 iv) 409600
v) $\frac{25}{49}$ vi) $\frac{961}{10201}$ vii) $\frac{1444}{1764}$ viii) $29\frac{30}{49}$

$$\begin{array}{r} 1.414 \\ 1 \overline{) 2.000000} \\ \underline{-1} \\ 100 \\ \underline{-96} \\ 400 \\ \underline{-281} \\ 11900 \\ \underline{-11296} \\ 604 \\ \dots \end{array}$$

ix) 0.0081

x) 11.56

xi) 8036.355

2 Find the square root of the following natural numbers by division method.

a) 4489

b) 9604

c) 7569

d) 28224

e) 1225

f) 72900

3 Find the square root of the following fractions by division method.

a) $11 \frac{14}{25}$

b) $\frac{1600}{169}$

c) $\frac{2916}{1444}$

d) $\frac{196}{484}$

e) $\frac{4225}{5476}$

4 Find the square root of the following decimal numbers by division method.

a) 0.1089

b) 0.0025

c) 92.16

d) 112.36

5 Find the square root of the following non-perfect squares up to three decimal places.

a) 5

b) 2

c) 7

d) 11

e) 3.1

6 Find the square root of the following decimal fractions up to 2 decimal places.

a) 3.21

b) 56.56

c) 46.5

d) 11.3

e) 5.776

4.2 Real-Life Problems of Square Root

Example 1:

9409 tiles are required for tiling the floor of a masjid. If the length and width of the masjid are equal, how many tiles are there in each row?

Solution:

As the length and width are equal, the masjid is in square shape. To find the number of tiles in each row, we have to find the square root of 9409.

So, there are 97 tiles in each row.

$$\begin{array}{r} 97 \\ 9 \overline{) 9409} \\ \underline{-81} \\ 1309 \\ 187 \overline{) 1309} \\ \underline{-1309} \\ 0 \end{array}$$

Example 2:

Area of a square piece of wood is equal to the 116.64 m².

Find the length of the side of the piece of wood.

Solution:

As the length of four sides of a square is equal, so:

Area of square = length \times length = r^2

$$116.64 \text{ m}^2 = r^2$$

Take the square root of both sides.

$$\sqrt{116.64} = \sqrt{\frac{11664}{100}}$$

Now find the square root of 11664 and 100.

$$\frac{\sqrt{11664}}{\sqrt{100}} = \frac{108}{10} = 10.8$$

So, the length of the side of the wooden piece is 10.8 m.

$$\begin{array}{r} 108 \\ 1 \overline{) 11664} \\ \underline{-1} \\ 016 \\ 20 \overline{) 016} \\ \underline{-0} \\ 1664 \\ 208 \overline{) 1664} \\ \underline{-1664} \\ 0 \end{array}$$

$$\begin{array}{r} 10 \\ 1 \overline{) 100} \\ \underline{-1} \\ 000 \\ 20 \overline{) 000} \\ \underline{-000} \\ 0 \end{array}$$

Example 3:

The area of a square shaped plot is 5929 square metres. Find its perimeter.

Solution:

We know that area of a square is obtained by multiplying its length by its length.

$$\text{Length} \times \text{length} = \text{Area}$$

$$\sqrt{\text{Length}^2} = \sqrt{5929}$$

$$\text{Length} = \sqrt{5929}$$

$$= 77 \text{ m}$$

So, length of the plot is 77 metres.

$$\text{Perimeter of the plot} = 4 \times \text{Length}$$

$$= 4 \times 77 \text{ m}$$

$$= 308 \text{ m}$$

So, the perimeter of the square shaped plot is 308 m.

$$\begin{array}{r} 77 \\ 7 \overline{) 5929} \\ \underline{-49} \\ 1029 \\ 147 \overline{) 1029} \\ \underline{-1029} \\ 0 \end{array}$$

Note it down

1. The area of the squared shaped ground of the Masjid is 1156 m². Find the length of the ground.
2. Area of the squared shaped room is 98.01 m². Find the perimeter of the room.



Make two groups of the students and instruct them to make real-life problems of finding square root. Ask them to solve them and share their answers with their class-fellows.

NOT FOR SALE

NOT FOR SALE

Example 4:

Arrange 2025 students in such a way that the number of rows and students in each row are equal. Find the number of students in each row.

Solution:

Since the number of students in a row is the same as the number of rows, square root of 2025 will be found.

$$\text{Since } \sqrt{2025} = 45$$

Hence, The number of students in each row = 45

$$\begin{array}{r} 45 \\ 4 \overline{) 2025} \\ \underline{-16} \\ 425 \\ \underline{-425} \\ 0 \end{array}$$

Exercise 4.2

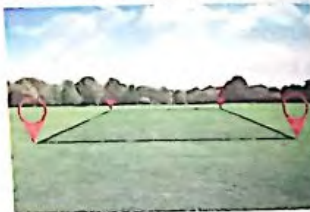
- 1 There are 102400 chairs in a hall arranged in such a way that the chairs in each row are equal to the total number of rows. Find the number of rows.



- 2 The area of a rectangular field is equal to the area of another square shaped field. Find the length of the square shaped field if the length and width of the rectangular field are 675 m and 147 m respectively.

- 3 Find the length of the square shaped masjid if its area is 722,500 square meter.

- 4 The length and width of a rectangular plot is 500 m and 125 m respectively. If a square shaped plot has the same area as the rectangular plot, find the cost of fencing the square plot at the rate of Rs 150 per metre.



- 5 The length of the base and altitude of a triangle is 220 cm and 55 cm respectively. If the area of a square is double the area of this triangle, find the length of the square.

- 6 The square shaped parking area is 10404 m². Find the cost of constructing a tiled path around it at the rate of Rs 435 per metre.

- 7 The area of a square shaped garden is 9801 square metres. Find the perimeter of the garden.

4.3 Cube and Cube Roots

4.3.1 Cube

Consider the following products;

$$2 \times 2 \times 2 = (2)^3 = 8 \text{ and}$$

$$3 \times 3 \times 3 = (3)^3 = 27 \text{ similarly}$$

$$4 \times 4 \times 4 = (4)^3 = 64$$

We observe that;

$2^3 = 8$ is obtained by multiplying 2 twice by itself. This means 8 is the cube of 2.

$3^3 = 27$ is obtained by multiplying 3 twice by itself. i.e 27 is the cube of 3.

The **cube** of a number is that number raised to the power 3.

4.3.2 Perfect cube

A natural number is said to be a perfect cube if it is the cube of some natural number. For example;

$$27 = (3)^3 \Rightarrow 27 \text{ is a perfect cube, similarly}$$

$$125 = (5)^3 \Rightarrow 125 \text{ is a perfect cube.}$$

The following table represent the values of perfect cubes of natural numbers from 1 - 20.

x	x ³	x	x ³
1	1	11	1331
2	8	12	1728
3	27	13	2197
4	64	14	2744
5	125	15	3375
6	216	16	4096
7	343	17	4913
8	512	18	5832
9	729	19	6859
10	1000	20	8000

Note it down



If volume of cube is a^3 , length of each of its dimensions is a .

Observe the above table, can you find some thing special for the cube of even and odd numbers?

To examine that a given number is a perfect cube or not, we find its prime factorization. If it is expressible as the product of cubes of the prime factors, the number is a perfect cube.

Example 1:

Examine that the numbers.

- a) 13824 b) 6125 are perfect cubes or not.

Solution:

a) 13824

Resolving 13824 into prime factors.

$$13824 = 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 3 \times 3 \times 3$$

$$= 2^8 \times 3^3$$

Since 13824 is expressible as the product of the cubes of prime factors.

∴ It is a perfect cube.

2	13824
2	6912
2	3456
2	1728
2	864
2	432
2	216
2	108
2	54
3	27
3	9
3	3
	1

b) 6125

Resolving 6125 into prime factors.

$$6125 = 5 \times 5 \times 5 \times 7 \times 7$$

$$= 5^3 \times 7^2$$

5	6125
5	1225
5	245
7	49
7	7
	1

Since 6125 cannot be expressed as the product of cubes of prime factors,

∴ 6125 is not a perfect cube.

4.3.3 Cube Root of a Number

The **cube root** of a number is a number whose cube is that number.

For example:

$$8 = (2)^3 \Rightarrow 2 \text{ is the cube root of } 8.$$

$$\text{Similarly } 27 = (3)^3 \Rightarrow 3 \text{ is the cube root of } 27.$$

The cube root of a number is denoted by the symbol $\sqrt[3]{\quad}$.

For any number n , cube root of n is denoted by $\sqrt[3]{n}$.

Here, $\sqrt{\quad}$ = radical

$\sqrt{\quad}$ = radical sign

3 = index of the radical

Math History

- Al-Uqlidisi worked on square and cube roots in Al-Fusul.
- Al-Baghdad, worked on cube roots in Al-Takmila.
- Al-Samawal, worked on nth roots in a treatise on Indian arithmetic.

Example 1:

Find the cube root of the following numbers.

- a) 2744 b) 46,656

Solution:

a) 2744

To find the cube root of 2744, we do the following steps;

i. Resolve the number into its prime factors.

ii. Form groups of three like factors.

iii. The product of single factor from each group is the required cube root.

$$2744 = 2 \times 2 \times 2 \times 7 \times 7 \times 7$$

$$= 2^3 \times 7^3$$

$$\therefore \sqrt[3]{2744} = \sqrt[3]{2^3 \times 7^3}$$

$$= 2 \times 7$$

$$= 14$$

2	2744
2	1372
2	656
7	343
7	49
7	7
	1

b) 46,656

$$\sqrt[3]{46,656}$$

$$46656 = 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 3 \times 3 \times 3 \times 3 \times 3 \times 3$$

$$= 2^6 \times 3^6$$

$$= \sqrt[3]{46656} = \sqrt[3]{2^6 \times 3^6}$$

$$= 2 \times 2 \times 3 \times 3$$

$$= 36$$

$$\therefore \sqrt[3]{46656} = 36$$

2	46656
2	23328
2	11664
2	5832
2	2916
2	1458
3	729
3	243
3	81
3	27
3	9
3	3
	1

Example 2:

Find the cube root of $\frac{125}{343}$.

Solution:

$$\sqrt[3]{\frac{125}{343}}$$

$$= \sqrt[3]{\frac{5 \times 5 \times 5}{7 \times 7 \times 7}}$$

5	125
5	25
5	5
	1

$$= \sqrt[3]{\frac{5^3}{7^3}} = \sqrt[3]{\left(\frac{5}{7}\right)^3} = \frac{5}{7}$$

$$\therefore \sqrt[3]{\frac{125}{343}} = \frac{5}{7}$$

Since $5 = \sqrt[3]{125}$

and $7 = \sqrt[3]{343}$

$$\therefore \sqrt[3]{\frac{125}{343}} = \frac{\sqrt[3]{125}}{\sqrt[3]{343}}$$

In general;

For rational number $\frac{a}{b}$, $\sqrt[3]{\frac{a}{b}} = \frac{\sqrt[3]{a}}{\sqrt[3]{b}}$

7	343
7	49
7	7
	1

Example 3:

Find the cube root of $\sqrt[3]{5.832}$.

Solution:

$$= \sqrt[3]{\frac{5832}{1000}}$$

$$= \frac{\sqrt[3]{5832}}{\sqrt[3]{1000}}$$

$$= \frac{\sqrt[3]{2 \times 2 \times 2 \times 3 \times 3 \times 3 \times 3 \times 3 \times 3}}{\sqrt[3]{2 \times 2 \times 2 \times 5 \times 5 \times 5}}$$

$$= \frac{2 \times 3 \times 3}{2 \times 5}$$

$$= \frac{18}{10} = 1.8$$

2	5832
2	2916
2	1458
3	729
3	243
3	81
3	27
3	9
3	3
	1

4.3.4 Properties of Cubes of Numbers

- i) The cube of an even number is even.
- ii) The cube of an odd number is odd.
- iii) The cube of a positive number is positive.
- iv) The cube of a negative number is negative.

v) For any rational number $\frac{a}{b}$, $\sqrt[3]{\frac{a}{b}} = \frac{\sqrt[3]{a}}{\sqrt[3]{b}}$.

vi) For any rational number $\sqrt[3]{a \times b} = \sqrt[3]{a} \times \sqrt[3]{b}$.

Example 1:

The volume of a cube shaped container is 17576 cubic metres. What is the length of the container?

Solution:

The volume of cube = length \times length \times length

To find the length, we need to find the cube root of its volume i.e. 17576 m^3 .

$$\text{Cube root of } 17576 = \sqrt[3]{17576}$$

$$= \sqrt[3]{2 \times 2 \times 2 \times 13 \times 13 \times 13}$$

$$= 2 \times 13$$

$$= 26$$

2	17576
2	8788
2	4394
13	2197
13	169
13	13
	1

So, the length of the container is 26 metre.

Note it down

Volume of solids is always measured in cubic units.

A cube has all edges of equal length i.e.

length = width = height

So, we can find the volume of a cube multiplying its length three time.

Volume of cube = length \times width \times height

Volume of cube = length \times length \times length

$$= l \times l \times l$$

$$= l^3$$

Example 2:

A wooden box is in cube shape. The volume of this box is 29791 cm^3 . What are the dimensions of this box?

Solution:

The dimensions of this cube will be equal as in a cube length = width = height. So, length can be found by finding the cube root of the volume 29791 cm^3 .

$$= \sqrt[3]{29791}$$

$$= \sqrt[3]{31 \times 31 \times 31}$$

$$= 31$$

31	29791
31	961
31	31
	1

So, the dimensions of this cube shapes wooden box are $31 \text{ cm} \times 31 \text{ cm} \times 31 \text{ cm}$.

Exercise 4.3

1 Find the cube root of the following if possible.

- | | | | | |
|------------------------|------------------------|---------------------|---------------------|-----------------------|
| a) 512 | b) 1728 | c) 6859 | d) 15625 | e) 21952 |
| f) 32768 | g) 46656 | h) 59319 | i) 85184 | j) $\frac{125}{4096}$ |
| k) $\frac{2197}{6859}$ | l) $\frac{1331}{8000}$ | m) $\frac{64}{729}$ | n) $\frac{27}{512}$ | o) 64×1728 |
| p) 12167 | q) 42875 | r) 1331 | s) 3375 | t) 1000 |

2 A cube shaped iron container has a volume of 6859 metre cube. Find the dimensions of the container.

3 The volume of a cube shaped hall is 74088 m³. Find the length of the hall.

4 The volume of a cubic room is 9261 m³. Find the area of the roof of this room.

5 Find the cubes of the following numbers:

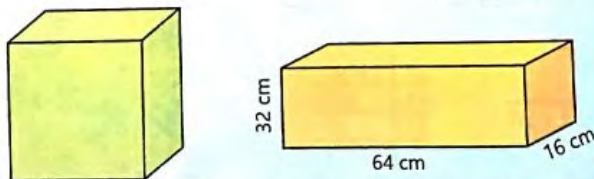
- | | | | | | |
|-------|-------|-------|-------|-------|-------|
| a) 67 | b) 12 | c) 98 | d) 50 | e) 44 | f) 73 |
|-------|-------|-------|-------|-------|-------|

Think Higher

a) Find out:

- the smallest number to be added to 1752 to make it a perfect square.
- the smallest number to be subtracted from 11500 to make it a perfect square.

b) The given cube and cuboid have the same volume. Find the dimensions of the cube.



Summary

Vocabulary

- When a number is multiplied by itself, the value (result) we get is called the square of that number.
- A number will be called a perfect square if it is a square of any number.
- A perfect square is also called a square number.
- All prime factors can be written in the form of square if a number is a perfect square.
- The square of an even number is even and an odd number is odd.
- When the numerator of a fraction is less than the denominator then that fraction is called a proper fraction.
- The square of a proper fraction is less than itself.
- The square of a decimal less than 1 is smaller than the decimal.
- We use the symbol $\sqrt{\quad}$ to represent square root. It is called the radical sign.
- There are two methods to find the square root of numbers.
 - i) Factorisation method
 - ii) Division method
- In division method the quotient is the square root of the given numbers.
- The cube of a number is that number raised to the power 3.
- A natural number is said to be a perfect cube if it is the cube of some natural number.
- If a number is expressible as the product of cubes of the prime factors, the number is a perfect cube.
- The cube root of a number is denoted by the symbol $\sqrt[3]{\quad}$.
- For any number n , cube root of n is denoted by $\sqrt[3]{n}$.
- Here, $\sqrt[n]{\quad}$ = radical
 $\sqrt{\quad}$ = radical symbol
 3 = radical index

- Square
- Square root
- Cube
- Cube root
- Radical
- Radicand
- Index

Review Exercise

1 Encircle the correct option.

a) If the prime factors of the given number cannot be written in pairs, the given number is:

- i) a perfect square
- ii) not a perfect square
- iii) a perfect cube
- iv) not a perfect cube

b) Finding square root is the inverse process of :

- i) adding a number
- ii) perfect cube of a number
- iii) squaring the number
- iv) division of a number

c) The square root of 4096 is:

- i) 54
- ii) 64
- iii) 74
- iv) 84

d) 104 is the square root of:

- i) 10806
- ii) 10836
- iii) 10826
- iv) 10816

e) Square root of 21609 is:

- i) 1047
- ii) 247
- iii) 107
- iv) 147

f) The cube of a number is that number raised to the power:

- i) 1
- ii) 2
- iii) 3
- iv) 9

g) $(17)^3 =$

- i) 4195
- ii) 4133
- iii) 4913
- iv) 4193

h) 40 is the cube root of:

- i) 64000
- ii) 6400
- iii) 16000
- iv) 80000

i) Cube root of 729 is:

- i) 6
- ii) 7
- iii) 8
- iv) 9

2 Find the square root of the following numbers by factorization method.

- a) 841
- b) 1225
- c) 1764
- d) $\frac{2500}{961}$
- e) $\frac{900}{9025}$
- f) $\frac{3366}{2401}$
- g) 1.2321
- h) 0.0676
- i) 6.4009

3 Find the square root of the following natural numbers by division method.

- a) 5929
- b) 4096
- c) 2704
- d) $\frac{1764}{676}$
- e) $\frac{1444}{81}$
- f) 47.61
- g) 146.41

4 Find the cube of the following numbers:

- a) 65
- b) 77
- c) 30
- d) 18
- e) 22
- f) 99

5 Find the cube root of the following numbers by factorization method.

- a) 64
- b) 125
- c) 512
- d) 1331
- e) 74088

6 Find the cube root of the following numbers by division method.

- a) 27
- b) 216
- c) 729
- d) 2744
- e) 13824

7 1681 blocks have been arranged in such a way that the number of rows and the number of blocks in each row are equal. Find the number of blocks in each row.

8 Area of a square field is 50625 m^2 . Find the total length of a fencing wire which is required to cover its boundary.



9 A positive number when multiplied by itself is 13225. Find the number.

10 Area of the floor of a square hall is 97344 m^2 . Find the dimensions of the floor.

11 A cube shaped plastic container has a volume of 8000 metre cube. Find the dimensions of the container.



12 The capacity of a cube shaped swimming pool is 85184 m^3 . What is the cost of tiling its base floor at the rate of Ra 850 per square metre.



Math Project

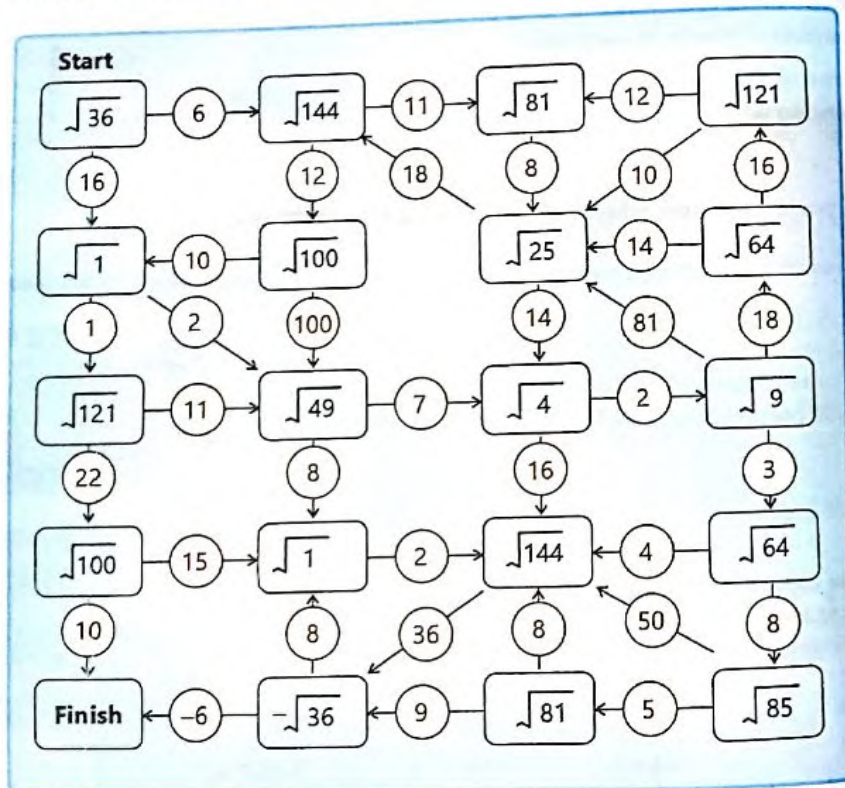


Material Required:

- Maze sheets (Unique one for each pair)
- Pencils
- Colours
- Scoring sheet

Procedure:

- Work in pairs.
- Each pair will be provided with a unique maze sheet (as the one sample shown in picture) containing questions regarding square root and cube roots.
- The members of the pair will calculate them and colour the correct path towards the end.
- The pair with accurate solutions done in minimum time wins.



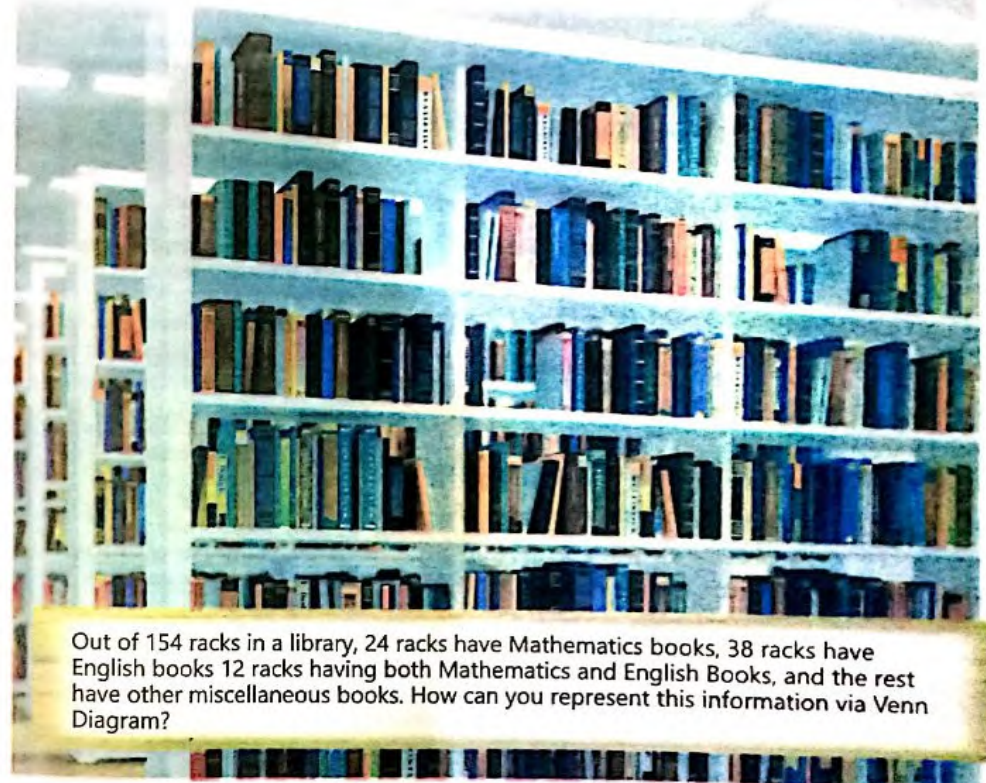
Unit 5

Sets

Student Learning Outcomes

After completing this unit, students will be able to:

- Describe sets using language (tabular, descriptive, and set-builder notation) and Venn diagrams.
- Find the power set (P) of set A where A has up to four elements.
- Describe operations on sets and verify commutative, associative, distributive laws with respect to union and intersection.
- Verify De Morgan's laws and represent through Venn Diagram.
- Apply sets in real-life word problems.



Out of 154 racks in a library, 24 racks have Mathematics books, 38 racks have English books 12 racks having both Mathematics and English Books, and the rest have other miscellaneous books. How can you represent this information via Venn Diagram?

Introduction

In the previous grade we have learnt about sets, its types and how we can represent different sets and their elements using Venn diagram. We learnt about few more topics related to sets such as, operations on sets such as union, intersection, difference and complement. Now we will learn how to describe sets using different language and by using Venn diagram. We will also learn about power set, some more operations on sets, verify some properties with respect to union and intersection and verify De Morgan's laws.

5.1 Set Notations

5.1.1 Set Builder Form

Set builder notation is another form to represent a set. In set builder form, instead of listing the elements, a common rule, characteristic or formula is written in the shortest possible form that describes the elements of the set.

For example,

"N = The set of natural numbers" is the descriptive form, whereas

$N = \{1, 2, 3, 4, 5, \dots\}$ is the tabular form.

The same set can be written in set builder form as:

$N = \{x \mid x \text{ is a natural number}\}$ or

$N = \{x \mid x \in \mathbb{N}\}$

We can read it as, "set N is a set of all elements x, such that x is a natural number".

Here, the elements of the set are denoted by the letter x followed by a vertical line "|" which is read as "such that". After the "such that" sign, the specific characteristic of the elements is written. Also the whole description is to be enclosed within the curly brackets "{ }". We can label it as:



Teaching Point: Write some examples of tabular and descriptive forms of sets on the board and ask the students to write them as set builder form.

Previous Knowledge Check

- Define equal and equivalent sets and give example.
- Write a set and write one subset and one super set of that set.
- Write an example of disjoint and overlapping set.

Quick Check

Are the following sets in descriptive form or tabular form?

A = {10, 20, 30, 40}

B = {0}

D = {1}

Note it down

| means "such that"
 \in means "belongs to"
 \notin means "does not belong to"
 \geq means "greater than or equal to"
 \leq means "less than or equal to"
 \wedge means "and"
 \vee means "or"

$$N = \{x \mid x \text{ is a natural number}\}$$

Condition

All elements Such that

Some examples are given below.

a) $Y = \{x \mid x \text{ is an integer, } x > -4\}$, or

$Y = \{x \mid x \in Y \wedge x > -4\}$

We read it as: "Y is the set of all elements x such that x is an integer and it is greater than -4". We can write it in tabular form as: $Y = \{-3, -2, -1, 0, +1, +2, +3, \dots\}$.

b) $T = \{x \mid x \text{ is an odd number, } 5 \leq x \leq 11\}$ or

$T = \{x \mid x \in T \wedge 5 \leq x \leq 11\}$

We read it as: "T is the set of all elements x such that x is an odd number which is greater than or equal to 5 and less than or equal to 11.

Thus, we write it in tabular form as: $T = \{5, 7, 9, 11\}$

c) $F = \{x \mid x = \text{vowels in the English alphabet}\}$

Thus, we write it in tabular form as: $F = \{a, e, i, o, u\}$

Quick Check

Express the following sets in set builder notation:

i. $\{3, 5, 7, 9, \dots, 21\}$ ii. $\{3, 6, 9, 12, \dots, 30\}$

iii. $\{2, 3, 5, 7, 11, 17\}$ iv. $\{a, e, i, o, u\}$

v. $\{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$

$$N = \{x \mid x \in \mathbb{N}\}$$

Condition

All elements Such that

Note it down

A set can be written by using ":" instead of "|". For example, set builder notation of natural numbers is: $N = \{x : x \text{ is a natural number}\}$.

Quick Check

Think of a set of your own choice and then write it in descriptive, tabular and set builder forms.

Quick Check

Which of the following sets is in set builder form?

U = {1, 2, 3, 4} V = $\{x \mid x \in \mathbb{Z} \wedge x > 3\}$

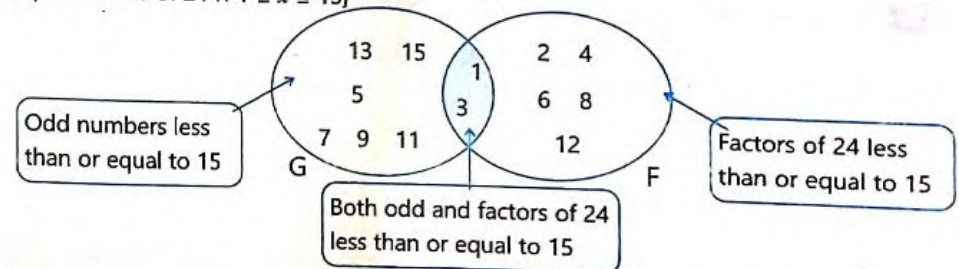
F = the set of seven colours of the rainbow

5.1.2 Set Notation through Venn Diagram

Let's represent one set contains odd numbers from 1 to 15 and factors of 24 between 1 to 15.

If $G = \{x \mid x \in \mathbb{O} \wedge 1 \leq x \leq 15\}$ and

$F = \{x \mid x \in \text{factors of 24} \wedge 1 \leq x \leq 15\}$



Similarly, we can represent different sets in Venn diagram.

Exercise 5.1

- 1 Write five sets of your own choice in the following three ways:
 - i. Descriptive form
 - ii. Tabular form
 - iii. Set builder notation
- 2 Express the following sets in tabular form and descriptive form:
 - i. $\{x|x \in N \wedge x \leq 9\}$
 - ii. $\{x|x \in P \wedge x < 20\}$
 - iii. $\{x|x \in E \wedge 2 < x < 15\}$
 - iv. $\{x|x \in C \wedge 1 < x < 15\}$
 - v. $\{x|x \in W \wedge x \leq 10\}$
 - vi. $\{x|x \in Z \wedge -10 \leq x < 10\}$
- 3 Write the following in set builder form.
 - a) E = Set of even numbers
 - b) T = {101, 103, 105, 107, 109}
 - c) A = {0, 1, 2, 3, 4, 5, ...}
 - d) V = {a, e, i, o, u}
 - e) D = {1, 4, 9, 16, 25}
 - f) F = Set of natural numbers greater than or equal to 11
 - g) G = {1, 3, 5, 7, 9}
 - h) B = {22, 33, 44, 55, ...}
 - i) E = {April, June, September, November}
 - j) The set of common factors of 18, 30, 42.
 - k) The set of numbers which are divisible by both 3 and 4.
 - l) The set of factors of 81 between 5 and 30.
 - m) The set of prime numbers between 11 and 13.
 - n) The set of even numbers less than or equal to 100.
 - o) The set of consonants in English alphabets.
 - p) The set of whole numbers between 20 and 50.
 - q) The set of factors of 24.

- 4 Represent the following pair of sets using Venn diagram.

- a) $A = \{x | x \in E \wedge 1 \leq x \leq 10\}$
 $B = \{x | x \in N \wedge 1 \leq x \leq 10\}$
- b) $F = \{x | x \in Z \wedge -10 \leq x \leq -5\}$
 $G = \{x | x \in W \wedge 1 \leq x \leq 15\}$
- c) $U = \{x | x \in P \wedge 1 \leq x \leq 20\}$
 $V = \{x | x \in Z' \wedge 1 \leq x \leq 20\}$

5.2 Some Common Sets

We have studied sets, types of sets and operations on sets in the previous classes. We have also learnt to represent basic operations on sets with the help of Venn diagrams. Let us define some common sets.

Sets	Description
i. $N = \{1, 2, 3, 4, 5, \dots\}$	Set of natural numbers
ii. $W = \{0, 1, 2, 3, 4, 5, \dots\}$	Set of whole numbers
iii. $Z = \{0, \pm 1, \pm 2, \pm 3, \dots\}$	Set of integers
iv. $Z^+ = \{+1, +2, +3, +4, +5, \dots\}$	Set of positive integers
v. $Z^- = \{-1, -2, -3, -4, -5, \dots\}$	Set of negative integers
vi. $E = \{0, \pm 2, \pm 4, \pm 6, \pm 8, \pm 10, \dots\}$	Set of even numbers
vii. $O = \{\pm 1, \pm 3, \pm 5, \pm 7, \pm 9, \dots\}$	Set of odd numbers
viii. $P = \{2, 3, 5, 7, 11, \dots\}$	Set of prime numbers

All the above sets are infinite sets. The set of integers Z can be separated into three sets;

- i. $Z^+ = \{+1, +2, +3, +4, +5, \dots\}$
- ii. $Z^- = \{-1, -2, -3, -4, -5, \dots\}$
- iii. Singleton set zero = {0}

Another set is the set of rational numbers.

$$Q = \left\{ \frac{p}{q} \mid p \in Z, q \in Z \wedge q \neq 0 \right\}$$

Can you list some elements of this set?

5.2.1 Subset of a Set

We know that a set A is called a subset of B if every element of A is also an element of B . We write $A \subseteq B$. If A is not a subset of a set B , we write $A \not\subseteq B$.

Example 1:

Find at least three subsets of $B = \{2, 3, 5, 7, 9\}$

Solution:

$A = \{2, 3, 5\}$ is a subset of B

$F = \{3, 7, 9\}$ is also a subset of B

Another subset of B is $D = \{2, 5, 7, 9\}$.

Subsets can be divided into proper and improper subsets.



Explain the set and subset of a set by giving different examples. Instruct each of them to write a set of your own choice and write two subsets of set.

Note it down

The numbers which can be written in the form of $\frac{p}{q}$ where p and q are integers and $q \neq 0$ are known as rational numbers, otherwise, they are called irrational numbers.

Let us now find all possible subsets of a set.

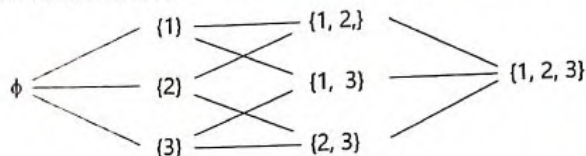
Example 2:

List all possible subsets of the set $X = \{1, 2, 3\}$

Solution:

- ϕ is the subset of the set X .
- Singleton subsets of X are, $\{1\}, \{2\}, \{3\}$
- Subset of X having two elements are;
 $\{1, 2\}, \{2, 3\}, \{1, 3\}$
- Since each set is an improper subset of itself, therefore the set itself i.e. $\{1, 2, 3\}$ is also a subset of itself.

Now count the subsets of X . These are 8. We can organise the subsets as follows;



Observe that the set X has 3 elements and the number of subsets of X is $2^3 = 8$.

Example 3:

Find all the possible subsets of the set $B = \{2, 4, 6, 8\}$

Solution:

All possible subsets of the set B are listed as;

$\phi, \{2\}, \{4\}, \{6\}, \{8\}, \{2, 4\}, \{2, 6\}, \{2, 8\}, \{4, 6\}, \{4, 8\}, \{6, 8\}, \{2, 4, 6\}, \{2, 4, 8\}, \{2, 6, 8\}, \{4, 6, 8\}, \{2, 4, 6, 8\}$.

These are 16. Observe that the set B has 4 elements and $2^4 = 16$

From the above two examples; we see that the set having

3 elements has $2^3 = 8$ subsets and the set having

4 elements has $2^4 = 16$ subsets.

In general;

A set having n elements has 2^n possible subsets.

Note it down

- An infinite set has an infinite number of subsets.
- ϕ is the subset of every set.

Quick Check

Find all the subsets of the following sets;

- $A = \{a, b, c\}$
- $B = \{2, 3, 5, 7\}$

5.2.2 Power Set

The set that consists of all the subsets of a set is called the power set of that set. For any set X ; the power set of X is written as $P(X)$.

Example 1:

Find the power set of the set $Y = \{0, 1\}$

Solution:

$Y = \{0, 1\}$

Power set of $Y = P(Y) = \{\phi, \{0\}, \{1\}, \{0, 1\}\}$

Note:

Number of elements of the set $Y = 2$

Number of possible subsets of $Y = 2^2 = 4$

Example 2:

Find the power set of the set $A = \{a, e, i\}$

Solution:

$P(A) = \{\phi, \{a\}, \{e\}, \{i\}, \{a, e\}, \{a, i\}, \{e, i\}, \{a, e, i\}\}$

Observe that the elements of power set are all subsets themselves.

Example 3:

Find the power set of the set $X = \{a, b, c, d\}$

Solution:

The power set of X is;

$P(X) = \{\phi, \{a\}, \{b\}, \{c\}, \{d\}, \{a, b\}, \{b, c\}, \{b, d\}, \{a, c\}, \{a, d\}, \{c, d\}, \{a, b, c\}, \{a, b, d\}, \{a, c, d\}, \{b, c, d\}, \{a, b, c, d\}\}$

Note it down

If A is an infinite set then $P(A)$ is also infinite.

Quick Check

Find the power set of;
 $M = \{ (r), (s), (t) \}$

Note it down

Symbol	Meaning
\in	belongs
\notin	does not belong
\subset	proper subset
\supset	improper subset
$\not\subset$	not a subset
\supseteq	equivalent to
\supset	superset

Quick Check

- Find the power set of the null set using the rule. Is it a null set?
- Find the power set of;
 - $\{0, 1, 2\}$
 - $\{1, 3, 5\}$
 - $J = \{y | y \in P \wedge y < 9\}$



Explain the power set of set to students by solving examples on the board and tell them the formula to find the number of possible subsets of set. Ask them to write a set that have 3 to 4 elements and then find the power set of that set.

Exercise 5.2

1 Write T for true statement and F for false statement.

- ϕ is the subset of every set. -----
- If a set A has n elements then $P(A)$ has 2^{n-1} elements. -----
- If $X \in X$ then $X \in P(X)$ -----
- If $A \subseteq B$, then $P(A) \subseteq P(B)$ -----
- The set $X = \{-1, 0, 1, 2\}$ has 12 possible subsets. -----

2 How many subsets can be made out of the following sets? Find all the subsets of each of the following sets.

- $A = \{2\}$
- $F = \{4, 8\}$
- $M = \{-1, 0, 1, 2\}$
- $S = \{ \}$
- $B = \{5, 10, 15\}$
- $X = \{2, 3, 5, 7, 11\}$
- $K = \{x | x \in P \wedge 5 \leq x \leq 13\}$
- $T = \{x | x \in Z \wedge -2 < x < 2\}$

3 Find the power sets of the following sets;

- $X = \{x, y, z\}$
- $A = \{+, -, \times, \div\}$
- $L = \{i, s, l, a, m\}$
- $B = \{0\}$
- $T = \{-3, -2\}$

5.3 Properties Involving Operations on Sets

We have learnt about the commutative, associative and distributive properties of whole numbers with respect to addition and multiplication. While manipulating sets the above properties also hold for the operations on union and intersection. Let us learn.

5.3.1 Commutative Property of Sets (with respect to Union)

For any two sets A and B, $A \cup B = B \cup A$

Example 1:

For the sets $S = \{1, 3, 5, 7, 9\}$ and $T = \{5, 6, 7, 8, 9\}$.

Prove that $S \cup T = T \cup S$

Solution:

$$S \cup T = \{1, 3, 5, 7, 9\} \cup \{5, 6, 7, 8, 9\}$$

$$S \cup T = \{1, 3, 5, 6, 7, 8, 9\} \dots \dots \dots (i)$$

and

$$T \cup S = \{5, 6, 7, 8, 9\} \cup \{1, 3, 5, 7, 9\}$$

$$T \cup S = \{1, 3, 5, 6, 7, 8, 9\} \dots \dots \dots (ii)$$

Equations (i) and (ii) imply $S \cup T = T \cup S$

5.3.2 Commutative Property of Sets (with respect to Intersection)

For any two sets A and B, $A \cap B = B \cap A$

Example 1:

For the sets $F = \{2, 3, 4, 5, 6\}$ and

$$G = \{2, 3, 5, 7, 11\}$$

Prove that $F \cap G = G \cap F$

Solution:

$$F \cap G = \{2, 3, 4, 5, 6\} \cap \{2, 3, 5, 7, 11\}$$

$$= \{2, 3, 5\} \dots \dots \dots (i)$$

and

$$G \cap F = \{2, 3, 5, 7, 11\} \cap \{2, 3, 4, 5, 6\}$$

$$= \{2, 3, 5\} \dots \dots \dots (ii)$$

Equations (i) and (ii) imply $F \cap G = G \cap F$.

5.3.3 Associative Property of Sets (with respect to Union)

For any three sets A, B and C, $A \cup (B \cap C) = (A \cup B) \cap C$

Example 1:

If $A = \{2, 3, 5, 7, 9\}$, $B = \{1, 2, 3, 4, 5\}$ and $F = \{2, 4, 6, 8\}$.

Prove the associative property with respect to union.

Solution:

We have to prove that $A \cup (B \cap F) = (A \cup B) \cap F$

$$\text{L.H.S} = A \cup (B \cap F)$$

$$B \cap F = \{1, 2, 3, 4, 5\} \cap \{2, 4, 6, 8\}$$

$$= \{1, 2, 3, 4, 5, 6, 8\}$$

So,

$$A \cup (B \cap F) = \{2, 3, 5, 7, 9\} \cup \{1, 2, 3, 4, 5, 6, 8\}$$

$$= \{1, 2, 3, 4, 5, 6, 7, 8, 9\} \dots \dots \dots (i)$$

$$\text{R.H.S} = (A \cup B) \cap F$$

$$A \cup B = \{2, 3, 5, 7, 9\} \cup \{1, 2, 3, 4, 5\}$$

$$= \{1, 2, 3, 4, 5, 7, 9\}$$

$$(A \cup B) \cap F = \{1, 2, 3, 4, 5, 7, 9\} \cap \{2, 4, 6, 8\}$$

$$= \{1, 2, 3, 4, 5, 6, 7, 8, 9\} \dots \dots \dots (ii)$$

Equation (i) and (ii) imply,

$$A \cup (B \cap F) = (A \cup B) \cap F$$



Make two groups of students. Instruct each group to write two sets of your own choice and then instruct first group to find the commutative property of sets with respect to intersection and the other group find the commutative property of set with respect to union.

5.3.4 Associative Property of Sets (With respect to Intersection)

For any three sets A, B and C, $A \cap (B \cap C) = (A \cap B) \cap C$

Example 1:
For the sets: $F = \{1, 5, 9, 13, 17\}$, $G = \{3, 6, 9, 12, 15\}$ and $H = \{2, 4, 6, 8, 10\}$

Prove that

$$F \cap (G \cap H) = (F \cap G) \cap H$$

Solution:

$$L.H.S = F \cap (G \cap H)$$

$$G \cap H = \{3, 6, 9, 12, 15\} \cap \{2, 4, 6, 8, 10\}$$

$$= \{6\}$$

$$F \cap (G \cap H) = \{1, 5, 9, 13, 17\} \cap \{6\}$$

$$= \{ \} \dots \dots \dots (i)$$

and

$$R.H.S = (F \cap G) \cap H$$

$$F \cap G = \{1, 5, 9, 13, 17\} \cap \{3, 6, 9, 12, 15\}$$

$$= \{9\}$$

$$(F \cap G) \cap H = \{9\} \cap \{2, 4, 6, 8, 10\}$$

$$= \{ \} \dots \dots \dots (ii)$$

Equation (i) and (ii) imply,

$$F \cap (G \cap H) = (F \cap G) \cap H$$

5.3.5 Distributive Property of Union of Sets Over Intersection

For any three sets A, B, and C, $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$

Example 1:

For the sets; $X = \{0, 3, 6, 9, 12, 15\}$, $Y = \{-5, -3, -1, 0, 1, 3, 5\}$ and $T = \{2, 4, 6, \dots, 16\}$

Prove that;

$$X \cup (Y \cap T) = (X \cup Y) \cap (X \cup T)$$

Solution:

$$L.H.S = X \cup (Y \cap T)$$

$$Y \cap T = \{-5, -3, -1, 0, 1, 3, 5\} \cap \{2, 4, 6, \dots, 16\}$$

$$= \{ \}$$

So,

$$X \cup (Y \cap T) = \{0, 3, 6, 9, 12, 15\} \cup \{ \}$$

$$= \{0, 3, 6, 9, 12, 15\} \dots \dots \dots (i)$$



Make two groups of students. Instruct each group to write three sets of your own choice and then instruct first group to find the associative property of sets with respect to intersection and the other group find the associative property of set with respect to union.

and

$$R.H.S = (X \cup Y) \cap (X \cup T)$$

$$X \cup Y = \{0, 3, 6, 9, 12, 15\} \cup \{-5, -3, -1, 0, 1, 3, 5\}$$

$$= \{-5, -3, -1, 0, 1, 3, 5, 6, 9, 12, 15\}$$

$$X \cup T = \{0, 3, 6, 9, 12, 15\} \cup \{2, 4, 6, \dots, 16\}$$

$$= \{0, 2, 3, 4, 6, 8, 9, 10, 12, 14, 15, 16\}$$

$$(X \cup Y) \cap (X \cup T) = \{-5, -3, -1, 0, 1, 3, 5, 6, 9, 12, 15\} \cap \{0, 2, 3, 4, 6, 8, 9, 10, 12, 14, 15, 16\}$$

$$= \{0, 3, 6, 9, 12, 15\}$$

(ii)

Equation (i) and (ii) imply,

$$X \cup (Y \cap T) = (X \cup Y) \cap (X \cup T)$$

5.3.6 Distributive Property of Intersection of Sets Over Union

For any three sets A, B, and C, $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$

Example 1:

For the sets; $O =$ Set of odd Integers, $W =$ Set of whole numbers, $P =$ Set of prime numbers

Prove that; $O \cap (W \cup P) = (O \cap W) \cup (O \cap P)$

Solution:

First we write the given sets in tabular form as;

$$O = \{\pm 1, \pm 3, \pm 5, \dots\}$$

$$W = \{0, 1, 2, 3, 4, \dots\}$$

$$P = \{2, 3, 5, 7, 11, \dots\}$$

Now; $L.H.S = O \cap (W \cup P)$

$$W \cup P = \{0, 1, 2, 3, 4, \dots\} \cup \{2, 3, 5, 7, 11, \dots\}$$

$$= \{0, 1, 2, 3, 4, \dots\}$$

$$O \cap (W \cup P) = \{\pm 1, \pm 3, \pm 5, \dots\} \cap \{0, 1, 2, 3, 4, \dots\}$$

$$O \cap (W \cup P) = \{1, 3, 5, 7, \dots\} \dots (i)$$

and

$$R.H.S = (O \cap W) \cup (O \cap P)$$

$$O \cap W = \{\pm 1, \pm 3, \pm 5, \dots\} \cap \{0, 1, 2, 3, 4, \dots\}$$

$$= \{1, 3, 5, 7, \dots\}$$

$$O \cap P = \{\pm 1, \pm 3, \pm 5, \dots\} \cap \{2, 3, 5, 7, 11, \dots\}$$

$$= \{3, 5, 7, 11, 13, 19, \dots\}$$

$$(O \cap W) \cup (O \cap P) = \{1, 3, 5, \dots\} \cup \{3, 5, 7, 11, 13, \dots\}$$

$$= \{1, 3, 5, 7, 9, \dots\} \dots (ii)$$

Equations (i) and (ii) imply

$$O \cap (W \cup P) = (O \cap W) \cup (O \cap P)$$



Make two groups of students. Instruct each group to write three sets of your own choice and then instruct first group to find the distributive property of union of sets over intersection of sets and the other group find the distributive property of intersection over union.

5.3.7 De-Morgan's Law

It has two parts: i) $(A \cup B)^c = A^c \cap B^c$

Example 1:

For $U = \{1, 2, 3, \dots, 12\}$

$A = \{7, 8, 9, 10, 11, 12\}$

$B = \{3, 5, 7, 9, 11\}$

$(A \cup B)^c$ and $A^c \cap B^c$ are shown below:

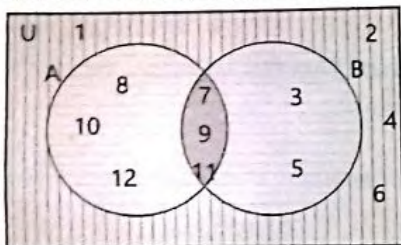


Fig. 1.15 $(A \cup B)^c$

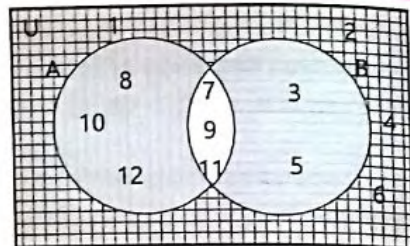


Fig. 1.16 $A^c \cap B^c$

The shaded region in above diagrams show that $(A \cup B)^c = A^c \cap B^c$.

ii) $(A \cap B)^c = A^c \cup B^c$

Example 2:

For the above sets A, B, C this law is demonstrated through the following figures:

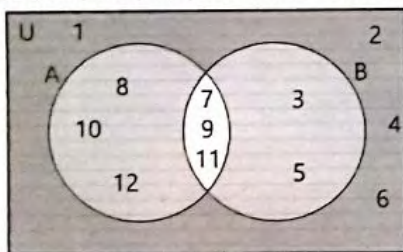


Fig. 1.17 $(A \cap B)^c$

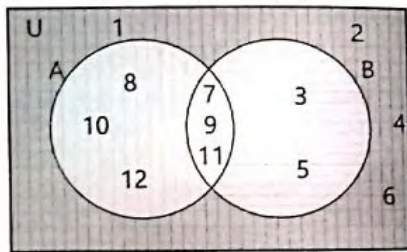


Fig. 1.18 $A^c \cup B^c$

The shaded region in both figures show that $(A \cap B)^c = A^c \cup B^c$.

Practice Exercise

If $A = \{x | x \in \mathbb{N} \wedge x < 6\}$, $B = \{x | x \in \mathbb{W} \wedge 3 \leq x \leq 5\}$, $F = \{x | x \in \mathbb{Z} \wedge -1 \leq x < 4\}$

Verify the following using the Venn diagrams.

- $A \cap (B \cap F) = (A \cap B) \cap F$
- $A \cup (B \cap F) = (A \cup B) \cap (A \cup F)$
- $A \cap (B \cup F) = (A \cap B) \cup (A \cap F)$

Math History

Augustus De-Morgan (1806 - 1871) a British Mathematician formulated the De-Morgan's Laws.

5.3.8 Real life Situation involving sets

There are many situations in our daily life where the concepts of sets can be used. Let's consider the example.

Example 1:

The data of 225 people who bought various ice cream flavors is given below.

People who bought only Vanilla	People who bought only strawberry	People who bought both
76	92	32

Create a Venn diagram to illustrate this information.

Solution:

Let's denote each set first.

A = People who bought only vanilla flavour

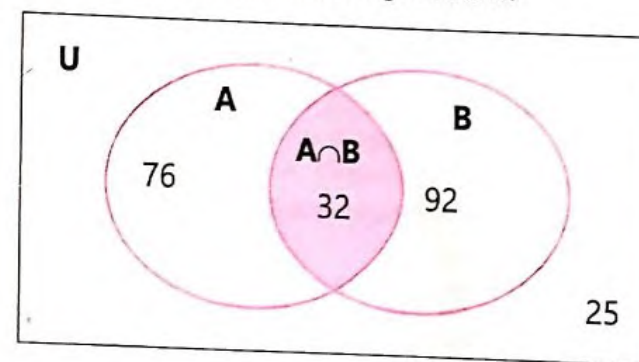
B = People who bought only strawberry flavour

Also, $A \cap B = 32$

Then finally, subtract the sum of these three sets from the total 225 people that will be shown outside the circles.

The quantity outside the circles = $225 - 200 = 25$

So, the Venn diagram for the above information is given below.



Teaching Point: Ask students to tell the uses of set and their operations in our daily life. Get students into small groups and ask them to make real word problem in which sets are involved. Ask them to share the problems with their class-fellows and teacher.

- 1 Given $A = \{1, 4, 7\}$, $B = \{5, 7, 11\}$, $D = \{3, 7, 10\}$

Use Venn diagrams to represent:

- i) $A \cup B \cap D$ ii) $A \cap B \cap D$ iii) $A \cup (B \cap D)$ iv) $A \cap (B \cup D)$

- 2 For the sets;

$A = \{1, 3, 5, 7, 9\}$, $B = \{1, 2, 3, 4, 5\}$ and $C = \{5, 6, 7, 8, 9\}$

Use Venn diagrams to verify:

- i) $A \cup (B \cap D) = (A \cup B) \cap D$ ii) $A \cap (B \cap D) = (A \cap B) \cap D$
 iii) $A \cup (B \cap D) = (A \cup B) \cap (A \cup D)$ iv) $A \cap (B \cup D) = (A \cap B) \cup (A \cap D)$

- 3 For the following sets verify the above mentioned properties with the help of Venn diagrams:

$X = \{0, 2, 4, 6, 8, 10\}$, $Y = \{4, 8, 12, 16, 20\}$, $T = \{6, 12, 18, 24, 30\}$

- 4 Let $A = \{1, 2\}$, $B = \{2, 3\}$, $D = \{3, 4\}$. Use Venn diagrams to represent:

- i) $A \cup B \cup D$ ii) $A \cap B \cap D$
 iii) $A \cup (B \cap D)$ iv) $A \cap (B \cup D)$

- 5 For the sets:

$A = \{1, 3, 5, 7, 9, 11, 13, 15\}$, $B = \{1, 2, 3, 4, 5, 6, 7\}$ and $D = \{7, 8, 9, 10, 11, 12\}$

Use Venn diagrams to verify:

- i) $A \cup (B \cap D) = (A \cup B) \cap D$ ii) $A \cap (B \cap D) = (A \cap B) \cap D$
 iii) $A \cup (B \cap D) = (A \cup B) \cap (A \cup D)$ iv) $A \cap (B \cup D) = (A \cap B) \cup (A \cap D)$

- 6 Find the complement of the following sets with respect to the universal set.

$U = \{-10, -9, -8, \dots, 0, \dots, +9, +10\}$

- i. $A = \{0, 1, 2, 3, 4, 5\}$ ii. $B = \{-10, -8, -6, \dots, +8, +10\}$
 iii. $X = \{1, 2, 3, 4, 5, 7, 8, 9\}$ iv. $Y = \{-5, -4, -3, -2, -1, 0, +1, +2, +3, +4, +5\}$
 v. $M = \{-1, 0, 1\}$ vi. $T = \{-10, -9, -8, \dots, -1, +1, +2, \dots, +10\}$

- 7 If $A = \{0, 4, 8, 12, 16, 20\}$, $B = \{0, 2, 4, 6, 8, 10\}$ and

$D = \{0, 2, 4, 6, 8, 10, 12, \dots, 20\}$. Verify the following:

- i. $A \cup B = B \cup A$ ii. $A \cap D = D \cap A$ iii. $A \cup (B \cap D) = (A \cup B) \cap D$
 iv. $B \cap (D \cap A) = (B \cap D) \cap A$ v. $B \cup D = D \cup B$ vi. $A - B \neq B - A$

- 8 State and verify the De Morgan's laws for the following sets:

- i. $U = \{1, 2, 3, 4, 5, \dots, 30\}$, $A = \{1, 3, 5, 7, \dots, 25\}$ and $B = \{2, 4, 6, 8, \dots, 26\}$
 ii. $U = Z$ (Set of integers), $A = E$ (Set of even integers) and $B = Z$ (Set of negative integers)

- 9 Verify:

- i. $X \cup (Y \cap T) = (X \cup Y) \cap (X \cup T)$
 ii. $X \cap (Y \cup T) = (X \cap Y) \cup (X \cap T)$

For $X = \{-9, -7, -5, -3, -1, 0, 1, 3\}$

$Y = \{-5, -3, -1, 0, 1, 3, 5\}$

and $T = \{-10, -5, 0, +5, +10\}$

- 10 State and verify the associative and distributive laws for the sets:

- i. $E^+ = \{2, 4, 6, 8, \dots\}$
 $O^+ = \{1, 3, 5, 7, \dots\}$
 $P = \{2, 3, 5, 7, 11, \dots\}$
 ii. $M = \{x \mid x \in W \wedge 1 \leq x < 10\}$
 $N = \{x \mid x \in P \wedge 2 < x \leq 13\}$
 $T = \{x \mid x \in N \wedge 1 \leq x \leq 8\}$

- 11 On a certain day 80 customers visited a stationary shop. 30 customer bought pencils, 25 customer bought erasers and 15 bought both pencils and erasers. Remaining customer bought other stationary items. Show this information using Venn diagram.

- 12 In a math book there are total 15 units, 6 units are based on algebra and 5 units are based on numbers and operation. 2 units are based on both algebra and numbers operation. Other units are based on other strands. Show this information through Venn diagram.

- 13 The data of 160 students who like math, science, and other subjects.

Students who like Maths	Students who like Science	Students who like Maths and science both	Students who like subjects other than math and science
70	50	20	20

Create a Venn diagram to illustrate this information.

Exercise 5.3

- 1 Given $A = \{1, 4, 7\}$, $B = \{5, 7, 11\}$, $D = \{3, 7, 10\}$

Use Venn diagrams to represent:

- i) $A \cup B \cap D$ ii) $A \cap B \cap D$ iii) $A \cup (B \cap D)$ iv) $A \cap (B \cup D)$

- 2 For the sets:

$A = \{1, 3, 5, 7, 9\}$, $B = \{1, 2, 3, 4, 5\}$ and $C = \{5, 6, 7, 8, 9\}$

Use Venn diagrams to verify:

- i) $A \cup (B \cap D) = (A \cup B) \cap D$ ii) $A \cap (B \cap D) = (A \cap B) \cap D$
 iii) $A \cup (B \cap D) = (A \cup B) \cap (A \cup D)$ iv) $A \cap (B \cup D) = (A \cap B) \cup (A \cap D)$

- 3 For the following sets verify the above mentioned properties with the help of Venn diagrams:

$X = \{0, 2, 4, 6, 8, 10\}$, $Y = \{4, 8, 12, 16, 20\}$, $T = \{6, 12, 18, 24, 30\}$

- 4 Let $A = \{1, 2\}$, $B = \{2, 3\}$, $D = \{3, 4\}$. Use Venn diagrams to represent:

- i) $A \cup B \cup D$ ii) $A \cap B \cap D$
 iii) $A \cup (B \cap D)$ iv) $A \cap (B \cup D)$

- 5 For the sets:

$A = \{1, 3, 5, 7, 9, 11, 13, 15\}$, $B = \{1, 2, 3, 4, 5, 6, 7\}$ and $D = \{7, 8, 9, 10, 11, 12\}$

Use Venn diagrams to verify:

- i) $A \cup (B \cap D) = (A \cup B) \cap D$ ii) $A \cap (B \cap D) = (A \cap B) \cap D$
 iii) $A \cup (B \cap D) = (A \cup B) \cap (A \cup D)$ iv) $A \cap (B \cup D) = (A \cap B) \cup (A \cap D)$

- 6 Find the complement of the following sets with respect to the universal set.

$U = \{-10, -9, -8, \dots, 0, \dots, +9, +10\}$

- i. $A = \{0, 1, 2, 3, 4, 5\}$ ii. $B = \{-10, -8, -6, \dots, +8, +10\}$
 iii. $X = \{1, 2, 3, 4, 5, 7, 8, 9\}$ iv. $Y = \{-5, -4, -3, -2, -1, 0, +1, +2, +3, +4, +5\}$
 v. $M = \{-1, 0, 1\}$ vi. $T = \{-10, -9, -8, \dots, -1, +1, +2, \dots, +10\}$

- 7 If $A = \{0, 4, 8, 12, 16, 20\}$, $B = \{0, 2, 4, 6, 8, 10\}$ and $D = \{0, 2, 4, 6, 8, 10, 12, \dots, 20\}$. Verify the following:

- i. $A \cup B = B \cup A$ ii. $A \cap D = D \cap A$ iii. $A \cup (B \cap D) = (A \cup B) \cap D$
 iv. $B \cap (D \cap A) = (B \cap D) \cap A$ v. $B \cup D = D \cup B$ vi. $A - B \neq B - A$

- 8 State and verify the De Morgan's laws for the following sets:

- i. $U = \{1, 2, 3, 4, 5, \dots, 30\}$, $A = \{1, 3, 5, 7, \dots, 25\}$ and $B = \{2, 4, 6, 8, \dots, 26\}$
 ii. $U = Z$ (Set of integers), $A = E$ (Set of even integers) and $B = Z$ (Set of negative integers)

- 9 Verify:

- i. $X \cup (Y \cap T) = (X \cup Y) \cap (X \cup T)$
 ii. $X \cap (Y \cup T) = (X \cap Y) \cup (X \cap T)$
 For $X = \{-9, -7, -5, -3, -1, 0, 1, 3\}$
 $Y = \{-5, -3, -1, 0, 1, 3, 5\}$
 and $T = \{-10, -5, 0, +5, +10\}$

- 10 State and verify the associative and distributive laws for the sets:

- i. $E^+ = \{2, 4, 6, 8, \dots\}$
 $O^+ = \{1, 3, 5, 7, \dots\}$
 $P = \{2, 3, 5, 7, 11, \dots\}$
 ii. $M = \{x \mid x \in W \wedge 1 \leq x < 10\}$
 $N = \{x \mid x \in P \wedge 2 < x \leq 13\}$
 $T = \{x \mid x \in N \wedge 1 \leq x \leq 8\}$

- 11 On a certain day 80 customers visited a stationary shop. 30 customer bought pencils, 25 customer bought erasers and 15 bought both pencils and erasers. Remaining customer bought other stationary items. Show this information using Venn diagram.

- 12 In a math book there are total 15 units, 6 units are based on algebra and 5 units are based on numbers and operation. 2 units are based on both algebra and numbers operation. Other units are based on other strands. Show this information through Venn diagram.

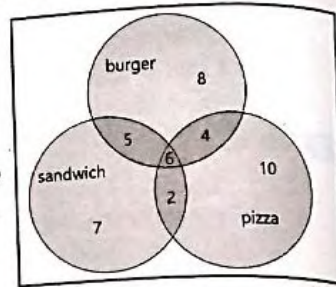
- 13 The data of 160 students who like math, science, and other subjects.

Students who like Maths	Students who like Science	Students who like Maths and science both	Students who like subjects other than math and science
70	50	20	20

Create a Venn diagram to illustrate this information.

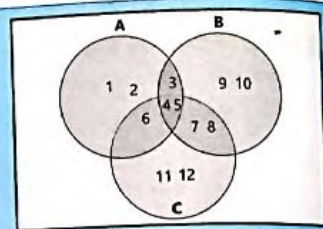
- 14 The following venn diagram shows the data of number of students and their favourite food. Look at it and answer the questions:

- How many students like only pizza?
- How many students like only sandwich?
- How many students like both pizza and burger?
- How many students like both sandwich and pizza?
- How many students like only pizza and sandwich?
- How many students like all three types of food?



Think Higher

Observe the given Venn diagram and write the information given below:
Write the elements of sets A, B and C.
Write sets of $A \cap B$, $B \cap C$, $A \cap C$ and $A \cap B \cap C$.



Summary

- In set builder form, instead of listing the elements, a common rule, characteristic or formula is written in the shortest possible form that describes the elements of the set.
- The set that consists of all the subsets of a set is called the power set of that set. For any set X, the power set of X is written as $P(X)$.

Vocabulary

- Set builder notation
- Tabular form
- Descriptive form
- Commutative property
- Associative property
- Distributive property
- De-Morgan's Law

Review Exercise

1 Choose the correct option.

- In _____ form of set characteristics of numbers or elements of a particular set are described.
i) descriptive ii) power iii) tabular iv) set builder

- The set of days of week is the form of set:
i) descriptive ii) tabular iii) set builder iv) Venn diagram
- If a set has three elements then the number of subsets of that set is:
i) 6 ii) 9 iii) 12 iv) 3
- The power set of the set $A = \{3, 4\}$ is:
i) $\{\emptyset, \{3\}, \{4\}, \{3, 4\}\}$ ii) $\{\{3\}, \{4\}, \{3, 4\}\}$
iii) $\{\emptyset, \{3\}, \{4\}, \{3, 3\}, \{3, 4\}, \{4, 4\}\}$ iv) $\{\emptyset, \{3\}, \{4\}, \{3, 4\}, \{4, 3\}\}$
- _____ is not the subset of prime numbers up to 10.
i) $\{5\}$ ii) $\{3\}$ iii) $\{2\}$ iv) $\{11\}$
- The tabular form of the set $A = \{x | x \in \mathbb{Z} \wedge x > -4\}$ is:
i) $\{-3, -2, -1, 0, 1, 2, 3, \dots\}$ ii) $\{\{3\}, \{4\}, \{3, 4\}\}$
iii) $\{\emptyset, \{3\}, \{4\}, \{3, 3\}, \{3, 4\}, \{4, 4\}\}$ iv) $\{\emptyset, \{3\}, \{4\}, \{3, 4\}, \{4, 3\}\}$
- The commutative property of sets with respect to intersection is:
i) $A \cup B = B \cup A$ ii) $A \cup B = A \cap B$
iii) $A \cap B = B \cup A$ iv) $A \cap B = B \cap A$
- If $U = \{1, 2, 3, 4, \dots, 10\}$ and $A = \{4, 6, 8\}$ then complement of A is:
i) $\{1, 2, 3, 5, 7, 9, 10\}$ ii) $\{3, 5, 7, 9\}$
iii) $\{1, 2, 3, \dots, 10\}$ iv) $\{1, 2, 3, 5, 6, \dots, 10\}$
- The De-Morgan's law is:
i) $(A \cap B)^c = A^c \cap B^c$ ii) $(A \cap B)^c = A^c \cup B^c$
iii) $(A \cap B)^c = A^c \cap B^c$ iv) $(A \cap B)^c = A^c \cap B^c$

2 Define the following:

- Tabular form
- descriptive form
- set builder notation
- union of sets

3 Write the given set in set builder notation.

- $\{2, 4, 6, 8, 10, \dots, 24\}$
- the set of prime numbers up to 50
- the set of natural numbers up to 20
- $\{5, 10, 15, 20, \dots, 50\}$

4 Write the given set in descriptive form.

- $\{x | x \in \mathbb{W} \wedge x < 20\}$
- $\{7, 9, 11, 13, \dots\}$
- $\{4, 8, 12, 16, 20\}$
- $\{y | y \text{ is a composite number, } 24 \geq y \leq 11\}$

5 Write the given sets into tabular form.

- a) The set of multiples of 9 greater than 12 and less than 100
b) $\{x \in \mathbb{Z} \mid -5 < x \leq 20\}$ c) Set of odd numbers up to 30

d) $\{x/x \in E\}$

6 Write the power sets of the given sets.

a) $\{1, 2, 3\}$

b) $\{5, 8, 9, 11\}$

c) $\{0\}$

d) $\{12, 13\}$

7 If $A = \{-4, -3, -2, \dots, 4\}$

$B = \{1, 2, 3, 4, \dots, 10\}$ and $U = \text{set of integers}$

Verify the De-Morgan's law and show it by using Venn diagram.

8 Verify the associative property of sets with respect to intersection if set are:

$M = \{2, 3, 5, 7, 11\}$ $T = \{1, 3, 5, 7, 9, 11\}$ and $S = \{2, 4, 6, 8, 10, 12\}$

9 Verify the commutative property of sets with respect to union if set are:

$X = \{1, 2, 3\}$ $Y = \{1, 2, 4, 5, 6, \dots, 10\}$

Show the commutative property through Venn diagram.

10 Verify distributive property of union of sets over intersection.

$G = \{4, 8, 12, 16, 20\}$

$D = \{1, 2, 3, 4, \dots, 11\}$

$F = \{-1, 0, 1, 2, 3, 4\}$

11 On a day during Ramazan 40 people bought different items from a store. 22 people bought dates and 8 people bought fruits and 6 bought both dates and fruits. Show this information through Venn diagram.

Math Project



Material Required:

- Required Material
- Two baskets
- Flash cards of different sets
- Flash cards of different operations on sets

Procedure:

- Make small groups of students.
- Put flash cards of sets in one basket and operation on set in the second basket.
- Instruct each group to select three cards from the set card basket and two cards from operations on sets.
- Instruct them apply the operations on that sets and then show that operations through Venn diagram.
- Then instruct them to share their findings with each other.
- The group with accurate working in short time is winner.

NOT FOR SALE

Unit 6

Sequences and Algebra

Student Learning Outcomes

After completing this unit, students will be able to:

- Differentiate between an arithmetic sequence and a geometric sequence.
- Find terms of an arithmetic sequence using:
 - term to term rule
 - position to term rule
- Construct the formula for the general term (nth term) of an arithmetic sequence.
- Solve real life problems involving number sequences and patterns.
- Recall the difference between:
 - open and close sentences
 - expression and equation
 - equation and inequality
- Recall the addition and subtraction of polynomials.
- Recall the multiplication of polynomials.
- Divide a polynomial of degree up to 3 by
 - a monomial
 - a binomial
- Simplify algebraic expressions involving addition, subtraction, multiplication and division.
- Recognise the following algebraic identities and use them to expand expressions:
 - $(a+b)^2 = a^2 + b^2 + 2ab$
 - $(a-b)^2 = a^2 + b^2 - 2ab$
 - $(a+b)(a-b) = a^2 - b^2$
- Apply algebraic identities to solve problems like $(103)^2$, $(1.03)^2$, $(99)^2$, 101×99 .
- Factorize the following types of expressions:
 - $ka + kb + kc$
 - $ac + ad + bc + bd$
 - $a^2 \pm 2ab + b^2$
 - $a^2 - b^2$
 - $a^3 \pm 2ab + b^2 - c^2$
- Manipulation of algebraic expressions
 - $(a+b)^3 = a^3 + 3a^2b + 3ab^2 + b^3$
 - $(a-b)^3 = a^3 - 3a^2b + 3ab^2 - b^3$



A car manufacturing company produced 800 cars in year one, 1250 cars in 2nd year, 1700 cars in 3rd year and so on. How many cars will it manufacture in 10th year?

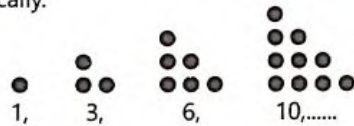
NOT FOR SALE

Introduction

In the previous classes we learnt about patterns, number patterns and algebraic expressions. Now in our daily life in many situations we will learn arithmetic and geometric sequences and will use them. We will also learn factorization of algebraic expressions.

6.1 Number Sequences

We are familiar with a variety of patterns. Each pattern follows a unique rule. Let us observe the following pattern. It is made by starting with a single dot, then adding two dots to the second row, three dots to the third row and so on. We can also list these triangular arrays using numbers numerically.



This list is known as a number sequence and the particular numbers are called triangular numbers.

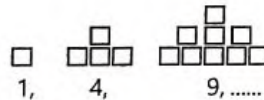
Can you describe the rule of this pattern?

Observe;

1, 3, 6, 10,
 $\rightarrow +2 \rightarrow +3 \rightarrow +4$

Begin with the number 1, the rule of the pattern is to add 2 to the first term to get the second term and then add 3 to the second term to get the third term, and so on.

Consider another sequence;



Extend the pattern to the next two terms. What is the rule of the sequence?

Observe that similar to the above sequence, each term is made by adding a row beneath the previous pattern.

We can restate the above sequence as;



explain students what is sequence. Write different sequence of numbers on the board and ask them to observe the sequence and tell the rule of this sequence and difference between two terms of sequence. Ask them use the same rule to make another sequence.

NOT FOR SALE

$\rightarrow +3 \rightarrow +5 \rightarrow +7$
 1, 4, 9, 16, ...

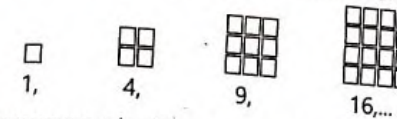
1, (1 + 3), (1 + 3 + 5), (1 + 3 + 5 + 7), ...

Can you describe the rule of this pattern now?

What will be the 7th term?

What will be the 10th term?

We can also present the pattern in another way, as;



The sequence can also be represented as squares of natural numbers.

$$1 = 1 \times 1 = 1^2$$

$$4 = 2 \times 2 = 2^2$$

$$9 = 3 \times 3 = 3^2$$

$$16 = 4 \times 4 = 4^2$$

Since the numbers forming the sequence can be represented as squares, so these numbers are called **square numbers**.

You can recognize the following interesting sequence;

1, 1, 2, 3, 5, 8, ...

This sequence is known as a **Fibonacci sequence**. Each term of the sequence is obtained by adding the two previous terms together.

Let us now recall the two major types of number sequences i.e arithmetic sequences and geometric sequences.

6.1.1 Arithmetic and Geometric Sequence

A sequence of numbers in which each new term is obtained by adding or subtracting a fixed number (known as the **common difference**) in the previous term is called an **arithmetic sequence**.

Example 1:

1. Sequence of even numbers:

2, 4, 6, 8, 10, ...

$\rightarrow +2 \rightarrow +2 \rightarrow +2 \rightarrow +2$

3. Sequence of multiples of 3.

3, 6, 9, 12, ...

$\rightarrow +3 \rightarrow +3 \rightarrow +3 \rightarrow +3$

2. Sequence of odd numbers:

1, 3, 5, 7, 9, ...

$\rightarrow +2 \rightarrow +2 \rightarrow +2 \rightarrow +2$

4. Sequence of decreasing numbers.

100, 97, 94, 91, ...

$\rightarrow -3 \rightarrow -3 \rightarrow -3 \rightarrow -3$

Quick Check:

Define the rules of the following patterns:

i. 1, 8, 27, 64, ...

ii. 3, 7, 10, 17, 27, ...

iii. A, C, F, J, ...

NOT FOR SALE

A sequence of numbers in which each term is obtained by multiplying or dividing the preceding term by a fixed number (known as the **common ratio**) is called a **geometric sequence**.

Note that in these examples of sequences, we have applied Term-to-Term rule to find each next term.

Example 2:

1. 5, 10, 20, 40,.....
 $\rightarrow \times 2 \rightarrow \times 2 \rightarrow \times 2$

Here, each term is obtained by multiplying the previous term by 2.

2. 4, 8, 16, 32,.....

i. Describe the rule of this pattern? ii. What will be the next term?

We know that **position to terms rule** helps us to find out what number is in a pattern if its position in the pattern is given. Using this rule, we can compute the value of any term in a sequence by finding the n th term.

Let's find the 10th term of this sequence.

Example 3:

Consider the sequence of even numbers;

2, 4, 6, 8, 10,.....

Here, First term = $a_1 = 2 \times 1$

Second term = $a_2 = 2 \times 2$

Third term = $a_3 = 2 \times 3$

General Term = n th term = $a_n = 2 \times n = 2n$

Since $a_n = 2n$, this mean;

i. We can generate the sequence by substituting $n = 1, 2, 3, 4, \dots$ in a_n .

ii. We can find any specific term of the sequence by substituting the term number instead of n , in the formula $2n$.

We have;

n	1	2	3	4	5	6
$2n$	2	4	6	8	10	12

Put $n = 10$ in $a_n = 2n$,

$a_{10} = 2(10) = 20$

So, the 10th term is 20.

Note it down

The term-to-term rule of a pattern describes how to get next term in that specific pattern.

Note it down

- The fixed number which is added or subtracted in an arithmetic sequence is called common difference because if we find the difference of any two successive terms, we will always get this common difference.
- The number multiplied to each term of the geometric sequence is known as the common ratio, because if we divide any two successive terms, we will always get this common ratio.

Example 4:

Similarly, the sequence of square numbers

i.e. 1, 4, 9, 16, 25,

or $(1)^2, (2)^2, (3)^2, (4)^2, (5)^2, \dots$

implies

$$a_1 = (1)^2$$

$$a_2 = (2)^2$$

$$a_3 = (3)^2$$

$$\vdots$$

$$a_n = n^2$$

Quick Check

If $a_n = n^2$, find the value of 13th and 20th terms of the sequence.

So, this sequence can be generated by putting different values of n in n^2 . We have;

n	1	2	3	4	5	6
n^2	1	4	9	16	25	36

We can find the 10th term of this sequence such as,

$$a_{10} = (10)^2 = 100$$

Similarly, the 17th term of this sequence is:

$$a_{17} = (17)^2 = 289$$

Exercise 6.1

1 Find the rules of the following patterns to justify whether the following sequences are arithmetic or geometric. Extend the patterns to the next 3 terms.

i. 2, 5, 8, 11,....

ii. 1, 4, 7, 10,....

iii. 4, 8, 12, 16,....

iv. 2, 4, 8, 16,....

v. 3, 9, 27, 81,....

vi. 2, 6, 18, 54,....

vii. 64, 60, 56, 52,....

viii. 729, 243, 81,....

ix. 6, 12, 18, 24, 30, 36,....

x. 25, 50, 100, 200,....

2 Using the given information, create the pattern and distinguish between arithmetic or geometric sequences.

Starting number	Pattern rule
i. 2	add 3
ii. 5	multiply by 2
iii. 3	multiply by 3
iv. 1	add 4
v. 80	subtract 8



Get students into two groups. Instruct them to write some arithmetic sequences and the other group write some geometric sequence and then tell the difference between arithmetic and geometric sequences.

3 Is the following sequence arithmetic or geometric?

3, 11, 19, 27, 35, ...

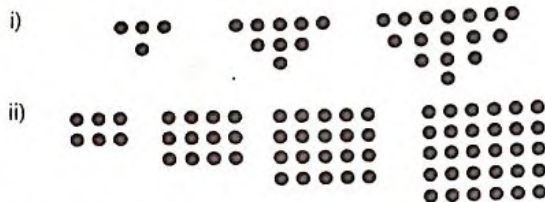
Find the 8th term of the sequence?

4 State the rule of the following sequence:

17, 28, 39, 50, ...

What is the 6th term of the sequence?

5 Write the numerical form of the following number patterns and apply the rule to find the 7th term of each.



6 Observe the following sequences, discuss the rules and sort out the sequences which are neither arithmetic nor geometric.

i) 4, 7, 10, 13, 16, ...

ii) 5, 9, 13, 17, 21, ...

iii) 6, 9, 14, 21, 30, ...

iv) 0, 3, 8, 15, 24, ...

v) 2, 5, 10, 17, 26, ...

vi) 5, 10, 15, 25, 40, ...

vii) 10, 12, 16, 22, ...

viii) 128, 64, 32, ...

ix) 1, 3, 4, 7, 11, ...

x) 15, 30, 60, 120, ...

7 Fill in the missing terms of the following sequences:

i) 5, 8, ..., 17, 20.

ii) 40, 34, ..., 22, 16.

iii) 1, 3, ..., 27, ..., 243

iv) 324, 108, ..., 12, 4.

v) 64, ..., 16, 8, 4, ...

8 Differentiate between an arithmetic and geometric sequence. Give examples.

9 What is the 7th term of the sequence having $a_n = 3n + 5$.

10 What is the 11th term of the sequence having $a_n = n(n - 1)$.

11 Find the 9th, 13th and 20th term of the sequence generated by $a_n = 4n + 1$.

NOT FOR SALE

6.2 General Term of an arithmetic Sequence

A sequence of numbers in which the difference between any two consecutive numbers is the same, is called an arithmetic sequence. For example, the sequence of odd numbers, i.e, 1, 3, 5, 7, 9, ... is an arithmetic sequence.

In the previous class, we have learnt to find the general term of various sequences by breaking out the terms in a uniform way. Let us now investigate the general term of an arithmetic sequence.

In the above sequence of odd numbers; 1, 3, 5, 7, 9, ...

First term = $a_1 = 1$

Common difference = $d = 3 - 1 = 5 - 3 = 7 - 5 = 2$.

We can write the terms of this sequence as;

1, $1 + 2$, $3 + 2$, $5 + 2$, $7 + 2$, ...

or $1, 1 + 1(2), 1 + 2(2), 1 + 3(2), 1 + 4(2), \dots$

Replacing the first term by a_1 and the common difference by d , we can write the arithmetic progression in general as;

$a, a + d, a + 2d, a + 3d, \dots$

What would be the n th term of the sequence?

We can write,

$a_1 = a$

$a_2 = a + d = a + (2 - 1)d$

$a_3 = a + 2d = a + (3 - 1)d$

$a_4 = a + 3d = a + (4 - 1)d$, etc.

In each of these terms, the co-efficient of d is 1 less than the index of the term. Therefore for the n th term a_n , the co-efficient of d would be $(n - 1)$, so we have,

$$a_n = a + (n - 1)d$$

which is the **general term of an arithmetic sequence**.

The following examples help to understand the concept.



write some arithmetic sequences on the board and ask students to find the general term of this sequence of these arithmetic sequences and also find the 23rd and 18th term of the given sequence.

NOT FOR SALE

Example 1:

Find the general term of the following arithmetic sequence,

-2, 3, 8, 13, 18, ...

Also find the 9th and the 19th terms.

Solution:

In the sequence,

-2, 3, 8, 13, 18, ...

$$a_1 = -2$$

$$d = 5 \quad (\because 3 - (-2) = 8 - 3 = 13 - 8 = 5)$$

Therefore,

$$a_n = a_1 + (n - 1)d$$

$$= -2 + (n - 1)(5)$$

$$= -2 + 5n - 5$$

$$a_n = 5n - 7$$

Hence, $a_9 = 5(9) - 7$

$$= 45 - 7$$

$$a_9 = 38$$

and

$$a_{19} = 5(19) - 7$$

$$= 95 - 7$$

$$a_{19} = 88$$

Example 3:

The given table shows the costs paid by a customer for renting a car for some period of time. If the sequence continues, how much would it cost the customer if he rent the car for 12 months?

Month	Charges (in Rs)
1	7500
2	9000
3	10500
4	12000

Example 2:

For the sequence given below,

3, -1, -5, -9, -13, ...

i. Find the general term.

ii. Find the 16th, 20th and 27th terms.

Solution:

i. Here $a_1 = 3$ and $d = -4$

Therefore,

$$a_n = a_1 + (n - 1)d$$

$$= 3 + (n - 1)(-4)$$

$$= 3 - 4n + 4$$

$$\Rightarrow a_n = 7 - 4n$$

ii. $a_{16} = 7 - 4(16)$

$$= 7 - 64$$

$$\Rightarrow a_{16} = -57$$

$$a_{20} = 7 - 4(20)$$

$$\Rightarrow a_{20} = 7 - 80$$

$$a_{20} = -73$$

$$a_{27} = 7 - 4(27)$$

$$a_{27} = 7 - 108$$

$$\Rightarrow a_{27} = -101$$

Solution.

Since the difference between any two successive costs is 1500, the costs form an arithmetic sequence with common difference 1500.

Here $a_1 = 7500$ and $d = 1500$

To find a_{12} the cost for twelve months.

Formula for nth term

$$a_n = a_1 + (n - 1)d$$

$$\text{Put } n = 12, a_1 = 7500, d = 1500$$

$$a_{12} = a_1 + (12 - 1)d$$

$$a_{12} = 7500 + (12 - 1)1500$$

$$= 24000$$

It would cost Rs 24000 to rent the car for twelve months.

Exercise 6.2

- 1 An arithmetic sequence is given by -1, 3, 7, 11, 15, ... What is the common difference?
- 2 Find the general term of each of the following arithmetic sequences. Also find the indicated terms.
 - a) 2, 6, 10, 14, ... (a_1).
 - b) 5, 11, 17, 23, 29, ... (a_8, a_{10}).
 - c) 18, 30, 42, 54, 66, ... (a_{11}, a_{14}).
 - d) 11, 10.5, 10, 9.5, 9, ... (a_{12}).
 - e) 7, -2, -11, -20, -29, ... (a_{24}).
 - f) 1, 11, 21, 31, 41, 51, ... (a_9, a_{17}).
- 3 Determine the common difference of an arithmetic sequence, whose first term is 2 and 6th term is 27.
- 4 Find the 17th term of an arithmetic sequence with first term 2 and common difference 7.
- 5 Madeeha's starting salary in 2012 was Rs 25000. The company raised the salary each year by an increment of Rs 4000. Find her salary in 11th year of her job.
- 6 In first year of tree planting campaign, 6000 plants were planted and each year there is an increase of 2200 plants. If the sequence continues, how many plants will be planted in 7th year of the campaign?



7. Each year, the average temperature of a region is increasing by 2.5°C . If the average temperature recorded in 2014 was 27°C , what will be the average temperature in the 10th year?



8. After every kilometer, there is an increase of 15 metre in the width of a canal. What is the width of canal after 7 kilometres?

6.3 Polynomials and Factorization

Let's recall some basic algebraic terms and the operation of polynomials before learning about further concepts related to algebra.

6.3.1 Open and close sentences

A mathematical sentence is called a closed sentence if that is known to be either true or false while an open sentence has variables whose values is not known.

For example, $60 - x = 2$. This is an open sentence as the value of 'x' is unknown and its not possible for us to say if it is true or false. Similarly, $x - 4 = 10$, $5 + x = 20$, $y - 7 = 22$ etc. are open sentences because we do not know the value of the unknown.. In the first example if we replace x with the number '58', it becomes true. i.e. $60 - 58 = 2$. In such cases, the sentence will be called a **closed sentence**.

Note it down

A statement (or sentence) that does not include enough information required to decide whether it is true or false is known as an open statement.

6.3.2 Algebraic Expression, Equation and Inequality

An algebraic expression is a combination of variables, constants, coefficients, exponents and symbols of operations i.e. + and -.

For example:

- a) $3x$ b) 9 c) $5x + 4y$ d) $2a - 3b + 4$, etc.

An equation is a mathematical statement that shows the equality of two expressions while an inequality shows that an expression is less than or more than the other. An equation shows the equality of two variables while an inequality shows the inequality of two variables. For example, $2x + 6 = 9$, $5x - 5 = 10$, $x = 3$ etc all are equations while $2x - 1 \leq 7$, $x + 3 < 9$, $x + 4 > 1$, $3x < 9$ etc all are inequalities.

Note it down

In an equation both sides of the equation must be same, if one side is not equal to the other

6.3.3 Polynomial

Polynomials are the algebraic expressions in which the powers of variables are whole numbers. It means that all of the algebraic expressions are polynomials except those which possess negative or fractional exponents of the variables involved. Consider the following examples. i. 5 ii. $x - y$ iii. $x^3 - 2x^2 + 3x - 5$

Degree of a Polynomial

Degree of a polynomial is the highest degree of the terms of that polynomial. For example; the degree of $7x^2 - 15$ is 2 and that of $2x^5 - 3x^2 + 4$ is 5.

Coefficients of a Polynomial

The coefficients of the terms of a polynomial are called the coefficients of that polynomial. For example; the coefficients of the polynomial $4x^3 - 3x^2 + 7x - 10$ are 4, -3, 7 and -10.

Polynomials in one, two and more Variables

Polynomials in one Variable

Polynomials that consist of only one variable are called polynomials in one variable. For example; the polynomials $2a - 3$ and $5x^3 - 7x^2 - 4x$ are in one variable.

Polynomials in two Variables

Polynomials that consist of two variables is called polynomials in two variables. The polynomials $2x + 3y$ and $x^2 + 20xy - y^2$ are the examples of polynomials in two variables.

Polynomial in more than two Variables

A polynomial can have more than two variables. A polynomial that has three variables is said to be a polynomial in three variables and so on. Some examples of this are given below.

- i. $2l + 3m - n$ ii. $mp + mq + nc + nd$
iii. $p^2 + pq - rs$ iv. $x^2 + y^2 + z^2 - 2xyz$

Polynomials of Various Degrees

A polynomial may have 0, 1, 2, 3 or even more variables. Similarly, there exist polynomials of various degrees. A polynomial may have 0, 1, 3, 4 or even a higher degree. Let us now study the various kinds of polynomials with respect to their degrees.

Linear Polynomials

A polynomial whose degree is one (1) is known as a linear polynomial.



Ask students to write some algebraic expression, equation and inequality in their notebooks.

For example, $2x + 3$, $x - 5$ and $z + 4$ are linear polynomials. A linear polynomial may be in one or more variables. The above mentioned three polynomials are linear polynomials in one variable. The polynomials $2x + 3y$, $2x + 5y - 3z$ and $x + y + z - p$ are the linear polynomials in two, three and four variables respectively. A linear polynomial may be even in more than four variables. The general form of a linear polynomial in one variable is $ax + b$ where 'a' and 'b' are real numbers and $a \neq 0$.

Quadratic Polynomials

A polynomial whose degree is two is known as a quadratic polynomial.

A quadratic polynomial may be in one, two or more variables. In the following, we shall discuss the quadratic polynomials in a bit more detail.

Quadratic Polynomials in One Variable

A polynomial with degree two and consisting of only one variable is known as a quadratic polynomial in one variable.

In general form of a quadratic polynomial in one variable is: $ax^2 + bx + c$ Where 'a', 'b' and 'c' are real numbers and 'a' can take any value except zero i.e., $a \neq 0$.

Quadratic Polynomial in more than one Variables

A polynomial with degree two and consisting of two variables is known as a quadratic polynomial in two variables.

A quadratic polynomial may comprise of more than two variables. Thus there exist innumerable quadratic polynomials in two or more variables. Some examples of such quadratic polynomials are $x^2 + y^2$, $x^2 + 2xy + y^2$, $x^2 - y^2$, $x^2 + y^2 + z^2$ and $p^2 + q^2 - 2pq$.

a) Monomials

Polynomials which contain only one term are known as monomials.

For example, $3x$, $4y$, and $6a^2$ are all examples of monomials.

b) Binomials

Polynomials which contain only two terms are known as binomials. For example: $4x + 3$ and $a^2 - b$ are examples of binomials.

Note it down

The concept of polynomials helps us to translate many mathematical and real life problems into algebraic expressions and equations. In this way, this concept helps us to solve the problems by using algebra.

Note it down

"Mono" means one.
"Bi" means two.
"Tri" means three.



Tell students about polynomial and its types. Instruct them to write two examples of each type of polynomial in their notebook. Also mention the polynomial types with each examples.

NOT FOR SALE

c) Trinomials

polynomials which contain three terms are known as trinomials. For example: $3u + 5v + 1$ and $x^2 - 7y + 1$ are examples of trinomials.

Quick Check

Read the following statements carefully and fill in the blanks.

- The polynomial $7x + 12y$ is a polynomial in -----variables.
- The polynomial $2xy + 3yz + 5xz$ is a polynomial in -----variables.
- The degree of the polynomial $x^3 + 12x^2 - 6x + 9$ is -----.
- The degree of the polynomial $x^2 - 7$ is -----.
- The polynomial $7x + 11$ is a ----- polynomial.
- The polynomial $x^2 - 2x - 3$ is a ----- polynomial.

Cubic Polynomials

A polynomial with degree three is known as a cubic polynomial.

For example; $x^3 + 5$, $x^3 + y^3$ and $2x^3 + 3x^2 + 5$ are cubic polynomials. Cubic polynomials, like quadratic polynomials, may be in one or more variables.

A cubic polynomial that is in only one variable is called a cubic polynomial in one variable. Examples of such polynomials are $x^3 + 2x^2 + 7x - 12$, $x^3 - 27$ and $4x^3 + 12x^2 - 17$. The polynomials $x^3 + y^3 + 3xy$ and $x^3 + y^3 + xz$ are examples of cubic polynomials in two or three variables respectively. Similarly, there exist certain innumerable cubic polynomials in more than 3 variables.

The general form of a cubic polynomial in one variable is $px^3 + qx^2 + rx + s$ where 'p', 'q', 'r' and 's' are real numbers but $p \neq 0$ i.e.; 'p' may take any real value except '0'. Can you explain what will happen if $p = 0$?

Biquadratic Polynomials

A polynomial with degree four is known as a biquadratic polynomial.

For example; $x^4 + 3$, $y^4 + 2y^2 - 7$ and $x^4 - y^4$ are biquadratic polynomials. A biquadratic polynomial may be in one or more variables. The polynomials $5x^4 + 2x^3 - 3x^2 + x - 15$, $x^4 - x^2 + 4$ and $x^4 + x^3 - 7$ are the examples of the biquadratic polynomials in one variable. The polynomials $x^4 - y^4$, $x^4 + y^4 - 2x^3 + 4y^2$ and $p^4 + 14 + 4pq$ are the examples of the biquadratic polynomials in two variables. Similarly, biquadratic polynomials may be in more than two variables.

NOT FOR SALE

Exercise 6.3

- 1** Observe the following algebraic expressions. Identify the variables and constants from each expression;

i) $4x + 3$ ii) $5y$ iii) $2x + 3y + z$ iv) $5a + 3b + 2$ v) $2p - 3$

- 2** Consider the following polynomials and answer the questions given below:

i) $a^2 + 2ab + b^2$ ii) $7x - 5$ iii) $3y^2 + 15y - 21$
 iv) $5x^2 + 2y$ v) $2a + 3b + 4c$ vi) $5x^4 + 2x^3 + 4x^2 + 7x - 20$
 vii) $\sqrt{2ab} + 3bc + 4cd$ viii) $5r^2 - 4b + 2t - 3u$

- a) Which of the above polynomials are in one variable?
 b) Which of the above polynomials are in two variables?
 c) Which of the above polynomials are in three variables?
 d) Which of the above polynomials are in more than three variables?

- 3** Read the following and classify as linear polynomials, quadratic polynomials, cubic polynomials or biquadratic polynomials

i) $5x - 2$ ii) $3x^2 - 15$ iii) $x^2 + y^2 + 2xy$
 iv) $y^2 - xz$ v) $\frac{2}{3}x^3 - \frac{4}{7}y^2 + 28yz$ vi) $5x^4 + 6x^3 + 2x^2 - 11x + 16$
 vii) $x^4 - y^4$ viii) $2a + 3b + 4c - 5d$ ix) $v^2 - u^2 + 2gs$
 x) $m^4 - n^4 + 4np$ xii) $x^3 - 6x^4 - 16$ xiii) $7.12x + 3.2y + 1.5z$
 xiii) $\sqrt{2} + \frac{1}{2}y^2 - yz$ xiv) $x^3 + y^3$ xv) $2x^3 + 4x^2 - 5x + 3$
 xvi) $x^4 + 15x^2 - 16$

- 4** Which of the above polynomials are quadratic polynomials in one variable?

- 5** Which of the above polynomials are quadratic in two variables?

- 6** Which of the cubic polynomials are in more than one variable?

- 7** Which of the above polynomials have the degree higher than a biquadratic polynomial?

6.4 Operations on Polynomials

The four most basic operations of arithmetic are addition (+), subtraction (-), multiplication (x) and division (÷). All of these basic operations are also applicable to polynomials.

6.4.1 Addition of Polynomials

We have learnt in grade 7th the addition of polynomials in the both ways; the vertical form and the horizontal form. Both of these methods are equally important. We can opt any of these methods as and when required. To refresh the concept of addition of polynomials, let us consider the following examples.

Example 1:

Find the sum of $2a - 3b + 4c$, $5b - 4c$ and $3a + 5c$.

Solution:

$$\begin{array}{r} 2a - 3b + 4c \\ 0a + 5b - 4c \\ + 3a + 0b + 5c \\ \hline 5a + 2b + 5c \end{array} \quad \text{(The vertical form)}$$

Also, $(2a - 3b + 4c) + (5b - 4c) + (3a + 5c)$ (The horizontal form)

$$\begin{aligned} &= 2a - 3b + 4c + 5b - 4c + 3a + 5c \\ &= 2a + 3a - 3b + 5b + 4c - 4c + 5c \\ &= 5a + 2b + 5c \end{aligned}$$

Example 2:

Find the sum of the following polynomials.

$$7x^4 - 5x^3 + 2x^2 - 5, 4x^4 - 2x^3 + 4x \text{ and } 7x^3 - 2x^2 + 3x + 11$$

Solution:

$$\begin{aligned} &= (7x^4 - 5x^3 + 2x^2 - 5) + (4x^4 - 2x^3 + 4x) + (7x^3 - 2x^2 + 3x + 11) \\ &= 7x^4 - 5x^3 + 2x^2 - 5 + 4x^4 - 2x^3 + 4x + 7x^3 - 2x^2 + 3x + 11 \\ &= 7x^4 + 4x^4 - 5x^3 - 2x^3 + 7x^3 + 2x^2 - 2x^2 + 4x + 3x - 5 + 11 \\ &= 11x^4 - 0x^3 + 0x^2 + 7x + 6 \\ &= 11x^4 + 7x + 6 \end{aligned}$$

Note it down

- Only like terms can be combined together to form a single term.
- Be careful about the signs of the various terms of the answer

Exercise 6.4

- 1** Find the sum of the polynomials in each of the following questions.

i. $x^2 - 2xy + y^2$, $x^2 - y^2$, $4xy + y^2$.
 ii. $p^3 + q^3 - 3p^2q + 5pq^2$, $4p^3 - q^3 + 6pq^2$, $2q^3 + 5p^2q - 11pq^2$
 iii. $7x^5 - 3x^4 + 11x^3 + 12$, $2x^5 - 4x^4 - 6x^2 + 5x - 10$, $6x^5 - 8x^3 + 7$
 iv. $12x^3 + 4x^2y - 5xy^2 + 15y^3$, $10x^3 - 11y^3$, $14x^2y - 5xy^2 - 4y^3$



Explain the sum of polynomial by solving different examples on the board. Put card of different polynomials and ask each student to come forward, select two cards and then add that polynomials.

Exercise 6.4

- 1** Find the sum of the polynomials in each of the following questions.

i. $x^2 - 2xy + y^2$, $x^2 - y^2$, $4xy + y^2$.
 ii. $p^3 + q^3 - 3p^2q + 5pq^2$, $4p^3 - q^3 + 6pq^2$, $2q^3 + 5p^2q - 11pq^2$
 iii. $7x^5 - 3x^4 + 11x^3 + 12$, $2x^5 - 4x^4 - 6x^2 + 5x - 10$, $6x^5 - 8x^3 + 7$
 iv. $12x^3 + 4x^2y - 5xy^2 + 15y^3$, $10x^3 - 11y^3$, $14x^2y - 5xy^2 - 4y^3$



Explain the sum of polynomial by solving different examples on the board. Put card of different polynomials and ask each student to come forward, select two cards and then add that polynomials.

- v. $4x^5 + 5x^4 - 3x^3$, $5x^5 - 2x^2 + 8x - 7$, $11x^5 - 3x^3 - 2x - 3$
 vi. $15x^3 - 8x^2y - 5xy^2 + 11y^3$, $12x^2y + 7xy^2 - 15y^3$, $-6x^3 + 4x^2y - 2xy^2$
 vii. $12p^4 - 3p^3q + 4p^2q^2$, $3p^4 - 4q^3$, $5p^3q - 9p^2q^2 + 5q^3$
 viii. $15u - 20u + 7w$, $10u - 15u + 12w$, $15u + 20u - 19w$

2 Simplify the following:

- i. $3(x^2 - xy + y^2) + 5(x^2 + 2xy + 3y^2)$
 ii. $7(pq + qr - rs) + 4(qr - rs - 7)$
 iii. $10(4x^3 - 3x^2 + 5x - 7) + 2(5x^2 - 12) + 3(7x^3 + 5x^2 - 4x)$

6.5 Subtraction of Polynomials

Polynomials can also be subtracted from one another. We have learnt the subtraction of polynomials in detail in grade VII. We learnt that like addition, subtraction of polynomials can also be performed in two ways; the vertical form and the horizontal form. Let us consider the following examples to refresh this concept.

Example 1:

Subtract $4x - 3y + 7z$ from $15x - 8y + 7z$

Solution:

$$\begin{array}{r} 15x - 8y + 7z \\ - 4x - 3y + 7z \\ \hline 11x - 5y + 0z \end{array}$$

Example 2:

Subtract the second polynomial from the first polynomial.

$$15x^5 - 3x^4 - 6x^2 + 10x + 25, \quad 7x^5 - 4x^3 + 12x^2 + 6x - 11$$

Solution:

$$\begin{aligned} & (15x^5 - 3x^4 - 6x^2 + 10x + 25) - (7x^5 - 4x^3 + 12x^2 + 6x - 11) \\ &= 15x^5 - 3x^4 - 6x^2 + 10x + 25 - 7x^5 + 4x^3 - 12x^2 - 6x + 11 \\ &= 15x^5 - 7x^5 - 3x^4 + 4x^3 - 6x^2 - 12x^2 + 10x - 6x + 25 + 11 \\ &= 8x^5 - 3x^4 + 4x^3 - 18x^2 + 4x + 36 \end{aligned}$$



Explain the multiplication of polynomial by solving different examples on the board. Put card of different polynomials and ask each student to come forward, select two or three cards and then find the multiply them to find the product. Repeat this activity to random students of the class.

NOT FOR SALE

Exercise 6.5

1 Subtract the first polynomial from the second in each of the following questions.

- i. $15a - 18b + 24c$, $35a - 40b + 34c$
 ii. $a^3 + a^2b + ab^2 + b^3$, $4a^3 + a^2b + 2ab^2 - b^3$
 iii. $p^4 - 5p^3 + 3p^2 - 15$, $p^5 + p^4 + 5p^3 + 3p^2 + 15$
 iv. $9x^4 - 16x^3 + 12x + 17$, $25x^5 - 36x^3 + 12x^2 + 20x + 40$
 v. $10x^5 - 13x^4y^3 + 33x^3y^2$, $37x^5 - 33x^4y^3 + 20xy^3y^2 + 31y^5$

6.6 Multiplication of Polynomials

We have learnt the multiplication of polynomials in detail in Grade VII. It is one of the most basic concepts of algebra. To recall the concept of multiplication of polynomial consider the following examples.

Example 1:

Multiply $(x - 2)$ and $(x + 3)$

Solution:

$$\begin{aligned} & (x - 2)(x + 3) \\ &= x(x + 3) - 2(x + 3) \\ &= x^2 + 3x - 2x - 6 \\ &= x^2 + x - 6 \end{aligned}$$

Example 2:

Multiply $(2x + 3)$ and $(x^2 - 7x + 5)$

Solution:

$$\begin{aligned} & (2x + 3)(x^2 - 7x + 5) \\ &= 2x(x^2 - 7x + 5) + 3(x^2 - 7x + 5) \\ &= 2x^3 - 14x^2 + 10x + 3x^2 - 21x + 15 \\ &= 2x^3 - 14x^2 + 3x^2 + 10x - 21x + 15 \\ &= 2x^3 - 11x^2 - 11x + 15 \end{aligned}$$

Note it down

- Two or more polynomials can be multiplied with each other.
- The answer obtained by multiplying two or more polynomials is called their product.
- The polynomials being multiplied are the factors of the product.
- The degree of a product of two or more polynomials can never be smaller than any of its factors.
- The degree of a product of two or more polynomials is equal to the sum of the degree of the polynomials being multiplied.

NOT FOR SALE

Example 3:

Simplify: $(2x+3y)(x^2-3x+4)-3(x-3y)^2$

Solution:

$$\begin{aligned} & (2x+3y)(x^2-3x+4)-3(x-3y)^2 \\ &= 2x(x^2-3x+4)+3y(x^2-3x+4)-3[x^2-2(x)(3y)+9y^2] \\ &= 2x^3-6x^2+8x+3x^2y-9xy+12y-3(x^2-6xy+9y^2) \\ &= 2x^3-6x^2+8x+3x^2y-9xy+12y-3x^2+18xy-27y^2 \\ &= 2x^3-6x^2-3x^2+8x+3x^2y-9xy+18xy+12y-27y^2 \\ &= 2x^3-9x^2+8x+3x^2y+9xy+12y-27y^2 \end{aligned}$$

Exercise 6.6

1 Simplify:

- $(2x+3)(3x+2y)$
- $(7f+5g)(7f-5g)$
- $(p^2+2p)(p^2+q^2-r^2)$

- $(5x-2y)(2x-3xy+5y)$
- $(2p+3q-r)(5p-4r)$
- $(x-2)^2(x^2-4x+4)$

2 Multiply the polynomials given in each of the following cases:

- $x, (x-5)$ and $(x+7)$
- $2b, (10c-a), (a-2c)$

- $3x, (x^2+y^2)$ and (x^2-y^2)
- $(p+q), (p-q)$ and (p^2-q^2)

3 Simplify the following:

- $5(a-2)(a+3)-2(a+2)(a-3)$
- $9(3p-1)(2p+2)-7(3p-2)^2$
- $4(2x+5y)^2-2(3x-2y)^2$

- $7x(x^2+4y-z)+4(x+2z)(x^2-x)$
- $5(x^2-2x+5)(x-5)-2(x+5)^2$
- $(2x+3y)^2+(3x-2y)^2-(4x+3y)^2$

6.7 Division of Polynomials

Division of a polynomial by another polynomial is the reverse process of the multiplication of two polynomials. The division of polynomials is a bit different from the division of numbers. However, the basic terms of 'dividend', 'quotient' and 'divisor' are also used in the same sense in algebra.

Note it down

- Division of polynomials is the reverse process of multiplication of polynomials.
- If we know that the product of any two polynomials and one of the two polynomials, we can find the other polynomial by dividing the product by the

It is the simplest form of the division of the polynomials. Any polynomial can be divided by a monomial. Actual working of this type of division is being explained with the help of the following examples.

Example 1: Divide $5x^2 - 15x$ by $5x$.

Solution:

$$\begin{array}{r} x-3 \\ 5x \overline{) 5x^2-15x} \\ \underline{+5x^2} \\ -15x \\ \underline{+15x} \\ 0 \end{array}$$

Thus, $(5x^2 - 15x) \div 5x = x - 3$

Here, the dividend = $(5x^2 - 15x)$

The divisor = $5x$

The quotient = $x - 3$

And the remainder = 0

Note it down

If the division is exact, the divisor and the quotient are the polynomials whose product is equal to the dividend. Hence, the divisor and the quotient are the factors of the dividend in such a case.

Dividend = divisor \times quotient

Example 2: Divide $14x^4 + 26x^3 - 12x^2 - 6x$ by $2x$.

Solution:

$$\begin{array}{r} 7x^3 + 13x^2 - 6x - 3 \\ 2x \overline{) 14x^4 + 26x^3 - 12x^2 - 6x} \\ \underline{+14x^4} \\ 26x^3 - 12x^2 - 6x \\ \underline{+26x^3} \\ -12x^2 - 6x \\ \underline{+12x^2} \\ -6x \\ \underline{+6x} \\ 0 \end{array}$$

Thus, $(14x^4 + 26x^3 - 12x^2 - 6x) \div 2x = (7x^3 + 13x^2 - 6x - 3)$

Note it down

- In each step, the lower polynomial is subtracted from the upper polynomial.
- Don't forget to change the sign(s) of the polynomial being subtracted.
- In case of exact division the remainder is zero.
- Zero (0) is also considered a polynomial in Algebra.



Explain the division of polynomial by solving different examples on the board. Put card of different polynomials and ask each student to come forward, select two cards and then find the division of polynomials.

Example 3: Divide $(3x^3 - 19x^2 + 10x + 50)$ by $(x - 5)$

$$\begin{array}{r} 3x^2 - 4x - 10 \\ x-5 \overline{) 3x^3 - 19x^2 + 10x + 50} \\ \underline{3x^3 - 15x^2} \\ -4x^2 + 10x + 50 \\ \underline{-4x^2 + 20x} \\ -10x + 50 \\ \underline{-10x + 50} \\ 0 \end{array} \quad \begin{array}{l} \leftarrow -3x^2(x-5) \\ \leftarrow -3x(x-5) \\ \leftarrow -10x(x-5) \end{array}$$

Thus, $(3x^3 - 19x^2 + 10x + 50) \div (x - 5) = (3x^2 - 4x - 10)$

Example 4: Divide $x^5 - y^5$ by $x - y$

$$\begin{array}{r} x^4 + x^3y + x^2y^2 + xy^3 + y^4 \\ x-y \overline{) x^5 - y^5} \\ \underline{x^5 - x^4y} \\ x^4y - y^5 \\ \underline{x^4y - x^3y^2} \\ x^3y^2 - y^5 \\ \underline{x^3y^2 - x^2y^3} \\ x^2y^3 - y^5 \\ \underline{x^2y^3 - xy^4} \\ xy^4 - y^5 \\ \underline{xy^4 - xy^4} \\ 0 \end{array} \quad \begin{array}{l} \leftarrow x^4(x-y) \\ \leftarrow x^3y(x-y) \\ \leftarrow x^2y^2(x-y) \\ \leftarrow xy^3(x-y) \\ \leftarrow y^4(x-y) \end{array}$$

Thus, $(x^5 - y^5) \div (x - y) = (x^4 + x^3y + x^2y^2 + xy^3 + y^4)$

Note that:

1. Only like terms can be written one below the other.
2. It is necessary to arrange the terms in descending order with respect to x . That is why x^4y has been written first instead of $-y^5$ in the third line. The same principle has been used in the 5th, 7th and the 9th lines.

Sometimes, while dividing a polynomial by another polynomial, we obtain a certain non-zero polynomial whose degree is smaller than that of the divisor. Hence the process of division can't be kept continued any more.

Note it down

When the division of polynomials is not exact, the divisor and the quotient are not the factors of the dividend.

Note it down

We stop the process of division when the degree of the remainder is smaller than the divisor.

Let us consider the following examples:

Example 5:

Divide $(x^3 - 6x^2 + 5x - 4)$ by x .

Solution:

$$\begin{array}{r} x^2 - 6x + 5 \\ x \overline{) x^3 - 6x^2 + 5x - 4} \\ \underline{\pm x^3} \\ -6x^2 + 5x - 4 \\ \underline{\mp 6x^2} \\ 5x - 4 \\ \underline{\pm 5x} \\ -4 \end{array}$$

Note that in the last step, the degree of the polynomial ' -4 ' is zero while the degree of the divisor is 1. So, we stop here and say that

$$(x^3 - 6x^2 + 5x - 4) \div x = (x^2 - 6x + 5) + \frac{(-4)}{x}$$

Where ' r ' denotes the remainder.

Now, the divisor \times the quotient + the remainder

$$\begin{aligned} &= x(x^2 - 6x + 5) + (-4) \\ &= x^3 - 6x^2 + 5x - 4 \\ &= \text{The dividend} \end{aligned}$$

This observation leads us to discover an important fact that

The dividend = The divisor \times the quotient + the remainder

Example 6:

Divide $x^2 - 5x + 11$ by $(x - 3)$

Solution:

$$\begin{array}{r} x - 2 \\ x-3 \overline{) x^2 - 5x + 11} \\ \underline{\pm x^2 \mp 3x} \\ -2x + 11 \\ \underline{\mp 2x \pm 6} \\ 5 \end{array} \quad \begin{array}{l} \leftarrow x(x-3) \\ \leftarrow -2(x-3) \end{array}$$

Thus, $(x^2 - 5x + 11) \div (x - 3) = (x - 2) + \frac{5}{x-3}$
i.e. Quotient = $x - 2$, remainder = 5

Quick Check

- Solve and verify that Dividend = Divisor \times Quotient
i) $(x^2 - 12x + 12) \div (x - 10)$
- Solve and verify that Dividend = Divisor \times Quotient + Remainder

Quick Check

- If the product of two polynomials is $4x^4 + 7x^3 - 15y^2$ and one of the polynomials is $(x+3)$, find the second polynomial.
- If the product of two polynomials is $2a^3 - 11a^2 - 11a + 15$ and one of the polynomials is $(2a+3)$, find the other.

Exercise 6.7

1 Divide the first polynomial by the second polynomial.

- | | |
|--------------------------------|------------------------------------|
| i. $2x^3 - 4x^2 + 5x, x$ | ii. $9a^2 + 30ab + 25b^2, 3a + 5b$ |
| iii. $8p^2 - 12pq, 4p$ | iv. $x^2 - 2xy + y^2, x - y$ |
| v. $x^2 + 2xy + y^2, x + y$ | vi. $x^3 - y^3, x - y$ |
| vii. $x^2 - y^2, x + y$ | viii. $6x^2 + 27x + 30, 3x + 6$ |
| ix. $y^3 + y^2 + y + 1, y + 1$ | |

2 Divide the second polynomial by the first polynomial.

- | | |
|--------------------------------|--|
| i. $x - 2, x^2 - 5x + 6$ | ii. $2x + 3y, 6x^2 + 19xy + 15y^2$ |
| iii. $a + b, a^3 + b^3$ | iv. $5x - 3, 20x^3 - 17x^2 + 34x + 15$ |
| v. $3p - 2q, 3p^2 + pq - 2q^2$ | |

3 Solve the following.

- | | |
|---|--|
| i. $x^2 + 5x + 6 \div (x + 2)$ | ii. $(2r^2 - 3rs - 20s^2) \div (r - 4s)$ |
| iii. $(x^5 - y^5) \div (x - y)$ | iv. $(p^4 - q^4) \div (p + q)$ |
| v. $(2x^3 - 5x^2y + 5xy^2 - 3y^3) \div (2x - 3y)$ | |

6.8 Simplification of Algebraic Expressions

Like whole numbers to simplify the algebraic expression involving brackets, we follow the BODMAS rule in order of operations so that no miscalculations are made. Let's recall simplification of algebraic expressions involving the 4 operations.

Example 1:

$$\begin{aligned}
 & 4x + [2(4x + 3) - \{(2 + 2x - 7x^2)(5 + 4x^2)\}] && \leftarrow \text{First solve the operation within parentheses.} \\
 & = 4x + [2(4x + 3) - \{(10 + 8x^2 + 10x + 8x^3 - 35x^2 - 28x^4)\}] && \leftarrow \text{Now solve the like terms in curly brackets} \\
 & = 4x + [8x + 6 - 10 - 8x^2 - 10x - 8x^3 + 35x^2 + 28x^4] && \leftarrow \text{Then open the curly bracket and change the sign.} \\
 & = 4x + [-4 - 2x - 8x^3 + 27x^2 + 28x^4] && \leftarrow \text{Then open the square bracket and change the sign.} \\
 & = 4x - 4 - 2x - 8x^3 + 27x^2 + 28x^4 && \leftarrow \text{Next simplify the like terms} \\
 & = -4 + 2x + 27x^2 - 8x^3 + 28x^4
 \end{aligned}$$

Example 2:

$$\begin{aligned}
 & 1 + 5x^2 - \{[(2x + 5x^2)(5x + 7)] - \{(1 + 3x) - (7x - 8x^2)\}\} && \leftarrow \text{First solve the operation within parentheses.} \\
 & = 1 + 5x^2 - \{[(10x^2 + 14x + 25x^3 + 35x^2) - (1 + 3x - 7x + 8x^2)]\} && \leftarrow \text{Now solve the like terms in curly brackets.} \\
 & = 1 + 5x^2 - \{[14x + 25x^3 + 45x^2] - [1 - 4x + 8x^2]\} && \leftarrow \text{Then open the curly bracket and change the sign.} \\
 & = 1 + 5x^2 - [14x + 25x^3 + 45x^2 - 1 + 4x - 8x^2] && \leftarrow \text{Then open the square bracket and change the sign.} \\
 & = 1 + 5x^2 - [18x + 25x^3 + 45x^2 - 1 - 8x^2] && \leftarrow \text{Next simplify the like terms} \\
 & = 1 + 5x^2 - [18x + 25x^3 + 37x^2 - 1] \\
 & = 1 + 5x^2 - 18x - 25x^3 - 37x^2 + 1 \\
 & = 2 - 18x - 32x^2 - 25x^3
 \end{aligned}$$

Oral Exercise:

Simplify the following.

- a) $2y - (2 + 4y^2) - 2[4(7x + 9y)]$
 b) $6ab - [2b^2 + 5a(7ab + 2b)(1 + b)]$

Example 3:

$$\begin{aligned}
 & 7 - [4x^2 + \{(4x + 7x^2)(-3x) - (8x + 3x^2)(1 + 6x^2)\}] && \leftarrow \text{First solve the operation within parentheses.} \\
 & = 7 - [4x^2 + \{-12x^2 + 21x^3 - (8x + 48x^3 + 3x^2 + 18x^4)\}] && \leftarrow \text{Now solve the like terms in curly brackets.} \\
 & = 7 - [4x^2 + \{-12x^2 + 21x^3 - 8x - 48x^3 - 3x^2 - 18x^4\}] && \leftarrow \text{Then open the curly bracket and change the sign.} \\
 & = 7 - [4x^2 + \{-8x - 15x^2 - 27x^3 - 18x^4\}] && \leftarrow \text{Then open the square bracket and change the sign.} \\
 & = 7 - [4x^2 - 8x - 15x^2 - 27x^3 - 18x^4] && \leftarrow \text{Next simplify the like terms} \\
 & = 7 - [-8x - 11x^2 - 27x^3 - 18x^4] \\
 & = 7 + 8x + 11x^2 + 27x^3 + 18x^4
 \end{aligned}$$



Write different questions related to BODMAS on the board and ask the students to simplify algebraic expressions according to the rule.

Example 4:

$$\begin{aligned}
 & 4t + 5s [3t + 4s^2 - \{(9s + 5t) + (3t^3 + 9ts^2) \div 3t\}] \\
 &= 4t + 5s [3t + 4s^2 - \{9s + 5t + t^2 + 3s^2\}] \\
 &= 4t + 5s [3t + 4s^2 - 9s - 5t - t^2 - 3s^2] \\
 &= 4t + 5s [-2t + s^2 - 9s - t^2 - 3s^2] \\
 &= 4t - 10st + 5s^3 - 45s^2 - 5st^2 \\
 &= 4t - 45s^2 + 5s^3 - 10st - 5st^2
 \end{aligned}$$

First solve the operation within parentheses.

Now solve the like terms in curly brackets.

Then open the curly bracket and change the sign.

Then open the square bracket and change the sign.

Next simplify the like terms

Exercise 6.8

1 Simplify the following.

- $2ab + 2b(5a + b)$
- $4x + [2y - \{(2xy + 5x)(1 + 2xy)\}]$
- $4c - 2d [4cd - 4c(1 + 2d)]$
- $6m - (2 + 5m) - \{(2m + n)(5m + 2n) - 4m\}$
- $2x [5y - \{3xy + (4y^2 + 3x^2)\}]$
- $9e^2 + 5f - \{4e(e + f)\}$
- $24a^3 - [4b^2 \{(a + b)(b^2 + a) - (a^2 + b^3 + c)\}]$
- $4y - 2yz [4 \div \{(8y + 2z^2) - 4y(5z + 2y)\}]$

6.9 Algebraic Identities

An equation is true conditionally, whereas the identity is always true. We know that an equation is true for specific values of the variable(s) involved. On the other hand, an identity is true for all values of the variable(s) involved. For example, $2x + 5 = 9$ is an equation. It is true only if $x = 2$. It cannot be true for values of x other than 2.

Here we will discuss the following 4 basic identities.

Identity I: $(x + a)(x + b) = x^2 + (a + b)x + ab$

Identity III: $(a - b)^2 = a^2 - 2ab + b^2$

Identity II: $(a + b)^2 = a^2 + 2ab + b^2$

Identity IV: $a^2 - b^2 = (a - b)(a + b)$

Identity I: $(x + a)(x + b) = x^2 + (a + b)x + ab$

Algebraic Proof:

$$(x + a)(x + b) = x^2 + (a + b)x + ab$$

$$\text{LHS} = (x + a)(x + b)$$

$$= x(x + b) + a(x + b)$$

$$= x^2 + bx + ax + ab$$

$$= x^2 + (b + a)x + ab$$

$$= x^2 + (a + b)x + ab = \text{RHS}$$

$$\text{Hence proved } (x + a)(x + b) = x^2 + (a + b)x + ab$$

$$\text{Thus LHS} = \text{RHS.}$$

Geometric Proof:

Look at the figure ABCD. It is divided into 4 portions.

$$\text{Now Area of ABCD} = \text{Length of ABCD} \times \text{Width of ABCD}$$

$$= (x + a)(x + b)$$

$$\text{But Area of ABCD} = \text{Area of EFJD} + \text{Area of FGCI} +$$

$$\text{Area of AHFE} + \text{Area of HBGF}$$

So,

$$(x + a)(x + b) = x^2 + ax + bx + ab$$

$$(x + a)(x + b) = x^2 + (a + b)x + ab$$

Hence proved.

Example 1:

Simplify the binomials by using the identity.

$$(x + 2)(x + 3)$$

Solution:

$$(x + 2)(x + 3)$$

By using the identity,

$$(x + a)(x + b) = x^2 + (a + b)x + ab$$

$$(x + 2)(x + 3) = x^2 + (2 + 3)x + (2 \times 3)$$

$$= x^2 + 5x + 6$$

Example 2:

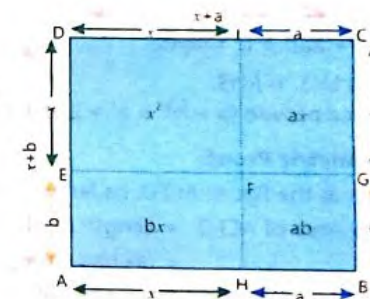
Find the unknown term.

$$(x - 1)(x - 2) = x^2 + (\quad) + ab$$

Solution:

By using the identity,

$$(x + a)(x + b) = x^2 + (a + b)x + ab$$



$$(x-1)(x-2) = x^2 + \{(-1) + (-2)\}x + (-1)(-2) \\ = x^2 - 3x + 2$$

So, $-3x$ is the required term.

Identity II: $(a+b)^2 = a^2 + 2ab + b^2$

Algebraic Proof:

$$(a+b)^2 = a^2 + 2ab + b^2$$

Proof:

$$\text{LHS} = (a+b)^2$$

$$= (a+b)(a+b)$$

$$= a^2 + ab + ba + b^2$$

$$= a^2 + ab + ab + b^2$$

$$= a^2 + 2ab + b^2 = \text{RHS.}$$

Thus LHS = RHS.

Hence proved $(a+b)^2 = a^2 + 2ab + b^2 = \text{R.H.S}$

Geometric Proof:

Look at the figure ACFD. Its length and width is $a+b$. It is divided into 4 portions.

Now Area of ACFD = Length of ACFD \times Width of ACFD

$$= (a+b)(a+b) = (a+b)^2$$

But Area of ACFD = Area of ABIG + Area of IHCB + Area of DEIG + Area of EFHI

So,

$$(a+b)^2 = a^2 + ab + ba + b^2$$

$$(a+b)^2 = a^2 + 2ab + b^2 \quad (\text{because } ab=ba)$$

Hence proved.

Example 3:

Find the squares of

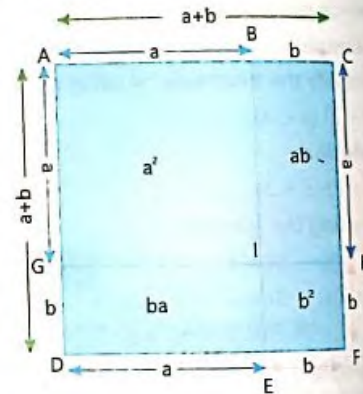
- i. $a+2b$ ii. $4p+8q$ iii. $2+5xy$

Solution:

$$\text{i. } (a+2b)^2 = a^2 + 2(a)(2b) + (2b)^2 \\ = a^2 + 4ab + 4b^2$$

$$\text{ii. } (4p+8q)^2 = (4p)^2 + 2(4p)(8q) + (8q)^2 \\ = 16p^2 + 64pq + 64q^2$$

$$\text{iii. } (2+5xy)^2 = (2)^2 + 2(2)(5xy) + (5xy)^2 \\ = 4 + 20xy + 25x^2y^2$$



Example 4:

Solve: i. 52×52 ii. $(102)^2$

Solution:

$$\text{i. } 52 \times 52 \\ = (52)^2 \\ = (50+2)^2 \\ = (50)^2 + 2(50)(2) + (2)^2 \\ = 2500 + 200 + 4 \\ = 2704$$

$$\text{ii. } (102)^2 \\ = (100+2)^2 \\ = (100)^2 + 2(100)(2) + (2)^2 \\ = 10000 + 400 + 4 \\ = 10404$$

Identity III: $(a-b)^2 = a^2 - 2ab + b^2$

Algebraic Proof:

$$(a-b)^2 = a^2 - 2ab + b^2$$

Proof:

$$\text{LHS} = (a-b)^2 \\ = (a-b)(a-b) \\ = a^2 - ab - ba + b^2 \\ = a^2 - ab - ab + b^2 \\ = a^2 - 2ab + b^2 \\ = \text{RHS}$$

Hence proved $(a-b)^2 = a^2 - 2ab + b^2$

Thus LHS = RHS

Geometric Proof:

Look at the figure ACFD. Its length and width is a . It is divided into 4 portions.

Now Area of ACFD = Length of ACFD \times Width of ACFD

$$= a \times a = a^2$$

But Area of ACFD = Area of ABIG + Area of IHCB +

Area of DEIG + Area of EFHI

So,

$$a^2 = (a-b)^2 + b(a-b) + b(a-b) + b^2$$

$$a^2 = (a-b)^2 + b^2 + 2b(a-b)$$

$$a^2 = (a-b)^2 + b^2 + 2ab - 2b^2$$

Example 5:

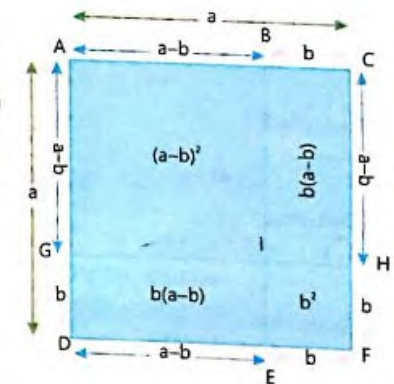
If $x + \frac{1}{x} = 4$, find the value $x^2 + \frac{1}{x^2}$

Solution: $x + \frac{1}{x} = 4$

$(x + \frac{1}{x})^2 = (4)^2$ (taking square on both sides)

$$x^2 + 2 + \frac{1}{x^2} = 16$$

$$x^2 + \frac{1}{x^2} = 16 - 2 = 14$$



$$a = (a-b) - b + 2ab$$

$$a = (a-b) - b + 2ab$$

$$(a-b) = a^2 + b^2 - 2ab$$

Hence proved.

Geometric Proof:

Look at the figure ACDF. Its length and width is a . It is divided into 4 portions.

Now Area of ACDF = Length of ACDF \times Width of ACDF

$$= a \times a = a^2$$

But Area of ACDF = Area of ABIG + Area of IHCB + Area of DEIG + Area of EFHI

So,

$$a^2 = (a-b)^2 + b(a-b) + b(a-b) + b^2$$

$$a^2 = (a-b)^2 + b^2 + 2b(a-b)$$

$$a^2 = (a-b)^2 + b^2 + 2ab - 2b^2$$

$$a^2 = (a-b)^2 - b^2 + 2ab$$

$$a^2 = (a-b)^2 - b^2 + 2ab$$

$$(a-b)^2 = a^2 + b^2 - 2ab$$

Hence proved.

Example 6:

Evaluate the following

i. $(p-2q)^2$ ii. $(x-4y)^2$

Solution:

$$i. (p-2q)^2 = (p)^2 - 2(p)(2q) + (2q)^2$$

$$= p^2 - 4pq + 4q^2$$

$$ii. (x-4y)^2 = (x)^2 - 2(x)(4y) + (4y)^2$$

$$= x^2 - 8xy + 16y^2$$

Example 7:

Solve:

i. $(97)^2$ ii. 988×988

Solutions:

$$i. (97)^2 = (100-3)^2$$

$$= (100)^2 - 2(100)(3) + (3)^2$$

$$= 10000 - 600 + 9$$

$$= 9409$$

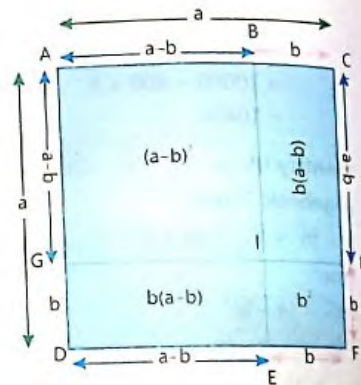
$$ii. (988 \times 988) = (998)^2$$

$$= (1000-2)^2$$

$$= (1000)^2 - 2(1000)(2) + (2)^2$$

$$= 1000000 - 4000 + 4$$

$$= 996004$$



Example 8:

If $x-y=4$ and $xy=2$, find x^2+y^2

Solution:

$$x-y=4 \quad (\text{given})$$

$$(x-y)^2 = (4)^2 \quad (\text{taking square})$$

$$x^2 - 2xy + y^2 = 16 \quad (\text{of both sides})$$

$$x^2 - 2(2) + y^2 = 16$$

$$x^2 - 4 + y^2 = 16$$

$$x^2 + y^2 = 16 + 4$$

$$x^2 + y^2 = 20$$

Identity IV: $a^2 - b^2 = (a-b)(a+b)$

Algebraic Proof:

$$\text{RHS.} = (a-b)(a+b)$$

$$= a(a+b) - b(a+b)$$

$$= a^2 + ab - ba - b^2$$

$$= a^2 + ab - ab - b^2$$

$$= a^2 - b^2$$

$$= \text{LHS}$$

Hence proved $(a-b)(a+b) = a^2 - b^2$

Thus, LHS. = RHS.

Geometric Proof:

Look at the figure ABCD. Its length and width is a . It is divided into 4 portions.

Now Area of ABCD = Length of ABCD \times Width of ABCD

$$= a \times a = a^2$$

But Area of ABCD = Area of EFHD + Area of FGCH + Area of ABGE

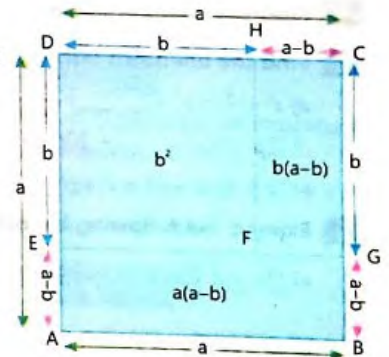
So,

$$a^2 = b^2 + b(a-b) + a(a-b)$$

$$a^2 = b^2 + (a+b)(a-b)$$

$$a^2 - b^2 = (a+b)(a-b)$$

Hence proved.



Example 9:

If $x - \frac{1}{x} = 4$, find $x^2 + \frac{1}{x^2}$

Solution:

$$x - \frac{1}{x} = 4$$

$$\left(x - \frac{1}{x}\right)^2 = (4)^2 \quad (\text{taking square of both sides})$$

$$x^2 - 2\left(x\right)\left(\frac{1}{x}\right) + \frac{1}{x^2} = 16$$

$$x^2 - 2 + \frac{1}{x^2} = 16$$

$$x^2 + \frac{1}{x^2} = 16 + 2$$

$$x^2 + \frac{1}{x^2} = 18$$

Outside World
If $x + \frac{1}{x} = 6$,
find the value of
 $x^2 + \frac{1}{x^2}$.



Explain the geometric and algebraic proofs of the identity with the help of examples. Make small groups of students and ask them to make one example of each identity and then solve it.

Example 10:Simplify the following: i. $(x+6)(x-6)$ ii. $(2p + \frac{1}{2q})(2p - \frac{1}{2q})$ **Solution:**

$$\begin{aligned} \text{i. } (x+6)(x-6) &= (x)^2 - (6)^2 \\ &= x^2 - 36 \end{aligned}$$

$$\begin{aligned} \text{ii. } (2p + \frac{1}{2q})(2p - \frac{1}{2q}) &= (2p)^2 - (\frac{1}{2q})^2 \\ &= 4p^2 - \frac{1}{4q^2} \end{aligned}$$

Example 11:Simplify: 108×92 with the help of formula.**Solution:**

$$\begin{aligned} 108 \times 92 &= (100 + 8)(100 - 8) \\ &= (100)^2 - (8)^2 \\ &= 10000 - 64 = 9936 \end{aligned}$$

Exercise 6.9**1** Simplify the following binomials by using the identity.

- i) $(x+5)(x+4)$ ii) $(y-1)(y-2)$ iii) $(a+6)(a+7)$
 iv) $(b+5)(b-7)$ v) $(2a+4)(2a-5)$ vi) $(5m+1)(5m+5)$
 vii) $(4z-1)(4z-4)$ viii) $(3n+6)(3n+3)$ ix) $(7l-1)(7l+9)$

2 Evaluate the following by using the algebraic identity.

- a) 34×34 b) 103×103 c) 121×121 d) 7.8×7.8
 e) $(204)^2$ f) $(1005)^2$ g) $(164)^2$ h) $(302)^2$

3 Find $a^2 + b^2$, if $a-b = 8$ and $ab = 3$.**4** Find $x^2 + \frac{1}{x^2}$ if $x - \frac{1}{x} = 4$.**5** Find the unknown terms.

- a) $x^2 + \frac{1}{x^2} = x^2 + \frac{1}{x^2} + \frac{1}{x^2}$ b) $a^2 - b^2 = (a+b)(a-b)$
 c) $(5p+3q)^2 = \frac{1}{x^2} + 30pq + 9q^2$ d) $(6a-4b)^2 = 36a^2 - 48ab + ()$
 e) $(x+4)(x+5) = x^2 + \frac{1}{x^2} + 20$

6 Expand the following by using basic algebraic formulae.

- a) $(2a + \frac{5}{7})^2$ b) $(3a+4b)^2$ c) $(t^2 - \frac{1}{x^2})^2$ d) $(\frac{3}{5}x - \frac{2}{3}y)^2$
 e) $(p^2 - 4q)^2$ f) $(x + \frac{1}{x})^2$ g) $(x - \frac{1}{x})^2$ h) $(x^2 + \frac{1}{x^2})^2$
 i) $(\frac{x}{2y} - \frac{2y}{x})^2$ j) $a^2 - 49$ k) $25x^2 - 9y^2$ l) $1 - 25t^2$
 m) $(x+5)^2 - (x+3)^2$ n) $81p^2 - 49q^2$

NOT FOR SALE

7 Find the value of $x^2 + \frac{1}{x^2}$ when $x + \frac{1}{x} = 3$.**8** Find the value of $x^4 + \frac{1}{x^4}$ when $x + \frac{1}{x} = 7$.**9** Find the value of $x^2 + \frac{1}{x^2}$ when $x - \frac{1}{x} = 10$.**10** Find the value of $b^4 + \frac{1}{b^4}$ when $b - \frac{1}{b} = 4$.**11** Prove that $x + \frac{1}{x} = x^2 + \frac{1}{x^2} = x^4 + \frac{1}{x^4}$ when $x + \frac{1}{x} = 2$.**12** Find the value of $p^4 + \frac{1}{p^4}$ when $p^2 + \frac{1}{p^2} = 30$.**13** Evaluate the following by using appropriate algebraic formulae.

- a) $(98)^2$ b) $(0.98)^2$ c) $(15)^2$ d) $(1.5)^2$
 e) $(0.05)^2$ f) $(1205)^2$ g) $(3200)^2$ h) $(3900)^2$
 i) $(1.10)^2$ j) 105×95 k) 325×275 l) 1010×990

6.10 Factorization

Factorization is the reverse process of multiplication. Like numbers, algebraic expressions can also be factorized. Factors of an algebraic expression are such algebraic expressions whose product is the given algebraic expression.

The process of finding factors of an algebraic expression is called factorization.

The process of writing an algebraic expression as a product of its factors is an important concept of algebra. In arithmetic, you have learnt that all composite numbers can be factorized. For example, factors of 21 are 3 and 7 and, therefore, we can write $21 = 3 \times 7$. In order to understand algebraic factorization, let us consider an example.

Example 1:

- (a) Find the product of $(x+2)$ and $(x+3)$.
 (b) Factorise $x^2 + 5x + 6$.

Solution:

$$\begin{aligned} \text{(a) } (x+2)(x+3) &= x(x+3) + 2(x+3) \\ &= x^2 + 3x + 2x + 6 \\ &= x^2 + 5x + 6 \end{aligned}$$

Note it down

- Factors of a number are numbers by which it is exactly divisible.
- A number is a multiple of all of its factors.
- The above two concepts are

NOT FOR SALE

(b) As, $(x+2)(x+3) = x^2 + 5x + 6$

So we can write $x^2 + 5x + 6 = (x+2)(x+3)$

As the product of $(x+2)$ and $(x+3)$ is equal to $x^2 + 5x + 6$, so factors of $x^2 + 5x + 6$ are $(x+2)$ and $(x+3)$.

Different methods, formulae and techniques are used to divide algebraic expressions into their factors. For the sake of simplicity, convenience and effective learning, let us discuss the factorization of different kinds of algebraic expressions one by one.

6.10.1 Factorization of Algebraic Expressions of the form $ka + kb + kc$

In this type of sentences, there is a number or a variable or a product of a number and a variable common to all terms of the given algebraic expression. For such type of algebraic expressions, there are two factors; one is the common factor of all terms and the other is the quotient obtained by dividing the given algebraic expression by the common factor.

Example 1: Factorize:

a) $2x + 2y + 2z$ b) $2xa + 6xb + 8xc$

Solution:

a) $2x + 2y + 2z$
 $= 2(x + y + z)$

By observation, we see that 2 is common to all three terms of the given algebraic expression. To find the other factor, we divided the given algebraic expression by 2 and get the quotient $(x + y + z)$, which is the other factor. Note that $(2x + 2y + 2z) = 2(x + y + z)$. You have learnt algebraic division in chapter 5 of this book. However, for your convenience, here is another way for division which is as under:

$$(2x + 2y + 2z) \div 2 = \frac{2x}{2} + \frac{2y}{2} + \frac{2z}{2} = (x + y + z)$$

b) $2xa + 6xb + 8xc$

Note that:

- The H.C.F of 2, 6 and 8 is 2.
- x is common to all terms of the given algebraic expression.

$$(2xa + 6xb + 8xc) \div 2x = \frac{2xa}{2x} + \frac{6xb}{2x} + \frac{8xc}{2x} = (a + 3b + 4c)$$

TEACHING POINT Explain the students how to factorize algebraic expression by solving different examples. Then make two groups of students and given each group 5 flash card of algebraic expression and instruct them to factorize and share the result with whole class.

Note it down

The product of all factors of a polynomials is always equal to the polynomial itself.

6.10.2 Factorization of Algebraic Expressions of the form $ac+ad+bc+bd$

For this type of algebraic expressions, all terms do not have a common factor. However, certain groups of terms possess a common factor. By taking such factors as common, there appears a common factor in all terms. The process of factorization is completed by taking common factors. The following example may help to understand the concept.

Example 1: Factorize:

$$ac + ad + bc + bd$$

Solution:

$$\begin{aligned} ac + ad + bc + bd \\ = a(c + d) + b(c + d) \\ = (c + d)(a + b) \end{aligned}$$

$$\therefore ac + ad + bc + bd = (c + d)(a + b)$$

Note it down

There are certain polynomials which do not have factors

Note that:

- All four terms of the given algebraic expressions do not possess a common factor.
- The first two terms possess 'a' as the common factor and the third and fourth terms possess 'b' as a common factor.
- By taking 'a' and 'b' as common, the given algebraic expression reduces to two terms (from 4 terms) and thus there appears $(c + d)$ as the common factor in these two terms.
- The process of factorization comes to its logical end when this newly appeared common factor ' $(c+d)$ ' is taken as common from the two terms.

Exercise 6.10

1 Factorize the following algebraic expressions.

a) $2a + 6$

b) $5x - 10$

c) $x^2 - x$

d) $3x^3 + 12x^2$

e) $7a - 7b + 7c$

f) $3b^2 + 6b - 15$

g) $5ax + 10ay - 25az$

h) $7p^3 - 14p^2 - 21p$

i) $10p^2q + 25pq^2 - 35pqr$

j) $11ab - 22bc + 33cd$

k) $a^2 + ab + 3a + 3b$

l) $4a^2 + 4b^2 + a^2c^2 + b^2c^2$

m) $3ax + 3ay - 14x - 14y$

n) $20xz - 10xy - 15yz$

o) $2tx + 6ty - 3qx - 9qy$

p) $4ay - 20by - az + 5bz$

q) $pr + ps + qr + qs$

r) $su - sv - tu + tv$

s) $6x^2 - 4xy - 9xy + 6y^2$

t) $15ax^2 + 21ay^2 - 20bx^2 - 28by^2$

$$(b) \text{ As, } (x+2)(x+3) = x^2 + 5x + 6$$

$$\text{So we can write } x^2 + 5x + 6 = (x+2)(x+3)$$

As the product of $(x+2)$ and $(x+3)$ is equal to $x^2 + 5x + 6$, so factors of $x^2 + 5x + 6$ are $(x+2)$ and $(x+3)$.

Different methods, formulae and techniques are used to divide algebraic expressions into their factors. For the sake of simplicity, convenience and effective learning, let us discuss the factorization of different kinds of algebraic expressions one by one.

6.10.1 Factorization of Algebraic Expressions of the form $ka + kb + kc$

In this type of sentences, there is a number or a variable or a product of a number and a variable common to all terms of the given algebraic expression. For such type of algebraic expressions, there are two factors; one is the common factor of all terms and the other is the quotient obtained by dividing the given algebraic expression by the common factor.

Example 1: Factorize:

a) $2x + 2y + 2z$ b) $2xa + 6xb + 8xc$

Solution:

a) $2x + 2y + 2z$
 $= 2(x + y + z)$

By observation, we see that 2 is common to all three terms of the given algebraic expression. To find the other factor, we divided the given algebraic expression by 2 and get the quotient $(x + y + z)$, which is the other factor. Note that $(2x + 2y + 2z) = 2(x + y + z)$. You have learnt algebraic division in chapter 5 of this book. However, for your convenience, here is another way for division which is as under:

$$(2x + 2y + 2z) \div 2 = \frac{2x}{2} + \frac{2y}{2} + \frac{2z}{2} = (x + y + z)$$

b) $2xa + 6xb + 8xc$

Note that:

- The H.C.F of 2, 6 and 8 is 2.
- x is common to all terms of the given algebraic expression.

$$(2xa + 6xb + 8xc) \div 2x = \frac{2xa}{2x} + \frac{6xb}{2x} + \frac{8xc}{2x} = (a + 3b + 4c)$$



Explain the students how to factorize algebraic expression by solving different examples. Then make two groups of students and given each group 5 flash card of algebraic expression and instruct them to factorize and share the result with whole class.

Note it down

The product of all factors of a polynomials is always equal to the polynomial itself.

6.10.2 Factorization of Algebraic Expressions of the form $ac+ad+bc+bd$

For this type of algebraic expressions, all terms do not have a common factor. However, certain groups of terms possess a common factor. By taking such factors as common, there appears a common factor in all terms. The process of factorization is completed by taking common factors. The following example may help to understand the concept.

Example 1: Factorize:

$$ac + ad + bc + bd$$

Solution:

$$ac + ad + bc + bd$$

$$= a(c + d) + b(c + d)$$

$$= (c + d)(a + b)$$

$$\therefore ac + ad + bc + bd = (c + d)(a + b)$$

Note it down

There are certain polynomials which do not have factors.

Note that:

- All four terms of the given algebraic expressions do not possess a common factor.
- The first two terms possess 'a' as the common factor and the third and fourth terms possess 'b' as a common factor.
- By taking 'a' and 'b' as common, the given algebraic expression reduces to two terms (from 4 terms) and thus there appears $(c + d)$ as the common factor in these two terms.
- The process of factorization comes to its logical end when this newly appeared common factor ' $(c+d)$ ' is taken as common from the two terms.

Exercise 6.10

1 Factorize the following algebraic expressions.

a) $2a + 6$

b) $5x - 10$

c) $x^2 - x$

d) $3x^2 + 12x^2$

e) $7a - 7b + 7c$

f) $3b^2 + 6b - 15$

g) $5ax + 10ay - 25az$

h) $7p^3 - 14p^2 - 21p$

i) $10p^2q + 25pq^2 - 35pqr$

j) $11ab - 22bc + 33cd$

k) $a^2 + ab + 3a + 3b$

l) $4a^2 + 4b^2 + a^2c^2 + b^2c^2$

m) $3ax + 3ay - 14x - 14y$

n) $20xz - 10xy - 15yz$

o) $2tx + 6ty - 3qx - 9qy$

p) $4ay - 20by - az + 5bz$

q) $pr + ps + qr + qs$

r) $su - sv - tu + tv$

s) $6x^2 - 4xy - 9xy + 6y^2$

t) $15ax^2 + 21ay^2 - 20bx^2 - 28by^2$

6.11 Factorization of Algebraic Expressions of the Form $a^2 \pm 2ab + b^2$

You have learnt three most basic algebraic formulae in Grade VII. You also revised them in the beginning of this chapter. Let us recall once again. These are:

$$1. (a + b)^2 = a^2 + 2ab + b^2 \quad 2. (a - b)^2 = a^2 - 2ab + b^2$$

These formulae can also be used to factorize an algebraic expression of the form $a^2 \pm 2ab + b^2$.

Consider,

$$(a + b)^2 = a^2 + 2ab + b^2$$

$$a^2 + 2ab + b^2 = (a + b)^2 \quad \text{---- (1)}$$

$$\text{Also, } (a + b)(a + b) = (a + b)^2 \quad \text{---- (2)}$$

Comparing equations (1) and (2), we can easily deduce that $a^2 + 2ab + b^2 = (a + b)(a + b)$. In this way, the algebraic expression $a^2 + 2ab + b^2$ has been expressed as the product of its factors $(a + b)$ and $(a + b)$. Similarly, it can also be proved that $a^2 - 2ab + b^2 = (a - b)(a - b)$.

Thus, if an algebraic expression is a perfect square, its factorization can be done with the help of the basic algebraic formulas. The following example may help you to understand the concept.

Example 1: Factorize the following algebraic expressions.

a) $4x^2 + 12x + 9y^2$

b) $25a^2 - 2ab + \frac{1}{25}b^2$

Solution:

$$\begin{aligned} \text{a) } 4x^2 + 12x + 9y^2 &= (2x)^2 + 2(2x)(3y) + (3y)^2 \\ &= (2x + 3y)^2 \\ &= (2x + 3y)(2x + 3y) \end{aligned}$$

$$\begin{aligned} \text{b) } 25a^2 - 2ab + \frac{1}{25}b^2 &= (5a)^2 - 2(5a)\left(\frac{1}{5}b\right) + \left(\frac{1}{5}b\right)^2 \\ &= \left(5a - \frac{1}{5}b\right)^2 \\ &= \left(5a - \frac{1}{5}b\right)\left(5a - \frac{1}{5}b\right) \end{aligned}$$

Note it down

$(a + b)^2$ is the perfect square of the sum of two numbers. Its expanded form is $a^2 + 2ab + b^2$.

Quick Check

1. Evaluate $\left(1 - \frac{1}{2}\right)^2$ by using an algebraic formula
2. Expand $\left(\frac{3}{2}x - \frac{2}{3}y\right)^2$
3. Factorize $25 - 10y + y^2$
4. Factorize $t^2 + 2tq + q^2$
5. Factorize $9x^2 + 30xy + 25y^2$

6.11.1 Factorization of Algebraic Expressions of Form $a^2 - b^2$

Recall the third basic algebraic formula which is as under:

$$a^2 - b^2 = (a - b)(a + b)$$

It can also be considered that $a^2 - b^2$ is an algebraic expression whose factors are $(a - b)$ and $(a + b)$. That is why this formulae helps us to factorize algebraic expressions of form $a^2 - b^2$.

The following example may further explain the idea.

Example 1: Factorize the following algebraic expressions.

a) $4a^2 - 25b^2$

b) $49p^2 - 100q^2$

Solution:

$$\begin{aligned} \text{a) } 4a^2 - 25b^2 &= (2a)^2 - (5b)^2 \\ &= (2a - 5b)(2a + 5b) \end{aligned}$$

Thus, factors of $4a^2 - 25b^2$ are $(2a - 5b)$ and $(2a + 5b)$.

$$\begin{aligned} \text{b) } 49p^2 - 100q^2 &= (7p)^2 - (10q)^2 \\ &= (7p - 10q)(7p + 10q) \end{aligned}$$

Thus, $49p^2 - 100q^2 = (7p - 10q)(7p + 10q)$

6.11.2 Factorization of Algebraic Expressions of Form $a^2 \pm 2ab + b^2 - c^2$

For this type of algebraic expressions, the part $a^2 \pm 2ab + b^2$ of the given algebraic expression is a perfect square and the last term c^2 is also a perfect square. Thus, such expressions can be reduced to form $a^2 - b^2$.

Consider the following example:

Example 1: Factorize the following algebraic expressions.

i. $a^2 + 10ab + 25b^2 - 49c^2$

ii. $36x^2 - 108xy + 81y^2 - 225z^2$

Solution:

$$\begin{aligned} \text{i. } a^2 + 10ab + 25b^2 - 49c^2 &= a^2 + 2(a)(5b) + (5b)^2 - (7c)^2 \\ &= (a + 5b)^2 - (7c)^2 \\ &= (a + 5b - 7c)(a + 5b + 7c) \end{aligned}$$

$$\begin{aligned} \text{ii. } 36x^2 - 108xy + 81y^2 - 225z^2 &= (6x)^2 - 2(6x)(9y) + (9y)^2 - (15z)^2 \\ &= (6x - 9y)^2 - (15z)^2 \\ &= (6x - 9y - 15z)(6x - 9y + 15z) \end{aligned}$$

Exercise 6.11

1 Factorize the following algebraic expressions.

a) $a^2 + 6a + 9$

b) $4a^2 + 20ab + 25b^2$

c) $9x^2 + 12xy + 4y^2$

d) $4a^2 - 12a + 9$

e) $9a^2 - 30ab + 25b^2$

f) $25p^2 + 70pq + 49q^2$

g) $49p^2 - 126pq + 81q^2$

h) $49t^2 + 154tv + 121v^2$

$$i) r^2 - \frac{4}{5}tv + \frac{4}{25}v^2$$

$$k) a^2 - 25$$

$$m) 9r^2 - 49q^2$$

$$o) 225x^2 - 256y^2$$

$$q) 9b^2 + 42b + 49 - d^2$$

$$j) 16x^2 - 56xy + 49y^2$$

$$l) 4b^2 - 81$$

$$n) \frac{9}{16}u^2 - \frac{25}{49}v^2$$

$$p) a + 5)^2 - c^2$$

$$r) 25p^2 - 70pq + 49q^2 - 25r^2$$

6.12 Some More Algebraic Formulae

You have learnt some most basic algebraic formulae in Grade VII. You not only revised these formulae in Grade VIII but also learnt the application of these formulae to solve certain problems involving evaluation and factorization of algebraic expressions. Let us learn two more algebraic formulae, which are:

$$a) (a + b)^3 = a^3 + 3a^2b + 3ab^2 + b^3$$

$$b) (a - b)^3 = a^3 - 3a^2b + 3ab^2 - b^3$$

6.12.1 Proof of the First Formula

We know that

$$(a+b)^2 = (a+b)(a+b)$$

$$(a+b)^3 = (a+b)(a^2 + 2ab + b^2) \quad [\because (a+b)^2 = a^2 + 2ab + b^2]$$

$$(a+b)^3 = a(a^2 + 2ab + b^2) + b(a^2 + 2ab + b^2)$$

$$(a+b)^3 = a^3 + 2a^2b + ab^2 + a^2b + 2ab^2 + b^3$$

$$(a+b)^3 = a^3 + 2a^2b + a^2b + ab^2 + 2ab^2 + b^3$$

$$(a+b)^3 = a^3 + 3a^2b + 3ab^2 + b^3$$

$$\text{Thus, } (a+b)^3 = a^3 + 3a^2b + 3ab^2 + b^3$$

6.12.2 Proof of the Second Formula

We also know that,

$$(a-b)^2 = (a-b)(a-b)$$

$$(a-b)^3 = (a-b)(a^2 - 2ab + b^2) \quad [\because (a-b)^2 = a^2 - 2ab + b^2]$$

$$(a-b)^3 = a(a^2 - 2ab + b^2) - b(a^2 - 2ab + b^2)$$

$$(a-b)^3 = a^3 - 2a^2b + ab^2 - a^2b + 2ab^2 - b^3$$

$$(a-b)^3 = a^3 - 2a^2b - a^2b + ab^2 + 2ab^2 - b^3$$

$$(a-b)^3 = a^3 - 3a^2b + 3ab^2 - b^3$$

$$\text{Thus, } (a-b)^3 = a^3 - 3a^2b + 3ab^2 - b^3$$

Note it down

Consider

$$(a+b)^2 =$$

$$a^2 + 3a^2b + 3ab^2 + b^2$$

$$(a+b)^3 =$$

$$a^3 + 3ab(a+b) + b^3$$

Similarly,

$$(a-b)^2 =$$

$$a^2 - 3a^2b + 3ab^2 - b^2$$

$$(a-b)^3 =$$

$$a^3 - 3ab(a-b) - b^3$$

Thus, these formulae can

also be written as,

$$(a+b)^3 =$$

$$a^3 + b^3 + 3ab(a+b)$$

$$(a-b)^3 =$$

$$a^3 - b^3 - 3ab(a-b)$$

These formulae can also help us to solve certain problems of different kinds, like evaluation and factorization. Let us study some examples to understand the application of these formulae.

Example 1:

Expand the following:

$$a) (2a + 3b)^3$$

$$b) (4x + y)^3$$

Solution:

$$a) (2a + 3b)^3$$

$$= (2a)^3 + 3(2a)^2(3b) + 3(2a)(3b)^2 + (3b)^3$$

$$= 8a^3 + 3(4a^2)(3b) + 6a(9b^2) + 27b^3$$

$$= 8a^3 + 36a^2b + 54ab^2 + 27b^3$$

$$b) (4x + y)^3$$

$$= (4x)^3 + 3(4x)^2(y) + 3(4x)(y^2) + y^3$$

$$= 64x^3 + 3(16x^2)(y) + 12xy^2 + y^3$$

$$= 64x^3 + 48x^2y + 12xy^2 + y^3$$

Example 2: Expand the following:

$$a) (5a - 3)^3$$

$$b) (4p - 5q)^3$$

Solution:

$$a) (5a - 3)^3$$

$$= (5a)^3 - 3(5a)^2(3) + 3(5a)(3)^2 - (3)^3$$

$$= 125a^3 - 3(25a^2)(3) + 15a \times 9 - 27$$

$$= 125a^3 - 225a^2 + 135a - 27$$

$$b) (4p - 5q)^3$$

$$= (4p)^3 - 3(4p)^2(5q) + 3(4p)(5q)^2 - (5q)^3$$

$$= 64p^3 - 3(16p^2)(5q) + 12p(25q^2) - 125q^3$$

$$= 64p^3 - 240p^2q + 300pq^2 - 125q^3$$

Example 3:

Find the value of $x^3 + \frac{1}{x^3}$ when $x + \frac{1}{x} = 3$

Solution:

$$\text{As, } x + \frac{1}{x} = 3$$

$$\text{So, } (x + \frac{1}{x})^3 = 3^3 \quad (\text{Take the cube of the both sides})$$

$$x^3 + 3(x)^2(\frac{1}{x}) + 3(x)(\frac{1}{x})^2 + (\frac{1}{x})^3 = 27$$

$$x^3 + 3 \times x \times x \times \frac{1}{x} + 3 \times x \times \frac{1}{x} \times \frac{1}{x} + \frac{1}{x^3} = 27$$

$$x^3 + 3x + 3 \times \frac{1}{x} + \frac{1}{x^3} = 27$$

$$x^3 + 3(x + \frac{1}{x}) + \frac{1}{x^3} = 27$$

$$x^3 + 3 \times 3 + \frac{1}{x^3} = 27 \quad [\text{Put } (x + \frac{1}{x}) = 3]$$

$$x^3 + 9 + \frac{1}{x^3} = 27$$

$$x^3 + \frac{1}{x^3} = 27 - 9$$

$$x^3 + \frac{1}{x^3} = 18$$

Note it down

The formulae of $(a+b)^3$ and $(a-b)^3$ are called perfect cubes. Be careful about signs of the terms while using the formula $(a-b)^3$.

Evaluate

$$a) (10+5)^3 \quad b) (10-5)^3$$

Note it down

The power or exponent 3 of a number is called cube. So, 5³ is read as 5 cube and $\frac{1}{x^3}$ is read as 1 cube.

Example 4:

Find the value of $x^3 + \frac{1}{x^3}$ when $x - \frac{1}{x} = 7$

Solution:

As, $x - \frac{1}{x} = 7$

$(x - \frac{1}{x})^3 = 7^3$ (Take cube of the both sides of the equation)

$x^3 - 3(x)(\frac{1}{x}) + 3(x)(\frac{1}{x})^2 - (\frac{1}{x})^3 = 243$

$x^3 - 3 \times x \times \frac{1}{x} + 3 \times x \times \frac{1}{x^2} - \frac{1}{x^3} = 243$

$x^3 - 3x + 3 \times \frac{1}{x} - \frac{1}{x^3} = 243$

$x^3 - 3(x - \frac{1}{x}) - \frac{1}{x^3} = 243$

$x^3 - 3 \times 7 - \frac{1}{x^3} = 243$ [Put $x - \frac{1}{x} = 7$]

$x^3 - 21 - \frac{1}{x^3} = 243$

$x^3 - \frac{1}{x^3} = 243 + 21$

Thus, $x^3 - \frac{1}{x^3} = 264$

Quick Check

Write and memorize the cubes of first 10 natural numbers.

Exercise 6.12

1 Expand the following algebraic expressions by using the algebraic formulae.

a) $(2a + 3b)^3$

b) $(x + 5y)^3$

c) $(a - 5b)^3$

d) $(3p + 4q)^3$

e) $(5p - 7q)^3$

f) $(2t - 3u)^3$

g) $(x + \frac{1}{x})^3$

h) $(x - \frac{1}{x})^3$

i) $(4p - \frac{1}{4})^3$

j) $(5x + \frac{1}{5x})^3$

k) Find the value of $x^3 + \frac{1}{x^3}$ when $x + \frac{1}{x} = 4$

l) Find the value of $x^3 + \frac{1}{x^3}$ when $x + \frac{1}{x} = 7$

m) Find the value of $x^3 - \frac{1}{x^3}$ when $x - \frac{1}{x} = 3$

n) Find the value of $x^3 - \frac{1}{x^3}$ when $x - \frac{1}{x} = 5$

o) Find the value of $x^3 + \frac{1}{x^3}$ when $x + \frac{1}{x} = 8$

p) Find the value of $x^3 - \frac{1}{x^3}$ when $x - \frac{1}{x} = 10$

NOT FOR SALE

Think Higher



- Observe the figure given above and find the area of the pink, green and orange shapes.
- Do orange and blue shapes have same area?
- Do green, pink and yellow shapes have same area?

Summary

- The term-to-term rule of a pattern describes how to get next term in that specific pattern.
- The fixed number which is added or subtracted in an arithmetic sequence is called common difference because if we find the difference of any two successive terms, we will always get this common difference.
- The number multiplied to each term of the geometric sequence is known as the common ratio, because if we divide any two successive terms, we will always get this common ratio.
- A statement (or sentence) that does not include enough information required to decide whether it is true or false is known as an open statement.
- The concept of polynomials helps us to translate many mathematical and real life problems into algebraic expressions and equations. In this way, this concept helps us to solve the problems by using algebra.

Vocabulary

- Arithmetic sequence
- Geometric sequence
- Polynomial
- Constant
- Variable
- Biquadratic

NOT FOR SALE

- "Mono" means one.
- "Bi" means two.
- "Tri" means three.
- Only like terms can be combined together to form a single term.
- Division of polynomials is the reverse process of multiplication of polynomials.
- If we know that the product of any two polynomials and one of the two polynomials, we can find the other polynomial by division.
- If the division is exact, the divisor and the quotient are the polynomials whose product is equal to the dividend. Hence, the divisor and the quotient are the factors of the dividend in such a dividing the product by the given polynomials.
- When the division of polynomials is not exact, the divisor and the quotient are not the factors of the dividend.
- Factors of a number are numbers by which it is exactly divisible.
- A number is a multiple of all of its factors.
- $(a+b)^2 = a^2 + b^2 + 2ab(a+b)$
- $(a-b)^2 = a^2 + b^2 - 2ab(a-b)$

Review Exercise

1. Choose the correct option.

- a) The general term for the sequence -5, 7, 19, 31, ...
- i. $a_n = 5 + 12(n+1)$ ii. $a_n = -5 + 12(n+1)$
 iii. $a_n = 5 - 12(n+1)$ iv. $a_n = -5 + 12(n-1)$
- b) The 11th term of the sequence 3, 11, 19, 27, ... is:
- i. 80 ii. 83 iii. 91 iv. 75
- c) A sequence of numbers in which each term is obtained by multiplying or dividing the preceding term by a fixed number is called:
- i. Arithmetic sequence ii. Geometric series
 iii. Fibonacci sequence iv. term to term rule
- d) A polynomial with degree three is known as a _____ polynomial.
- i. linear ii. quadratic iii. cubic iv. biquadratic
- e) The result of multiplication of $(2x + 3)$ and $(1 + 4x)$ is:
- i. $8x^2 + 14x + 3$ ii. $14x + 3$ iii. $8x^2 + 14x - 3$ iv. $8x^2 - 14x + 3$

f) $(2a - 4)^2$ is equal to:

- i. $4a^2 - 8a + 16$ ii. $4a^2 - 16a + 16$
 iii. $a^2 - 16a + 16$ iv. $4a^2 - 16a - 16$

g) The factorization of $x^2 + 7x + 12$ is:

- i. $(x + 4)(x - 3)$ ii. $(x - 4)(x - 3)$
 iii. $(x + 4)(x + 3)$ iv. $(x - 4)(x + 3)$

h) $(m - n)^3$ is equal to:

- i. $m^3 - 3m^2n + 3mn^2 - n^3$ ii. $m^3 + 3m^2n + 3mn^2 - n^3$
 iii. $m^3 - 3m^2n - 3mn^2 - n^3$ iv. $m^3 - 3m^2n + 3mn^2 + n^3$

i) An _____ is a combination of variables, constant, coefficient, exponent and symbol of operations.

- i. expression ii. equation iii. equality iv. inequality

j) _____ are the algebraic expressions in which the powers of variables are whole numbers.

- i. expressions ii. arithmetic sequences
 iii. open sentences iv. polynomials

2. Find the 27th and 34th term of the given sequence 31, -21, -11, -1, 9, ..

3. Rehan bought a bike in Rs 87800 on installment. If he pays Rs 1500 in one month, 2200 on the second month, 2900 on third month. If he keeps increasing the installment money. How much will he pay on 11th month and 15th month?

4. Wasif wants to participate in a race. For this, he is going to nearby park for running practice. First day he runs for 15 minutes, second day he runs for 30 minutes, third day he runs for 45 minutes. He keeps increasing the time for practice. For how much time will he run on 12th and 19th day.

5. Rabia gave donation to an Eidi centre. First month she donated Rs 3000, next month she donate the money with increase of Rs750 that is Rs 3750, third month she donated Rs 4500. If she keeps donating on next months in the same pattern, how much she will donate in 7th month and 11th month.

6 Find the 21st term of an arithmetic sequence with first term -9 and the common difference is 11.

7 Find the sum of $5x - 3y + 5z$, $2x + 7y$ and $7y + 12z$

8 Find the difference of $15x^4 + 4y^3 + 10x + 5$ and $7x^4 - 8x^2 + 7y^3 - 8$

9 Find the product of $(x^3 + yz + 2y^2)$ and $x^2 + 2y$

10 Simplify the following.

a) $5v - v[4u^2 + 5w(4v + 2w)(u^2 + 2v)]$

b) $6y + 3xy[7y^2 - (5x^3 + (6y + 3x^2y))]$

11 Divide $6x^2y + 3xy + 3y^2$ by $3xy$

12 Simplify the following.

a) $(y + 5)(y - 5)$

b) $(4t + 5u)^2$

c) $(102)^2$

13 Find the value of $x^2 + \frac{1}{x^2}$ when $x + \frac{1}{x} = 4$

14 Factorize the following expression.

a) $9x^2 + 6xy + 6xy + 6y^2$

b) $3s + 6t$

c) $b^2 + 6ab + (3a)^2$

15 1. Find the value of $x^3 + \frac{1}{x^3}$ if:

a) $x + \frac{1}{x} = 6$

b) $x - \frac{1}{x} = 12$

16 An arithmetic sequence has the first term 7 and the common difference between the terms is 9, find the general term, also find the 11th and 12th term.

Math Project

Material Required:

- Flash cards of algebraic identities
- Blank papers
- Scoring sheet

Identity I

$$(a + b)^2 = a^2 + 2ab + b^2$$

Identity II

$$(a - b)^2 = a^2 - 2ab + b^2$$

Identity III

$$a^2 - b^2 = (a + b)(a - b)$$

Identity IV

$$(x + a)(x + b) = x^2 + (a + b)x + ab$$

Procedure:

- Get students into 4 small groups.
- Put the card upside down on the table. Instruct each group to come forward and choose one card and go back to their position.
- When each group has one card of algebraic identities. Now instruct them to show their cards.
- Ask them to show algebraic and geometric proof of these identities with the help of example.
- Each group will complete the challenge and then get it checked by the other group.
- Record the time to complete the task for each group.
- The group with most accurate and quick response wins.

Unit 7

Exponents and Laws of Exponents

Student Learning Outcomes

After completing this unit, students will be able to:

- Identify base, index/ exponent and its value.
- Deduce and apply the following laws of Exponents/ Indices:
 - Product Law
 - Quotient Law
 - Power Law



Planet Mars is at an estimated average distance of 228,000,000 million kilometer. Can this distance be written in any shorter form?

Introduction

In many of our life situation we use exponent where we want to make lengthy calculation much easier. We use it when we find the area of something. In this unit we will learn about base, exponent and its value. we will also learn about some laws of exponent.

7.1 Base, Exponent and Value

Several mathematical calculations require writing a number in the form of repeated multiplication when it is being multiplied by itself multiple times. To write lengthy repeated multiplication, we need a lot of space and it's a cumbersome activity. To make repeated multiplication easier to express, we use exponential notation. It shows how many times a number, variable or expression is being multiplied by itself using an exponent that is raised a little when writing it.

For example:

16 can be written as:

$$2 \times 2 \times 2 \times 2 = 2^4 \text{ (2 to the power 4)}$$

The numbers on the left side of the above example are in expanded form while the number on the right is in exponential form. It represents the same repeated multiplication in short and precise form.

$$\text{In } 2^4 = 16,$$

2 is called the base of 2^4

4 is called the exponent of 2^4

16 is the value of 2^4

Similarly,

25 can be written as:

$$5 \times 5 = 5^2 \text{ (5 to the power 2).}$$

$$\begin{array}{c} \text{exponent (power)} \\ \text{4 times 2} \downarrow \\ 2 \times 2 \times 2 \times 2 = 2^4 = 16 \leftarrow \text{value} \\ \uparrow \\ \text{base} \end{array}$$

Note it down

Exponents are also called Powers or Indices.

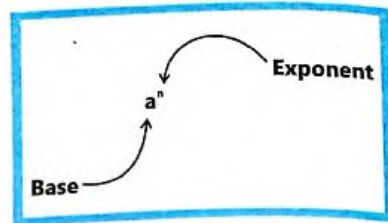
In the same way, $\frac{8}{125}$ can be written as:

$$\frac{2}{5} \times \frac{2}{5} \times \frac{2}{5} = \left(\frac{2}{5}\right)^3 \text{ (read as } \frac{2}{5} \text{ to the power 3)}$$

In general, we can write:

$$a \times a \times a \times \dots \times a \text{ (n times)} = a^n$$

It says: When a number **a** is multiplied by itself **n** number of times, it can be written in exponential form as **aⁿ** where **a** is the **base** and the superscript number **n** written above the base is called the **exponent**.



Example 1:

Identify the base and exponent in each of the following.

a) 6^7

Solution: 6 = base

7 = exponent

b) b^t

b = base

t = exponent

c) $\left(\frac{a}{b}\right)^4$

$\frac{a}{b}$ = base

4 = exponent

Example 2:

Write the following in exponential form and identify its base, exponent, and value. Also tell the way to read the exponential form.

Solution:

a) 7×7

$$7 \times 7 = 7^2 = 49$$

7 is the base, 2 is the exponent and 49 is the value of 7^2 .

In words 7^2 is read as: "7 to the power 2" or simply "7 squared".

b) $3 \times 3 \times 3$

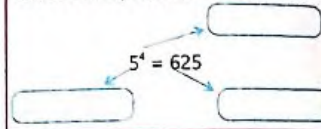
$$3 \times 3 \times 3 = 3^3 = 27$$

3 is the base, 3 is the exponent, and 27 is the value of 3^3 .

In words 3^3 is read as: "3 cubed" or "3 to the power 3".

Quick Check

Label the exponent, base and value.



Note it down

The result of any calculation is called its **value**.



Write 4 to 5 numbers with powers on the board. Now call the students one by one and ask them to identify the base and exponent of the numbers.

c) $\left(\frac{3}{5}\right) \times \left(\frac{3}{5}\right) \times \left(\frac{3}{5}\right)$

$$\left(\frac{3}{5}\right) \times \left(\frac{3}{5}\right) \times \left(\frac{3}{5}\right) = \left(\frac{3 \times 3 \times 3}{5 \times 5 \times 5}\right) = \left(\frac{3}{5}\right)^3 = \frac{27}{125}$$

$\left(\frac{3}{5}\right)$ is the base, 3 is the exponent, and $\frac{27}{125}$ is the value of $\left(\frac{3}{5}\right)^3$.

In words $\left(\frac{3}{5}\right)^3$ is read as: " $\frac{3}{5}$ cubed" or " $\frac{3}{5}$ to the power 3".

Similarly, 169 can be written as 13×13 or 13^2 .

Also, $\left(\frac{81}{64}\right) = \left(\frac{9 \times 9}{8 \times 8}\right) = \left(\frac{9}{8}\right)^2$.

Exponential expressions can also be written as rational numbers.

$$\left(\frac{-3}{4}\right)^3 = \left(\frac{-3}{4}\right) \times \left(\frac{-3}{4}\right) \times \left(\frac{-3}{4}\right) = \left(\frac{-27}{64}\right)$$

Outside Circle

Write in exponential form.

General form	Exponential form
$5 \times 5 \times 5 \times 5 \times 5$	
$b \times b \times b \times b \times b \times b$	
$\frac{7}{9} \times \frac{7}{9} \times \frac{7}{9}$	

Exercise 7.1

1 Identify the exponent and base of the following terms. Then write the value of each.

a) 7^2

b) $\left(\frac{1}{2}\right)^3$

c) $(5)^2$

d) 3^4

e) $(1)^9$

f) $(8)^2$

g) $\left(\frac{7}{13}\right)^8$

h) $(12)^2$

i) $\left(\frac{1}{3}\right)^3$

j) $\left(\frac{22}{133}\right)^2$

k) 144^2

l) $\left(\frac{-3}{4}\right)^5$

2 Write the following in exponential form. Also tell the way to read the exponential form.

a) $4 \times 4 \times 4 \times 4 \times 4$

b) $n \times n \times n \times n \times n \times n$

c) $ab \times ab \times ab$

d) $e \times e \times e \times e$

e) $\left(\frac{-1}{2}\right) \times \left(\frac{-1}{2}\right) \times \left(\frac{-1}{2}\right)$

f) $\left(\frac{1}{9}\right) \times \left(\frac{1}{9}\right) \times \left(\frac{1}{9}\right) \times \left(\frac{1}{9}\right)$

g) $\left(\frac{a}{b}\right) \times \left(\frac{a}{b}\right) \times \left(\frac{a}{b}\right) \times \left(\frac{a}{b}\right)$

h) $(-by) \times (-by) \times (-by) \times (-by) \times (-by) \times (-by)$

i) $(-9) \times (-9) \times (-9) \times (-9)$

j) $\left(\frac{1}{10}\right) \times \left(\frac{1}{10}\right) \times \left(\frac{1}{10}\right) \times \left(\frac{1}{10}\right) \times \left(\frac{1}{10}\right)$

3 Prove the following equalities.

a) $\left(\frac{5}{2}\right)^5 = \frac{3125}{32}$

b) $(7)^6 = 117649$

c) $\left(\frac{4}{5}\right)^2 = \frac{16}{25}$

d) $(5)^3 = 125$

e) $(\frac{2}{6})^1 = (-\frac{8}{216})$ f) $(4)^1 = 64$ g) $(\frac{1}{10})^4 = (\frac{1}{10000})$ h) $(1)^8 = 1$
 i) $(\frac{7}{6})^5 = (-\frac{16807}{7776})$ j) $(12)^3 = 1728$

4 Express the following rational numbers in exponential form.

a) 196 b) 64 c) 729 d) $\frac{1}{100}$ e) $\frac{36}{441}$ f) $\frac{1}{16}$ g) 27 h) $\frac{64}{216}$

7.2 Laws of Exponents

The laws of exponents help us to simplify expressions involving exponents. We will learn about the following laws of exponents in detail.

1. Product Law
2. Quotient Law
3. Power Law
4. Zero Exponent Law
5. Negative Exponent Law

1) Product Law

This law is also called the law for multiplication of exponents. This law addresses the following two cases.

Case I: When bases are the same but exponents are different.

Case II: When bases are different but exponents are the same.

Case I: When bases are the same but exponents are different.

Example 1: Consider the product of 5^4 and 5^2 .

$$\begin{aligned} 5^4 \times 5^2 &= (5 \times 5 \times 5 \times 5) \times (5 \times 5) \\ &= 5 \times 5 \times 5 \times 5 \times 5 \times 5 \\ &= 5^6 \end{aligned}$$

We can observe that the exponent 6 of the product 5^6 is the sum of the exponents 4 and 2, of 5^4 and 5^2 .

So, we can write that $5^4 \times 5^2 = 5^{4+2} = 5^6$

Example 2: Consider $(\frac{4}{7})^2 \times (\frac{4}{7})^3$

$$\begin{aligned} (\frac{4}{7})^2 \times (\frac{4}{7})^3 &= [(\frac{4}{7}) \times (\frac{4}{7})] \times [(\frac{4}{7}) \times (\frac{4}{7}) \times (\frac{4}{7})] \\ &= \frac{4}{7} \times \frac{4}{7} \times \frac{4}{7} \times \frac{4}{7} \times \frac{4}{7} \\ &= (\frac{4}{7})^5 \end{aligned}$$

Note it down

If the exponents are negative integers, they will be added according to the laws of addition of integers.

TEACHING POINT Write the examples of product law with both conditions on the board. Now call the students one by one and ask them to differentiate them according to both conditions and then solve the examples using the laws.

NOT FOR SALE

So, we can say that when multiplying two or more rational numbers with the same base but different exponents, we can add the exponents while keeping the base same.

General form:

If a is any rational number and $m, n \in \mathbb{Z}$, then $a^m \times a^n = a^{m+n}$.

Case II: When bases are different but exponents are the same.

Example 1: Consider the product of 3^2 and 2^2 .

$$\begin{aligned} 3^2 \times 2^2 &= (3 \times 3) \times (2 \times 2) \\ &= 3 \times 3 \times 2 \times 2 \\ &= (3 \times 2) \times (3 \times 2) \\ &= (3 \times 2)^2 \end{aligned}$$

We can observe that the product of two numbers 3^2 and 2^2 having the same exponents can be written as the product of numbers with the common exponent i.e. 2.

Example 2: $(\frac{1}{3})^3 \times (\frac{2}{3})^3$

$$\begin{aligned} (\frac{1}{3})^3 \times (\frac{2}{3})^3 &= (\frac{1}{3} \times \frac{1}{3} \times \frac{1}{3}) \times (\frac{2}{3} \times \frac{2}{3} \times \frac{2}{3}) \\ &= (\frac{1}{3} \times \frac{2}{3}) \times (\frac{1}{3} \times \frac{2}{3}) \times (\frac{1}{3} \times \frac{2}{3}) \\ &= (\frac{1}{3} \times \frac{2}{3})^3 \end{aligned}$$

Quick Check

Apply the product law and find the value of the following.

- | | |
|---------------------|---|
| a) 3 times 3^4 | d) $2^3 \times 2^4$ |
| b) $5^4 \times 5^8$ | e) $2^5 \times 2^3$ |
| c) $2^3 \times 2^6$ | f) $(\frac{2}{3})^2 \times (\frac{1}{4})^2$ |

So, we can say that when multiplying two or more rational numbers having the same exponents but different bases, we can write it as the product of the bases with the common exponent.

General form:

If a and b are any two rational numbers and $m \in \mathbb{Z}$, then $a^m \times b^m = (a \times b)^m$.

2) Quotient Law

This law is also called the law for division because it deals with division of exponents and addresses the following two cases.

Case I: When bases are the same but exponents are different.

Case II: When bases are different but exponents are the same.

Case I: When bases are the same but exponents are different.

Example 1: Consider the division of 6^5 and 6^3 .



Write the examples of quotient law with both conditions on the board. Now call the students one by one and ask them to differentiate them according to both conditions and then solve the examples using the laws.

NOT FOR SALE

$$6^5 \div 6^3 = \frac{6 \times 6 \times 6 \times 6 \times 6}{6 \times 6 \times 6}$$

$$= 6 \times 6$$

$$= 6^2$$

We can observe that the exponent 2 of the product 6^2 is the difference of the exponents 5 and 3, of 6^5 and 6^3 . So, we can write $6^5 \div 6^3 = 6^{5-3} = 6^2$.
When dividing two rational numbers with the same base but different exponents, we subtract the exponent of the denominator from the exponent of the numerator while keeping the base same.

General form:
If a is any non-zero rational number and $m, n \in \mathbb{Z}$, then $a^m \div a^n = a^{m-n}$.

Case II: When bases are different but exponents are the same.

Example 1: Divide 2^5 by 4^5 .

Solution: $2^5 \div 4^5 = (2 \times 2 \times 2 \times 2 \times 2) \div (4 \times 4 \times 4 \times 4 \times 4)$

$$= \frac{2 \times 2 \times 2 \times 2 \times 2}{4 \times 4 \times 4 \times 4 \times 4}$$

$$= \frac{2}{4} \times \frac{2}{4} \times \frac{2}{4} \times \frac{2}{4} \times \frac{2}{4}$$

$$= \frac{2}{4}$$

$$= \left(\frac{2}{4}\right)^5$$

We can observe that the quotient of 2^5 and 4^5 can be written as the quotient of 2 and 4 with the same exponent 5.

Example 2: $\left(\frac{1}{4}\right)^3 \div \left(\frac{2}{5}\right)^3$

Solution: $\left(\frac{1}{4}\right)^3 \div \left(\frac{2}{5}\right)^3 = \left(\frac{1}{4} \times \frac{1}{4} \times \frac{1}{4}\right) \div \left(\frac{2}{5} \times \frac{2}{5} \times \frac{2}{5}\right)$

$$= \left(\frac{1}{4} \div \frac{2}{5}\right) \times \left(\frac{1}{4} \div \frac{2}{5}\right) \times \left(\frac{1}{4} \div \frac{2}{5}\right)$$

$$= \left(\frac{1}{4} \div \frac{2}{5}\right)^3$$

So, we observed that when dividing two rational numbers with the same exponents but different bases, we can write it as the quotient of the bases with the common exponent.

General form:

If a and b are any two rational numbers and $m \in \mathbb{Z}$, then $a^m \div b^m = (a \div b)^m$.

Quick Check

Solve:

- i) $x^{20} \div x^{15}$ ii) $a^{10} \div a^{40}$
iii) $p^{56} \div p^{43}$ iv) $2^9 \div 2^6$
v) $4^1 \div 4^5$

Quick Check

Apply the quotient law and find the value of the following.

- a) $x^{19} \div y^{19}$ f) $4^5 \div 6^5$
b) $a^{49} \div b^{49}$ g) $\left(\frac{1}{6}\right)^5 \div \left(\frac{1}{7}\right)^5$
c) $p^{56} \div q^{56}$ h) $\left(\frac{3}{4}\right)^9 \div \left(\frac{2}{3}\right)^9$
d) $2^9 \div 3^9$
e) $4^5 \div 5^5$

3) Power Law

This law explains how to solve an expression involving multiple powers (exponents).

Example 1: Simplify the expression $(5^2)^3$.

First we consider 5^2 as the base with exponent 3.

Solution: $(5^2)^3 = 5^2 \times 5^2 \times 5^2$

$$(5^2)^3 = 5^{2+2+2} \quad (\text{applying product law})$$

$$(5^2)^3 = 5^6$$

We can note that $(5^2)^3 = 5^6 = 5^{2 \times 3}$

It shows that if a number has two or more exponents (powers), we multiply the powers together while keeping the same base.

Example 2: Simplify $\left[\left(-\frac{1}{3}\right)^2\right]^2$

Solution: $\left[\left(-\frac{1}{3}\right)^2\right]^2 = \left(-\frac{1}{3}\right)^{2 \times 2}$

$$= \left(-\frac{1}{3}\right)^4$$

General form:

If a is any rational number and $m, n \in \mathbb{Z}$, then $(a^m)^n = a^{m \times n} = a^{mn}$

4) Zero Exponent Law

Example 1: Divide 2^4 by 2^4 .

Solution: $2^4 \div 2^4 = (2 \times 2 \times 2 \times 2) \div (2 \times 2 \times 2 \times 2)$

$$= \frac{(2 \times 2 \times 2 \times 2)}{(2 \times 2 \times 2 \times 2)}$$

$$= 1$$

But according to quotient law, $2^4 \div 2^4 = 2^{4-4} = 2^0$

So, we can see that $2^0 = 1$.

So, we observed that a number having 0 as its exponent is always equal to 1.

General form:

If a is any non-zero rational number, then $a^0 = 1$.

5) Negative Exponent Law

Example 1: Divide 3^3 by 3^5 .

Solution: $3^3 \div 3^5 = \frac{(3 \times 3 \times 3)}{(3 \times 3 \times 3 \times 3 \times 3)}$

Note it down

When multiplying two numbers m and n , $m \times n$ is same as $m.n$ or mn .

Quick Check

Find the value of the following.

- a) 101^0 b) 2^0 c) $\left(\frac{22}{9}\right)^0$
d) $(1)^0$ e) 100^0 f) $5^4 \div 5^4$



Write the examples of power law, zero exponent law and negative exponent law on the board. Now call the students one by one and ask them to solve the questions using the laws.

$$= \frac{1}{3 \times 3}$$

$$= \frac{1}{3^2} \text{ ----- (A)}$$

But according to quotient law:

$$3^3 \div 3^5 = 3^{3-5} = 3^{-2} \text{ ----- (B)}$$

From (A) and (B), we can observe that: $3^{-2} = \frac{1}{3^2}$

So, in a rational number, if the numerator having a negative exponent moves to the denominator, its exponent becomes positive (and vice versa).

General form:

If a is any non-zero rational number and $m \in \mathbb{Z}$, then $a^{-m} = \frac{1}{a^m}$ and $a^m = \frac{1}{a^{-m}}$.

Reciprocal of numbers with positive exponents

The reciprocal of 3 is $\frac{1}{3}$.

The reciprocal of $(\frac{1}{4})^a$ is $(\frac{4}{1})^a$ or 4^a .

The reciprocal of 2^1 is $\frac{1}{2^1}$.

The reciprocal of $(\frac{3}{2})^5$ is $(\frac{2}{3})^5$.

The reciprocal of $(-\frac{6}{7})^7$ is $(-\frac{7}{6})^7$.

Reciprocal of numbers with negative exponents

The reciprocal of 5^{-2} is $\frac{1}{5^{-2}}$.

$$\frac{1}{5^{-2}} = \frac{5^0}{5^{-2}}$$

$$= 5^{0-(-2)} \text{ (Applying quotient law)}$$

$$= 5^2$$

So reciprocal of 5^{-2} is 5^2 .

Similarly, the reciprocal of 5^2 is 5^{-2} .

The reciprocal of $(\frac{1}{2})^5$ is $(\frac{2}{1})^5$.

The reciprocal of $(-\frac{2}{5})^9$ is $(-\frac{5}{2})^9$.

Quick Check

Find the value of the following.

a) $(3x^6y^3)^7 \times (x^5y^2)^{-1}$ b) $(\frac{4}{9})^3 \div (\frac{4}{9})^7$

Note it down

- One raised to any power is always equal to one.
- Any number raised to the power 1 is the number itself.

Quick Check

Find the reciprocal of the following.

a) $\frac{2}{3^2}$ b) 7^{-5} c) $(-\frac{4}{5})^7$

Note it down

A number having a negative exponent is equal to its reciprocal having a positive exponent i.e. $a^{-n} = \frac{1}{a^n}$.

Example 1:

Simplify $(\frac{3}{4})^{-3} \div (\frac{4}{5})^{-2}$

Solution:

$$(\frac{3}{4})^{-3} \div (\frac{4}{5})^{-2} = (\frac{4}{3})^3 \div (\frac{5}{4})^2$$

$$= \frac{4 \times 4 \times 4}{3 \times 3 \times 3} \times \frac{4 \times 4}{5 \times 5}$$

$$= \frac{1024}{675}$$

6) Power (exponent) of Negative Integers (bases)

Multiplication of negative integers can also be written in exponential form. Consider these examples:

$$(-2) \times (-2) = (-2)^2 = 4 \text{ (minus} \times \text{minus} = \text{plus)}$$

$$(-2) \times (-2) \times (-2) = (-2)^3 = -8$$

(Here, minus \times minus = plus. Another minus turns the result into minus).

Similarly,

$$(-2) \times (-2) \times (-2) \times (-2) = (-2)^4 = 16$$

$$(-2) \times (-2) \times (-2) \times (-2) \times (-2) = (-2)^5 = -32$$

From these examples, we can observe that:

- If a negative base has an even exponent, the value (answer) will always be positive.
- If a negative base has an odd exponent, the value (answer) will always be negative.

Example 1: Find the value of $(-4)^3$.

$$(-4)^3 = (-4) \times (-4) \times (-4) = -64$$

Example 2: Find the value of $(-3)^5 \times (-3)^7$.

$$(-3)^5 = (-3) \times (-3) \times (-3) \times (-3) \times (-3) = -243$$

$$(-3)^7 = (-3) \times (-3) \times (-3) \times (-3) \times (-3) \times (-3) \times (-3) = -2187$$

So,

$$(-3)^5 \times (-3)^7 = (-3)^{12} = 531441$$

Example 3: Find the value of $(-2)^3 \times (-2)^2$.

$$(-2)^3 \times (-2)^2 = (-2)^{3+2} = (-2)^5 = -32$$

Quick Check

Solve:

- a) $(-9) \times (-9) \times (-9)$
 b) $(-5) \times (-5) \times (-5) \times (-5) \times (-5)$
 c) $(-6)^4$ d) $(-4)^4$
 e) $(-7)^3$

Note it down

Following are the sign rules:

$$\oplus \times \oplus = \oplus$$

$$\ominus \times \ominus = \oplus$$

$$\oplus \times \ominus = \ominus$$

$$\ominus \times \oplus = \ominus$$

Exercise 7.2

1 Simplify the following by applying the product law of exponents.

- a) $(-8)^3 \times (-8)^1$ b) $x^3 \times x^1$ c) $(4)^2 \times (5)^2$
 d) $(3)^7 \times (6)^7$ e) $5^2 \times 3^2 \times (5 \times 3)^3$ f) $(x^2 y^3 z^7) \times (x^7 y^4 z^3)$
 g) $4^7 \times 4^5$ h) $(\frac{5}{8})^2 \times (\frac{5}{8})^3$ i) $(\frac{6}{5})^2 \times (\frac{6}{5})^4$
 j) $(3)^6 \times (3)^5$ k) $(2^{-2} \times 4^3 \times 5^{-7}) \times (2^7 \times 4^{-4} \times 5^3)$ l) $(\frac{2}{5})^4 \times (\frac{2}{5})^5 \times (\frac{2}{5})^{-6}$

2 Simplify using the quotient law.

- a) $3^7 \div 3^5$ b) $(-7)^9 \div (-7)^5$ c) $(5)^{12} \div (5)^8$ d) $(\frac{4}{10})^{15} \div (\frac{4}{10})^7$
 e) $(\frac{3}{7})^{21} \div (\frac{3}{7})^7$ f) $(\frac{1}{12})^9 \div (\frac{1}{12})^3$ g) $(3^4 \div 3^2)$ h) $(\frac{21}{37})^{16} \div (\frac{21}{37})^{16}$
 i) $(x)^4 \div (x)^b$ j) $(\frac{4}{5})^3 \div (\frac{4}{5})^3$

3 Express the following as single exponents.

- a) $[(-n)^7]^3$ b) $(-3)^4$ c) $[(\frac{3}{7})^4]^7$ d) $(6^3)^4$ e) $[(-4)^7]^7$
 f) $(\frac{4}{6})^{11}$ g) $[(-a)^3]^2$ h) $(a^b)^c$ i) $[(-\frac{a}{b})^7]^d$

4 Simplify using laws of exponents.

- a) $(\frac{-a}{b})^{11} \div (\frac{-a}{b})^8$ b) $(-3^2)^4$ c) $(2)^0 \times (2)^6$ d) $(\frac{2}{4})^{-8} \times (\frac{12}{5})^{-1} \times (671)^0$
 e) $\frac{1}{9} \times \frac{1}{9} \times \frac{1}{9} \times \frac{1}{9}$ f) $(2^7 \times 2^5)$ g) $6^3 \times \frac{6^7}{6^2}$ h) $\frac{6^{-3} \times 5^{-3}}{5^{-4} \times 6^{-4}}$
 i) $xy^8 \times xy^7$ j) $(4^0 \times 5^0 \times 6^0) + (7^0 \times 8^0 \times 9^0)$ k) $(-4)^3 \times (-4)^2$
 l) $[(\frac{-3}{4})^4]^2$ m) $(7^4)^2$ n) $(\frac{2}{5})^{-4} \div (\frac{6}{4})^{-3}$ o) $(\frac{3}{7})^{-2} \div (\frac{6}{5})^{-1}$
 p) $(-4)^6 \times (-4)^8$ q) $(-7)^4 \times (-7)^6$

7.3 Application of Exponent Laws

We know how to simplify expressions. Here we will apply the laws of exponents to evaluate various expressions. Consider the following examples.

Example 1: Evaluate the following by applying laws of exponents.

a) $(-12x^3)^0$

$(-12x^3)^0 = 1$

(Applying zero exponent law)

TEACHING POINT Write different laws on the board and ask the students to identify them. Then ask them to make one sum on each law and solve them. Lastly ask them to share their finding with the others in the class and ask them if it is right or wrong.

b) $36x^6y^3z^2 \div 12x^4y^3z$

$\frac{36x^6y^3z^2}{12x^4y^3z} = \frac{36}{12} x^{6-4} y^{3-3} z^{2-1}$

(Applying quotient law)

$= 3x^2y^0z^1$

(Applying zero exponent law)

$= 3x^2z$

c) $(5^5 \div 5^3) \times 3^2$

$(5^5 \div 5^3) \times 3^2 = (5^5 \div 5^3) \times 3^2$

$= (5)^{5-3} \times 3^2$

$= (5)^2 \times 3^2$

$= (5 \times 3)^2$

$= (15)^2$

$= 225$

(Applying quotient law; same base

different exponents)

(Applying product law of same exponents, different bases)

d) $(\frac{3}{6})^{-4} \times (\frac{3}{6})^4 + (\frac{4}{7})^5 \times (\frac{4}{7})^{-4}$

$(\frac{3}{6})^{-4} \times (\frac{3}{6})^4 + (\frac{4}{7})^5 \times (\frac{4}{7})^{-4} = (\frac{3}{6})^{-4+4} + (\frac{4}{7})^{5-4}$

(Applying product law; same base different exponents)

$= (\frac{3}{6})^0 + (\frac{4}{7})^1$

$= (\frac{3}{6})^0 + (\frac{4}{7})^1$

$= 1 + \frac{4}{7}$

(applying zero exponent law)

$= \frac{7+4}{7}$

$= \frac{11}{7}$

e) $(\frac{-6}{7})^6 \times (\frac{-6}{7})^{-3} \times [(\frac{-6}{7})^{-2}]^{-1}$

$(\frac{-6}{7})^6 \times (\frac{-6}{7})^{-3} \times [(\frac{-6}{7})^{-2}]^{-1} = (\frac{-6}{7})^{6+(-3)} \times (\frac{-6}{7})^{-2 \times -1}$

(Applying product law)

(Applying power law)

$= (\frac{-6}{7})^3 \times (\frac{-6}{7})^2$

$= (\frac{-6}{7})^{3+2}$

(Applying product law)

$= (\frac{-6}{7})^5$

$= \frac{-7776}{16807}$

Quick Check

Solve:

a) $(4a^3b) \times (-7a^4b)$

b) $7x^4 (-x^5)$

Exercise 7.3

1 Simplify the following using laws of exponents.

a) $\frac{ab^{-2}}{b^{-3}}$ b) $\frac{11ac^{-2}}{(abc)^{-1}}$ c) $\frac{1}{6a} \times a^{-3}$ d) $\frac{5a^5}{3a^4}$
 e) $\frac{8x^4 \times b^3 \times c^4}{16x^2 \times b^2 \times c^2}$ f) $\left(\frac{16 \times 27}{x^4 \times y^4}\right)$ g) $\left[\frac{x^{-3} \times b^6}{x^{-1} \times b^{-3}}\right] \times \left(\frac{a^{-2} \times b^{-2}}{a^{-1} \times b^{-2}}\right)$

2 Prove the following by evaluating both sides.

a) $9^6 \div 9^2 = 9^4$ b) $4^6 \div 6^6 = \left(\frac{4}{6}\right)^6$ c) $(-6)^4 \div (7)^4 = \left(-\frac{6}{7}\right)^4$
 d) $\left(-\frac{22}{23}\right)^9 \div \left(-\frac{22}{23}\right)^4 = \left(-\frac{22}{23}\right)^5$ e) $\left(-\frac{4}{12}\right)^9 \div \left(-\frac{4}{12}\right)^3 = \left(-\frac{4}{12}\right)^6$ f) $x^2 \div y^2 = \left(\frac{x}{y}\right)^2$

3 Evaluate the following using the laws of exponents.

a) $\frac{6^5 \times 6^4}{6^8}$ b) $\frac{2^3 \times 4^{-4} \times 3^{-9}}{2^{-6} \times 4^3 \times 3^{-8}}$ c) $\frac{1}{6} \times \frac{2}{2^{-2}} \div 3$ d) $\left(\frac{1}{3^{-2}}\right)^3$
 e) $\left(\frac{3^2}{5^{-2}}\right) \times 3 \times 5^{-1}$ f) $4(-2)^2 \times \frac{(-2)^5 \times 3^6}{(-2)^{-2} \times 3^2}$ g) $\frac{(4^{-1} \times 3^3)}{4^{-4} \times 3^2}$ h) $\frac{(2 \times y^{-3})^0}{(x^4 y^{-3} z^{-3})^2}$
 i) $(4^4 \times 2^{-5} \times 7^{-2}) \div 4^3 \times 2^{-4} \times 7$ j) $-(4^6 \times 5^2 \times 4^3 \times 5^{10})$ k) $(6^0 + 3^0) \div (2^0 + 10^0)$

4 Prove that:

a) $3a^{3(b+y)} \div 3a^{3(y+r)} = 1$ b) $p^{(a-b)} \times p^{(b-c)} \times p^{(c-a)} = 1$
 c) $\left(\frac{x^a}{x^b}\right)^c \times \left(\frac{x^b}{x^c}\right)^a \times \left(\frac{x^c}{x^a}\right)^b = 1$ d) $(x^6 y^{-3}) \times (x^{-1/2} y^{-2}) \times (x^7 y^5) = 1$

5 Express the following expressions having only positive exponents.

a) $(-9)^5 \times (-4)^7$ b) $\left(\frac{2}{3}\right)^{-5}$ c) $\frac{1}{x^{-6} \times y^{-8}}$
 d) $\frac{x^{-4} \times y^{-5} \times z^{-7}}{x^{-4} \times y^{-7} \times z^{-5-9}}$ e) $(4^{-3} \times 5^{-7})$ f) $(-6rs)^{-6}$

Think Higher

1. Evaluate x.

a) $4^2 = 2^x$
 b) $9^x = 3^8$
 c) $412^x = 1^5$

2. Prove that: $\left[\left(\frac{1}{3}\right)^{-1} - \left(\frac{1}{4}\right)^{-1}\right]^{-1} = -1$

3. Umair is making different objects using lego blocks. He used 31 blocks in first row, 32 blocks in 2nd row, 33 blocks in third row and so on. Keeping the same pattern in mind, calculate how many blocks will he use in the 11th row?

Summary

- To make repeated multiplication easier to express, we use exponential notation.
- When a number a is multiplied by itself n number of times, it can be written in exponential form as a^n where a is the base and the superscript number n written above the base is called the exponent.

Following are the basic exponential laws:

➤ Product Law

$$a^m \times a^n = a^{m+n}$$

$$a^m \times b^n = (ab)^n$$

➤ Quotient Law

$$a^m \div a^n = a^{m-n}$$

$$a^m \div b^n = \left(\frac{a}{b}\right)^n$$

➤ Power Law

$$(a^m)^n = a^{mn}$$

➤ Zero Exponent Law

$$a^0 = 1$$

➤ Negative Exponent Law

$$a^{-m} = \frac{1}{a^m}$$

- One raised to any power is always equal to one.
- Any number raised to the power 1 is the number itself.

Review Exercise

1 Choose the correct option.

- a) $a^n \times b^n =$ _____
 i. $(ab)^n$ ii. $(b)^n$ iii. $(a+b)^{m+n}$ iv. $(a+b)^n$
- b) $4^3 =$ _____
 i. 16 ii. 12 iii. 64 iv. 32
- c) $x^{-3} \times x^4 =$ _____
 i. x^{-7} ii. x^{-1} iii. $\frac{1}{x}$ iv. x
- d) $(7)^0 =$ _____
 i. 0 ii. 1 iii. 7 iv. 0.7
- e) $5^{-3} =$ _____
 i. $\frac{1}{125}$ ii. $\frac{1}{225}$ iii. $\frac{1}{325}$ iv. $\frac{1}{425}$
- f) $(-4a^2)^3 =$ _____
 i. $16a^6$ ii. $-64a^6$ iii. $-64a^5$ iv. $-16a^4$
- g) The reciprocal of $(\frac{a}{b})^{-2}$ is _____
 i. $(\frac{b}{a})^{-2}$ ii. $\frac{1}{a}$ iii. $(\frac{a}{b})^2$ iv. 0

2 Define exponent, base and value using examples.

3 Find the value of the following exponential expressions.

- a) 4^1 b) 5^7 c) 7^2 d) 11^0 e) $(\frac{6}{9})^4$ f) 3^{-5}

4 Apply the law of exponents to simplify the following.

- a) $7^1 \times 7^4$ b) $(x^2 \times y^{-1})^3$ c) $(4b^3)(4b^4)$ d) $(\frac{4}{3} xy^{-3})^2$

5 Evaluate the following using laws of exponents.

- a) $(4^4 \div 4^3) \times 2^2$ b) $(\frac{4}{3})^{-3} \times (\frac{4}{3})^3 + (\frac{4}{7})^5 \times (\frac{4}{7})^{-4}$
- c) $(\frac{-2}{5})^6 \times (\frac{-2}{5})^{-3} \times [(\frac{5}{-2})^{-2}]^{-1}$ d) $[(\frac{1}{100})^2]^3 \times [(\frac{1}{100})^3]^2 \div (\frac{1}{100})^{15}$
- e) $[(\frac{1}{9})^4]^3 \times [(\frac{1}{9})^6] \div (\frac{1}{9})^{21}$ f) $(\frac{4}{6})^8 \div (\frac{4}{6})^4$

Math Project

Material Required:

- Exponent sheet for each pair (as shown in image)
- Question chits (same questions as in exponentials sheet)
- Basket for each pair
- Markers (2 colours for each pair)

Procedure:

- Work in pairs.
- Teacher will provide exponent sheet to each pair.
- Same question in chits will be put in basket for.
- Members of each pair will pick a chit randomly turn by turn and solve the question.
- After solving it correctly they will write the answer on the chit and cross the relevant box in the exponent sheet with marker (colour assigned).
- Repeat it turn by turn.
- The member who filled four in the same row, column or diagonal wins.
- If no row of 4 can be made, keep solving till all squares are crossed.

$(x^5y^5)^{-1}$	$(ab^2)^3$	$\frac{(3x^5y^2)^3}{2x^4y^4}$	$(2p^2)(3p^4)$
$(7x)(11x^7)$	$2(a^5)^3$	$x^3(-x^7)^2$	$(17^0)^9$
$(\frac{2}{5}ab^{-1})^2$	$(4a^3)^2$	$\frac{(-9bc)^2}{c^6}$	$(-y^5)^3 \div y^5$
$\frac{7^3x^4y^5}{7x^3y}$	$\frac{-10x^2y^4}{5x^3y^4}$	$(m^5 \times n^{-2})^6$	$5a^3(-a^{-5})$

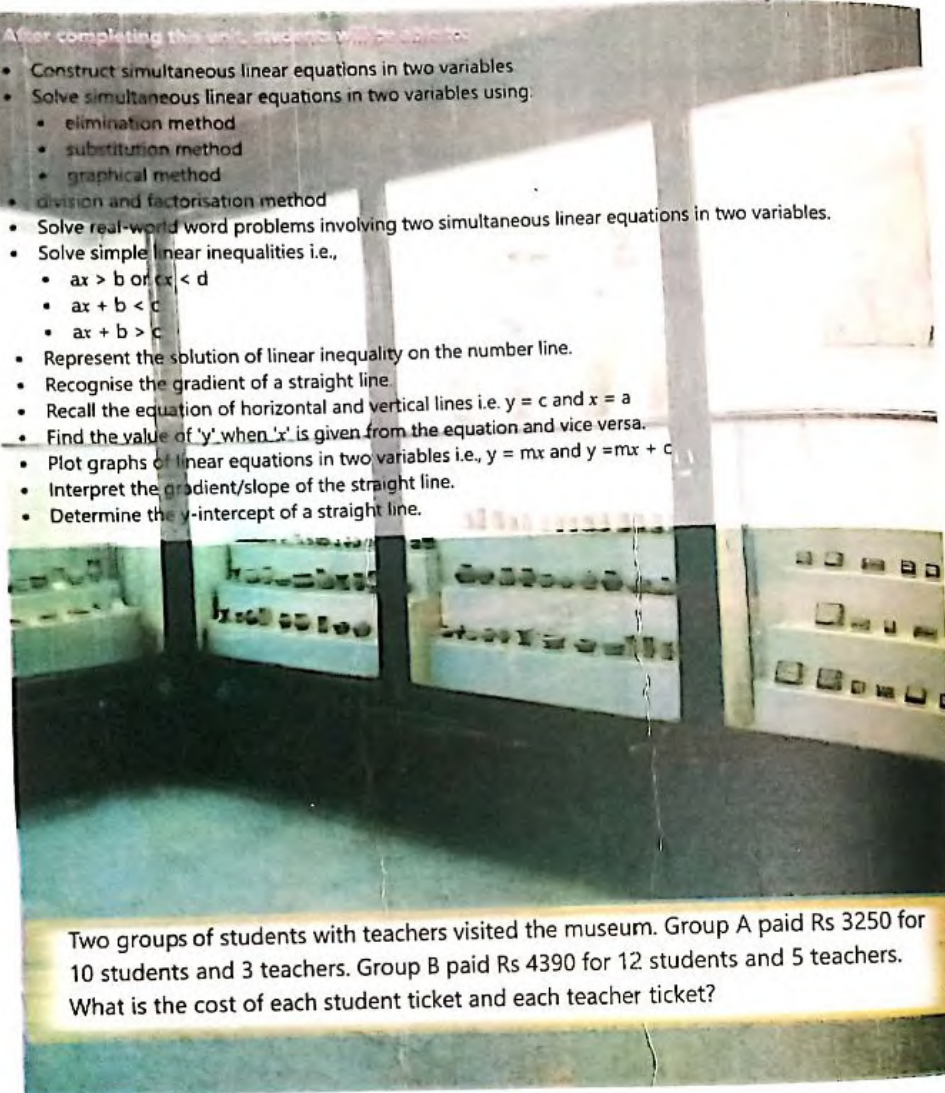
Unit 8

Simultaneous Linear Equations

Student Learning Outcomes

After completing this unit, students will be able to:

- Construct simultaneous linear equations in two variables
- Solve simultaneous linear equations in two variables using:
 - elimination method
 - substitution method
 - graphical method
- division and factorisation method
- Solve real-world word problems involving two simultaneous linear equations in two variables.
- Solve simple linear inequalities i.e.,
 - $ax > b$ or $cx < d$
 - $ax + b < c$
 - $ax + b > c$
- Represent the solution of linear inequality on the number line.
- Recognise the gradient of a straight line.
- Recall the equation of horizontal and vertical lines i.e. $y = c$ and $x = a$
- Find the value of 'y' when 'x' is given from the equation and vice versa.
- Plot graphs of linear equations in two variables i.e., $y = mx$ and $y = mx + c$
- Interpret the gradient/slope of the straight line.
- Determine the y-intercept of a straight line.



Two groups of students with teachers visited the museum. Group A paid Rs 3250 for 10 students and 3 teachers. Group B paid Rs 4390 for 12 students and 5 teachers. What is the cost of each student ticket and each teacher ticket?

NOT FOR SALE

Introduction

We have learnt how to construct linear equations in two variables, solving linear equations in one and two variables and plotting the graph of the linear equation. Now we will learn about simultaneous linear equations and their solutions through various methods. We will also learn about simple inequalities and their solution.

8.1 Simultaneous Linear Equations

We know that $ax + by = c$ is the linear equation in two variables. Now we will learn about simultaneous linear equations (or system of linear equations) in two variables. It includes two linear equations in two (or more) variables which are solved to find a common solution (if any).

We know that the general form of a linear equation in two variables is:

$ax + by = c$, where x and y are variables, a and b are non-zero coefficients and c is a constant.

Now observe these equations:

$$a_1x + b_1y = c_1$$

$$a_2x + b_2y = c_2$$

These two are a pair of simultaneous linear equations in two variables, x and y .

8.1.1 Constructing Simultaneous Linear Equations

We can construct simultaneous linear equations in two variables for many real-life situations.

Observe these examples.

Example 1:

Arslan is 4 times as old as his son. After 12 years the sum of their ages will be 69 years. Construct simultaneous linear equations for the given conditions.

Note it down

A solution for a single equation is any point that lies on the line for that equation. A solution for a system of equations is any point that lies on each line in the system.



NOT FOR SALE

Solution:

Let Arsalan's age be = x years

Let his son's age be = y years

According to the given conditions:

$$x = 4y \text{ -----(i)}$$

$$x + 12 + y + 12 = 69$$

or

$$x + y = 69 - 24$$

or

$$x + y = 45 \text{ -----(ii)}$$

So, (i) and (ii) are the required simultaneous linear equations for the given conditions.

Example 2:

The cost of 3 textbooks and 5 notebooks is Rs 1325 and the cost of 4 textbooks and 2 notebooks is Rs 1230. Construct simultaneous linear equations for the given condition.

Solution:

Let the price of a textbook be = Rs x

Let the price of a notebook be = Rs y

According to the given conditions:

$$3x + 5y = 1325 \text{ -----(i)}$$

$$4x + 2y = 1230 \text{ -----(ii)}$$

So, (i) and (ii) are the required simultaneous linear equations for the given condition.

Exercise 8.1

1. The cost of a bag is three times the cost of a geometry box. The cost of 2 bags and 3 geometry boxes is Rs 3960. Construct simultaneous linear equations for these conditions.
2. The volume of apple juice is 3 liters less than the column of orange juice and the total volume of apple and orange juice is 11 liters. Construct simultaneous linear equations for these conditions.
3. The sum of two numbers is 82 and their difference is 23. Construct simultaneous linear equations for these conditions.

Note it down

The general form of simultaneous linear equations is:

$$a_1x + b_1y = c_1$$

$$a_2x + b_2y = c_2$$

where x and y are variables, a_1, b_1, a_2, b_2 are non-zero coefficients and c_1, c_2 are the constants.

8.2 The Solution of Two Simultaneous Linear Equations in Two Variables

The discussion of the previous section leads to the result that a linear equation in two variables is satisfied by as many values as required. Let us consider two linear equations.

$$2x - y = 4 \text{ -----(1)}$$

$$3x - 2y = 5 \text{ -----(2)}$$

As we have discussed earlier, the equation $2x - y$ is satisfied by the following values of x and y .

x	1	2	3	4	5	6	7
y	-2	0	2	4	6	8	10

Some values of x and y for which the equation $3x - 2y = 5$ is satisfied have been displayed in the following table.

x	1	2	3	4	5	6	7
y	-1	$\frac{1}{2}$	2	$\frac{7}{2}$	5	$\frac{13}{2}$	8

By studying both tables, it reveals that $(x, y) = (3; 2)$ is the only pair of values of x and y that satisfies both equations. Thus, we say that the pair of equations (1) and (2) constitute a pair of simultaneous linear equations and their solution is $x = 3$ and $y = 2$. This discussion leads us to an important result that a pair of simultaneous linear equations always has a unique solution i.e; they are satisfied by a single pair of values of the variables involved.

To solve simultaneous linear equations means to find the unique pair of values of the variables involved that satisfy both equations and this unique pair of values of variables is known as the solution of the simultaneous equations.

8.2.1 Methods of Solving Simultaneous Linear Equations

There are three common methods of solving a pair of simultaneous linear equations. These are;

1. Elimination Method
2. Substitution Method
3. Graphical Method
4. Division Method
5. Factorization Method

Note it down

The values of x and y can also be written in the form of an ordered pair.
For example;
 $(x, y) = (5, -3)$ means $x = 5$ and $y = -3$.

8.2.2 Elimination Method

In this method, we eliminate one of the variables in the given simultaneous linear equations and hence get a third linear equation in only one variable. By solving the third linear equation, we get the value of one of the variables involved. By substituting this value of one variable in any of the given equations, the value of the other variable can be found. The following examples may help you to learn this method.

Example 1:

Solve the following simultaneous equations.

$$4x + 3y = 26, \quad 2x - 3y = 4$$

Solution: $4x + 3y = 26$ — (1)

$$2x - 3y = 4$$
 — (2)

Add equations (1) and (2),

$$(4x + 3y) + (2x - 3y) = 26 + 4$$

$$4x + 3y + 2x - 3y = 30$$

$$6x = 30$$

$$x = \frac{30}{6}$$

$$x = 5$$

Substitute this value of x in equation (1),

$$4(5) + 3y = 26$$

$$20 + 3y = 26$$

$$3y = 26 - 20$$

$$3y = 6$$

$$y = \frac{6}{3}$$

$$y = 2$$

Thus $x = 5$ and $y = 2$ is the solution of the given simultaneous equations.

Example 2:

Solve the following simultaneous linear equations.

$$x + y = 28, \quad x + 3y = 52$$

Solution:

$$x + y = 28$$
 — (1)

$$x + 3y = 52$$
 — (2)

Note it down

In example 1, we found $x = 5$ and $y = 2$. It can be written as $(x, y) = (5, 2)$ and is called the solution of the given equations. Sometimes, this solution is written in the form of a set and is called the solution set. The solution set of example 1 is $\{(5, 2)\}$.

Subtract equation (1) from equation (2),

$$(x + 3y) - (x + y) = 52 - 28$$

$$x + 3y - x - y = 24$$

$$y = \frac{24}{2}$$

$$y = 12$$

put $y = 12$ in equation (1),

$$x + 12 = 28$$

$$x = 28 - 12$$

$$x = 16$$

Thus $x = 16$ and $y = 12$ is the solution of the given equations.

Example 3:

Solve the following simultaneous equations. $2x + 5y = 16$, $3x - 2y = 5$

Solution:

$$2x + 5y = 16$$
 — (1)

$$3x - 2y = 5$$
 — (2)

Multiply equation (1) by 3 and equations (2) by 2,

$$3(2x + 5y) = 3 \times 16$$

$$2(3x - 2y) = 2 \times 5$$

$$6x + 15y = 48$$
 — (3)

$$6x - 4y = 10$$
 — (4)

Subtract equation (4) from equation (3),

$$(6x + 15y) - (6x - 4y) = 48 - 10$$

$$6x + 15y - 6x + 4y = 48 - 10$$

$$19y = 38$$

$$y = \frac{38}{19}$$

$$y = 2$$

Substitute $y = 2$ in equation (2),

$$3x - 2(2) = 5$$

$$3x - 4 = 5$$

$$3x = 5 + 4$$

$$3x = 9$$

$$x = \frac{9}{3}$$

$$x = 3$$

Thus $x = 3$ and $y = 2$ is the solution of the given equations.

Exercise 8.2

1 In each of the following questions, you have been given a pair of linear equations in one variable. Solve both equations separately and tell which of them are simultaneous equations and which are not.

- | | | |
|-----------------|-----------------|------------------|
| a) $a + 3 = 9$ | b) $b + 2 = 11$ | c) $p - 15 = 15$ |
| $a - 2 = 5$ | $b - 5 = 4$ | $p + 7 = 37$ |
| d) $x - 3 = 10$ | e) $z - 8 = 12$ | |
| $x - 7 = 6$ | $z - 4 = 11$ | |

2 Solve the following simultaneous equations by using the elimination method:

- | | | |
|-------------------|-------------------|-------------------|
| a) $a + b = 7$ | b) $m + n = 10$ | c) $p + q = 14$ |
| $a - b = 3$ | $m - n = 4$ | $p - q = 4$ |
| d) $x + y = 25$ | e) $x + y = 11$ | f) $2x + 3y = 65$ |
| $x - y = 5$ | $x + 3y = 25$ | $5x + 3y = 95$ |
| g) $2p - 3q = 13$ | h) $5a + 2b = 23$ | i) $7x - 6y = 54$ |
| $7p + 2q = 8$ | $2a + 7b = 3$ | $2x + 3y = 39$ |

8.3 Substitution Method

In this method, we take any one of the given equations first and find the value of one of the variables in terms of the other. Then we substitute this value of the variable in the second equation. In this way, the second equation reduces to a third equation which is a linear equation in one variable. Hence, the value of one of the variables is found by solving the third equation. Finally, we substitute (put) this value of one variable in any of the given equations that gives us the value of the other variable on solving it. This method has been further explained with the help of the following examples.

Example 1:

Solve the following simultaneous equations:

$$x + y = 12, \quad x - y = 2$$

Solution:

$$x + y = 12 \quad \text{--- (1)}$$

$$x - y = 12 \quad \text{--- (2)}$$

From equation (2), we get

$$x = 2 + y \quad \text{--- (3)}$$

Put this value of x in equation (1),

$$(2 + y) + y = 12$$

$$2 + y + y = 12$$

$$\begin{aligned} 2 + 2y &= 12 \\ 2y &= 12 - 2 \\ 2y &= 10 \\ y &= \frac{10}{2} \\ y &= 5 \end{aligned}$$

Substitute $y = 5$ in equation (3),

$$\begin{aligned} x &= 2 + 5 \\ x &= 7 \end{aligned}$$

Thus, $x = 7$ and $y = 5$ is the solution of the given equations.

Example 2:

Solve the following simultaneous equations.

$$7x - 4y = 10, \quad 2x + 3y = 65$$

Solution:

$$7x - 4y = 10 \quad \text{--- (1)}$$

$$2x + 3y = 65 \quad \text{--- (2)}$$

From equation (1), we get,

$$7x = 10 + 4y$$

$$\text{or } x = \frac{10 + 4y}{7} \quad \text{--- (3)}$$

Substitute this value of x in equation (2)

$$2 \left(\frac{10 + 4y}{7} \right) + 3y = 65$$

$$\frac{20 + 8y}{7} + 3y = 65$$

$$20 + 8y + 21y = 65 \times 7$$

$$20 + 29y = 455$$

$$29y = 455 - 20$$

$$29y = 435$$

$$y = \frac{435}{29}$$

$$y = 15$$

Substitute $y = 15$ in equation (3),

$$x = \frac{10 + 4(15)}{7}$$

$$x = \frac{10 + 60}{7} = \frac{70}{7} = 10$$

Thus, $x = 10$ and $y = 15$ is the solution of the given simultaneous equations.

Note it down

The subject of mathematics has played a pivotal role in the advancement of science and technology. That is why it is said that mathematics is the language of science. However, it is also said that algebra is the grammar of mathematics. The solution of equations is an important field of algebra.

Exercise 8.3

1 Solve the following simultaneous equations by using the substitution method.

- | | | | |
|----------------|------------------|-------------------|-------------------|
| a) $a + b = 9$ | b) $2a + b = 6$ | c) $x + y = 7$ | d) $x + y = 16$ |
| $a - b = 5$ | $a - 3b = 17$ | $x - y = 3$ | $x - y = 4$ |
| e) $x + y = 4$ | f) $p + q = -12$ | g) $2u + 3v = 52$ | h) $15x + 6y = 9$ |
| $x - y = -10$ | $p - q = 2$ | $5u - 2v = 16$ | $10x - 18y = 28$ |

2 Solve the following simultaneous equations by using either the elimination or substitution method.

- | | | | |
|--------------------|-------------------|---------------------|--------------------|
| a) $12p - 7q = 20$ | b) $5l + 2m = 54$ | c) $13u - 10v = 11$ | d) $2y + z = 30$ |
| $17p + 13q = 16$ | $3l + 5m = 59$ | $5u - 7v = 20$ | $3y - 2z = 17$ |
| e) $5x + 3y = 95$ | f) $8x - 3y = 6$ | g) $11x + 2y = 73$ | h) $4x + 3y = -31$ |
| $3x + 5y = 105$ | $15x - 7y = -8$ | $15x - 7y = 12$ | $3x + 15y = 15$ |

8.4 Graphical Method

Solving simultaneous equations using graphical method means to solve two or more simultaneous linear equations that share a common point drawing their graphs. The point (or points) of intersection represents the solution(s) of the given simultaneous linear equations.

To solve Simultaneous Linear Equations Using Graphical Method, follow these steps:

1. Write the first linear equation.
2. Use Trial and error method and put any value of x (or y) and solve the equation. This will give the value of y (or x). This will give us the values of the coordinates of the first point.
3. Similarly, put a few more values of x or y and get corresponding values of y or x .



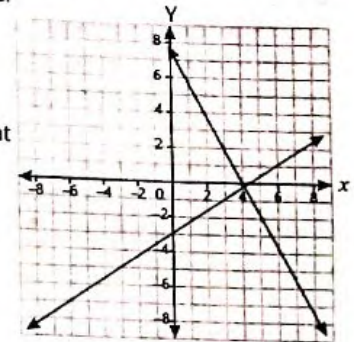
Tell the students that there are a few other methods of solving simultaneous linear equations for example the cross-multiplication method, the division method, and the factorization method.

NOT FOR SALE

4. In the same way, find coordinates of at least 3 points for the second equation.
5. Plot the points for both equations and if needed, extend the length of the lines until they meet.
6. Look at the point at which the 2 lines meet. These coordinates of this point will give us the solution of the given simultaneous linear equations.

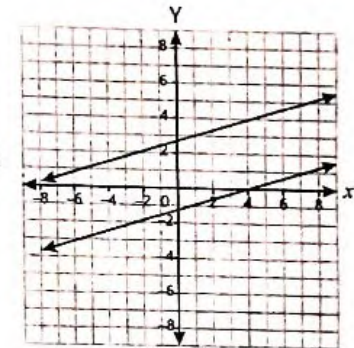
One point of intersection- One solution

If the lines intersect at one common point, this shows that there is only one solution for the simultaneous linear equations.



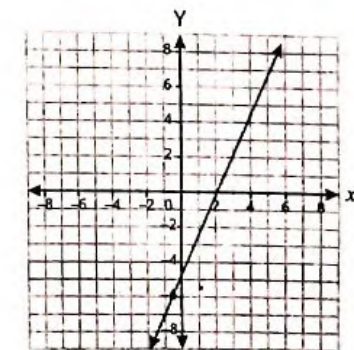
No point of Intersection (parallel lines) - No Solution

If the lines do not intersect at all i.e. they are parallel to each other, then there is no solution.



If same lines (coincident lines) - Many Solutions

Both equations give the same line, then there are infinitely many solutions.



NOT FOR SALE

Example 1:

Arham and Ahad have 8 marbles. Ahad has 2 more marbles than Arham.

- Construct a simultaneous equation for this condition.
- Find how many marbles does each of them have.

Solution:

a) Let the number of marbles Ahad has be = x

Let the number of marbles Arham has be = y

According to the given condition:

$$x + y = 8 \text{ -----(i)}$$

$$x = y + 2 \text{ or } x - y = 2 \text{ -----(ii)}$$

So, (i) and (ii) are the required equations for this condition.

b) The equations are:

$$x + y = 8 \text{ -----(i)} \quad x - y = 2 \text{ -----(ii)}$$

Put $x = 0$ in (i)

$$x + y = 8$$

$$0 + y = 8$$

$$y = 8$$

$$\therefore (x, y) = (0, 8)$$

Put $y = 0$ in (i)

$$x + y = 8$$

$$x + (0) = 8$$

$$x = 8$$

$$\therefore (x, y) = (8, 0)$$

Put $x = 1$ in (i)

$$x + y = 8$$

$$1 + y = 8$$

$$y = 8 - 1 = 7$$

$$\therefore (x, y) = (1, 7)$$

So, we get the coordinates of $x + y = 8$ as follows.

x	0	8	1
y	8	0	7

Now calculate values of x and y for equation $x - y = 2$.

Put $x = 0$ in (ii)

$$x - y = 2$$

$$(0) - y = 2$$

$$y = -2$$

$$\therefore (x, y) = (0, -2)$$

Put $y = 0$ in (ii)

$$x - (0) = 2$$

$$x = 2$$

$$\therefore (x, y) = (2, 0)$$

Put $x = 1$ in (i)

$$x - y = 2$$

$$(1) - y = 2$$

$$-y = 2 - 1$$

$$-y = 1$$

$$y = -1$$

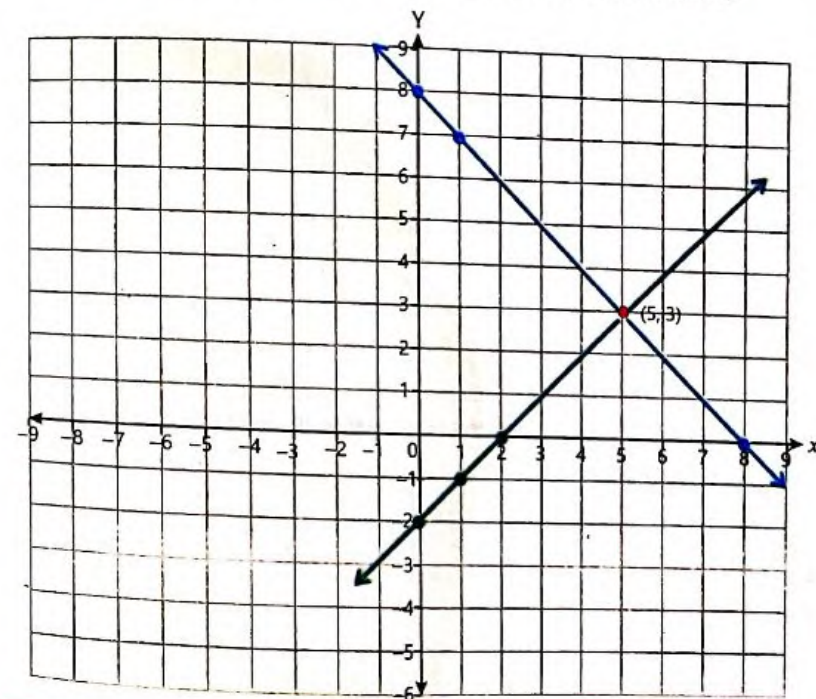
$$\therefore (x, y) = (1, -1)$$

So, we get the coordinates as follows.

x	0	2	1
y	-2	0	-1

Now plot the coordinates of points we get from the two equations.

The lines intersect at point $(5, 3)$, so this is the solution of the given simultaneous linear equations. This shows that Ahad has 5 marbles and Arham has 3 marbles.



Verification:

We must verify the solution to avoid any mistake.

Put $x = 5$ and $y = 3$ in both equations.

$$\begin{array}{rcl} x + y & = & 8 \\ 5 + 3 & = & 8 \\ 8 & = & 8 \end{array} \quad \begin{array}{rcl} x - y & = & 2 \\ 5 - 3 & = & 2 \\ 2 & = & 2 \end{array}$$

As the solution satisfies both the equations, so $(8, 5)$ is the solution for the given simultaneous linear equation.

Example 2:

Solve the following system of linear equations graphically.

Solution:

$$2x + y = 4 \quad \text{-----(i)}$$

$$x + 4y = 2 \quad \text{-----(ii)}$$

Using trial and error method, put values of x or y to get the corresponding values.

Put $x = 0$ in (i) we get:

$$2(0) + y = 4$$

$$y = 4$$

Put $y = 0$ in (i), we get:

$$2x + 0 = 4$$

$$2x = 4$$

$$x = 2$$

Put $x = 1$ in (i), we get:

$$2(1) + y = 4$$

$$2 + y = 4$$

$$y = 2$$

So, we get $(0, 4)$, $(2, 0)$ and $(1, 2)$ as the coordinates of the line in equation (i).

Similarly find three points for (ii).

Put $x = 0$ in (ii)

$$x + 4y = 2$$

$$0 + 4y = 2$$

$$y = \frac{1}{2} \text{ or } 0.5$$

NOT FOR SALE

Put $y = 0$ in (ii)

$$x + 4(0) = 2$$

$$x = 2$$

Put $x = 1$ in (ii)

$$x + 4y = 2$$

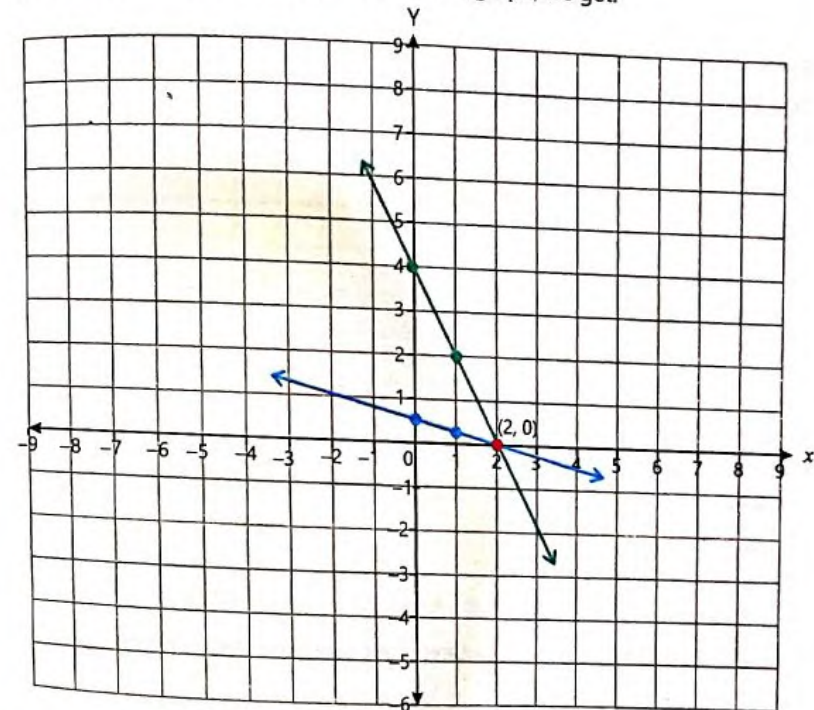
$$1 + 4y = 2$$

$$4y = 1$$

$$y = \frac{1}{4}$$

So, we get $(0, \frac{1}{2})$, $(2, 0)$ and $(1, \frac{1}{4})$ as the coordinates of the line in equation (ii).

Plotting the coordinates of both equations on the graph, we get.



The lines intersect at point $(2, 0)$, so this is the solution of the given simultaneous linear equations.

NOT FOR SALE

Verification:

We must verify the solution to avoid any mistake.

Put $x = 2$ and $y = 0$ in both equations.

$$\begin{array}{rcl} 2x + y & = & 4 \\ 4 + 0 & = & 4 \\ 4 & = & 4 \end{array} \quad \begin{array}{rcl} x + 4y & = & 2 \\ 2 + 0 & = & 2 \\ 2 & = & 2 \end{array}$$

As the solution satisfies both the equations, so $(2, 0)$ is the solution for the given simultaneous linear equation.

Example 3:

Solve the following system of linear equations graphically.

$$3x + 6y = 9$$

$$x + 2y = 3$$

Solution:

First find the ordered pairs satisfying these equations.

Put $x = 0$ in $3x + 6y = 9$

$$3(0) + 6y = 9$$

$$6y = 9$$

$$y = \frac{3}{2}$$

Put $y = 0$ in $3x + 6y = 9$

$$3x + 6(0) = 9$$

$$3x = 9$$

$$x = 3$$

Put $x = -1$ in $3x + 6y = 9$

$$3(-1) + 6y = 9$$

$$-3 + 6y = 9$$

$$6y = 12$$

$$y = 2$$

So, $(0, \frac{3}{2})$, $(3, 0)$ and $(-1, 2)$ are the ordered pairs satisfying the equation $3x + 6y = 9$.

Now put $x = 0$ in $x + 2y = 3$.

$$0 + 2y = 3$$

$$2y = 3$$

$$y = \frac{3}{2}$$

Put $y = 0$ in $x + 2y = 3$

$$x + 2(0) = 3$$

$$x = 3$$

Put $x = -1$ in $x + 2y = 3$

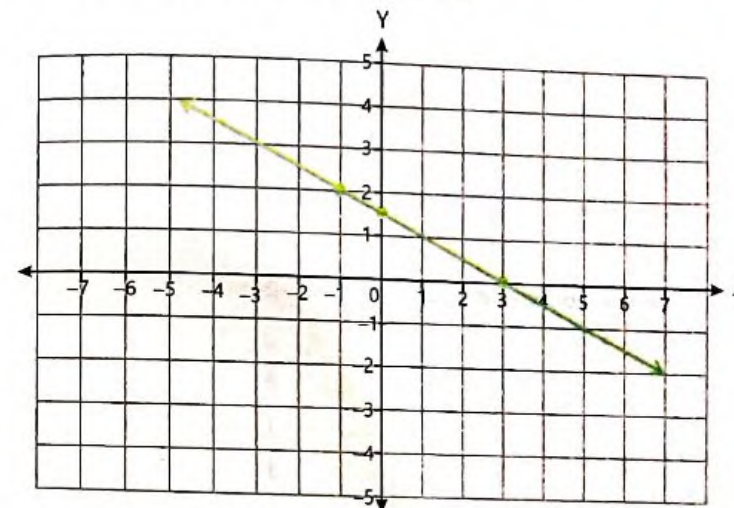
$$-1 + 2y = 3$$

$$2y = 4$$

$$y = 2$$

So, $(0, \frac{3}{2})$, $(3, 0)$ and $(-1, 2)$ are the ordered pairs satisfying the equation $x + 2y = 3$.

Now plot all the points of both equations on the graph.



We can see that both lines resulted in same straight line. So, there are infinite solutions for these simultaneous equations.

Example 4:

Solve the following system of linear equations graphically.

$$2x - 4y = 5$$

$$x - 2y = 5$$

Solution:

Let's find some ordered pairs satisfying equation $2x - 4y = 5$

Put $x = 0$ in $2x - 4y = 5$.

$$2(0) - 4y = 5$$

$$-4y = 5$$

$$y = -\frac{5}{4}$$

$$\text{Put } y = 0 \text{ in } 2x - 4y = 5.$$

$$2x - 4(0) = 5$$

$$2x = 5$$

$$x = \frac{5}{2}$$

$$\text{Put } x = 1 \text{ in } 2x - 4y = 5.$$

$$2(1) - 4y = 5$$

$$2 - 4y = 5$$

$$-4y = 3$$

$$y = -\frac{3}{4}$$

So, $(0, \frac{5}{4})$, $(\frac{5}{2}, 0)$ and $(1, -\frac{3}{4})$ are the ordered pairs satisfying the equation

$$2x - 4y = 5.$$

Now find some ordered pairs satisfying equation $x - 2y = 6$

$$\text{Put } x = 0 \text{ in } x - 2y = 6.$$

$$0 - 2y = 6$$

$$y = -3$$

$$\text{Put } y = 0 \text{ in } x - 2y = 6.$$

$$x - 2(0) = 6$$

$$x = 6$$

$$\text{Put } x = 1 \text{ in } x - 2y = 6.$$

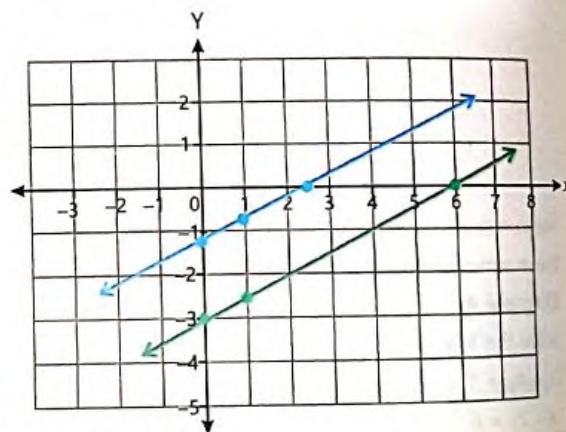
$$1 - 2y = 6$$

$$-2y = 5$$

$$y = -\frac{5}{2}$$

So, $(0, -3)$, $(6, 0)$ and $(1, -\frac{5}{2})$ are

the ordered pairs satisfying the equation $x - 2y = 6$



We can see that the lines of the given equations are parallel to each other and there is no point of intersection. Hence these equations have no solution.

NOT FOR SALE

Exercise 8.4

1 Solve the following system of linear equations graphically.

a) $x + y = 7$

b) $2x - y = 4$

c) $3x - y = 0$

d) $3x - 5y = 6$

$x - y = 5$

$x - 2y = 7$

$2x + y = 5$

$2x - 3y = 5$

e) $\frac{x}{3} + \frac{y}{5} = 2$

f) $\frac{x}{3} + 5 = 16$

g) $2x + 5y = 12$

h) $5x - y = 4$

$2x - y = 1$

$x + y = 6$

$4x + 3y = -4$

$10x - 4y = 6$

8.5 Factorization and Division Method

Let us solve the given simultaneous linear equation by factorization method.

Example 1: $2x + 6y = 10$; $3x + 9y = 12$

Solution:

$$2x + 6y = 10 \quad \text{--- (i)}$$

In this equation 2 is the common factor so take 2 common.

$$2(x + 3y) = 10$$

Dividing both side by 2 we get:

$$\frac{2}{2}(x + 3y) = \frac{10}{2}$$

$$x + 3y = 5 \quad \text{--- (ii)}$$

Now take second equation. In this equation 3 is a common factor.

$$3(x + 3y) = 12$$

Dividing both side by 3 we get:

$$\frac{3}{3}(x + 3y) = \frac{12}{3}$$

$$x + 3y = 4 \quad \text{--- (iii)}$$

Now subtract equations (ii) and (iii):

$$x + 3y - (x + 3y) = 5 - 4$$

$$x + 3y - x - 3y = 1$$

$$-y = 1 \text{ or } y = -1$$

Put the value of x in the equation (ii) we have

$$x + 2(-1) = 5$$

$$x - 2 = 5$$

adding 2 on both side we have:

$$x - 2 + 2 = 5 + 2$$

$$x = 7$$

NOT FOR SALE

Example 2: Let us solve the simultaneous linear equation by division method.

$$x + 2y = 2; \quad x - 3y = 6$$

Divide first equation by second equation.

$$\frac{x + 2y}{x - 3y} = \frac{2}{6}$$

By cross multiplying we get:

$$6(x + 2y) = 2(x - 3y)$$

$$6x + 12y = 2x - 6y$$

$$6x - 2x = -12y - 6y$$

$$4x = -18y$$

$$x = \frac{-9}{2}y$$

Put the value of x in first equation we have:

$$\frac{-9}{2}y + 2y = 2$$

Dividing by 2 we have:

$$-9y + y = 1$$

$$-8y = 1$$

$$y = \frac{-1}{8}$$

Put the value of y in the value of x we have:

$$x = \frac{-9}{2} \times \frac{-1}{8} = \frac{9}{16}$$

Exercise 8.5

1 Solve the following simultaneous linear equation using division method.

a) $5x + 10y = 15; 3x + 12y = 9$

b) $4x + 8y = 12; 7x + 14y = 14$

c) $9x + 9y = 18; 2x + 8y = 4$

d) $3x + 12y = 3; 6x + 12y = 24$

e) $11x + 33y = 44; 5x + 15y = 30$

f) $x + y = 2; 4x + 2y = 4$

2 Solve the following simultaneous linear equation using factorization method.

a) $x + y = 4; x - y = 8$

b) $3x + 5y = 2; 4x - 3y = 5$

c) $x + 4y = 1; 3x - 7y = 5$

d) $6x + 7y = 10; 2x - 5y = 7$

8.6 Word Problems

The formation of linear equations and the solution of simultaneous equations is a very important concept of algebra which has wide application in different branches of mathematics as well as in solving certain real life problems. In this section, we shall only discuss that how simultaneous linear equations in two variables can help us to solve certain real life problems. The following examples may further help you to understand this concept.

Example 1:

The sum of two numbers is 30 and their difference is 16. Find the numbers.

Solution:

Step I: Conversion of the given information into a pair of simultaneous equations:

Let the required larger number = x

And let the required smaller number = y

According to the statement of the question,

(The sum) $x + y = 30$ — (1)

(The difference) $x - y = 16$ — (2)

Step II: Solve the simultaneous linear equations formed in step I. The solution of these equations gives us the required answer. To solve, we add equations (1) and (2),

$$(x + y) + (x - y) = 30 + 16$$

$$x + y + x - y = 46$$

$$2x = 46$$

$$x = \frac{46}{2}$$

$$x = 23$$

Put $x = 23$ in equation (1),

$$23 + y = 30$$

$$y = 30 - 23$$

$$y = 7$$

Thus, the required numbers are 23 and 7.

Example 2:

Sohail bought four chairs and a table for Rs. 1800 while Ehtisham bought six chairs and two tables of the same kind for Rs. 3000. Find the price of a chair and a table.

Solution:

Step I: Conversion of the given information into simultaneous linear equations:

Note it down

Whenever you want to solve a real life problem with the help of the solution of simultaneous linear equations, the first and the most basic step is to translate correctly the problems into an algebraic equation.

Let the price of 1 chair = Rs x
 Then the price of 4 chairs = Rs $x \times 4 = \text{Rs } 4x$
 And the price of 6 chairs = Rs $x \times 6 = \text{Rs } 6x$
 Also let the price of 1 table = Rs y
 Then the price of 2 tables = Rs $y \times 2 = \text{Rs } 2y$
 According to the statement of the question,
 (The price of 4 chairs) + (The price of 1 table) = Rs 1800
 Rs $4x + \text{Rs } y = \text{Rs } 1800$

$$4x + y = 1800 \text{ — (1)}$$

Also, (The price of 6 chairs) + (The price of 2 tables) = Rs 3000
 Rs $6x + \text{Rs } 2y = \text{Rs } 3000$

$$6x + 2y = 3000 \text{ — (2)}$$

Step II: To find the required prices, we solve the equation,
 $y = 1800 - 4x$ — (3)

Put this value of 'y' in equation (2),

$$6x + 2(1800 - 4x) = 3000$$

$$6x + 3600 - 8x = 3000$$

$$3600 - 2x = 3000$$

$$3600 = 3000 + 2x$$

$$3600 - 3000 = 2x$$

$$600 = 2x$$

$$\frac{600}{2} = 2x$$

$$x = 300$$

Put $x = 300$ in equation (3),

$$y = 1800 - 4(300)$$

$$y = 1800 - 1200$$

$$y = 600$$

Thus, the price of one chair = Rs 300

and the price of one table = Rs 600

Note: While actually solving this type of problems, it is not compulsory to show the steps I and II separately.

NOT FOR SALE

Example 3:

Rs 3280 is divided between two people in such a way that one quarter of one person's share is equal to $\frac{1}{6}$ of the other. Find their shares.

Solution:

Let the shares = x and y

As per given conditions

$$x + y = 3280 \text{ — (i)}$$

$$\frac{1}{4}x = \frac{1}{6}y$$

$$6x = 4y \text{ (cross-multiplication)}$$

$$6x - 4y = 0$$

$$2(3x - 2y) = 0$$

$$3x - 2y = 0 \text{ — (ii)}$$

$$x + y = 3280$$

$$3x - 2y = 0$$

$$2x + 2y = 6560 \text{ (multiplying equation (i) by (ii))}$$

$$5x = 6560$$

$$x = \frac{6560}{5} = 1312$$

substituting the value of x in equation (i)

$$x + y = 3280$$

$$y = 3280 - x$$

$$= 3280 - 1312$$

$$= 1968$$

Shares = Rs 1312 and Rs. 1968

Example 4:

In an isosceles triangle, the length of equal sides are given as shown in the figure. Find the value of x and y , if the sum of these sides is 12cm.

Solution:

Then

$$3x - 2y + 2 = x - y + 7$$

$$3x - x - 2y + y + 2 - 7 = 0$$

$$2x - y - 5 = 0$$

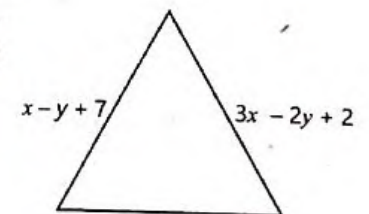
$$2x - y = 5 \text{(I)}$$

Sum of the given sides is 12cm.

Then

$$3x - 2y + 2 + x - y + 7 = 12$$

$$4x - 3y + 9 = 12$$



NOT FOR SALE

$$4x - 3y = 12 \quad \text{--- (i)}$$

$$4x - 3y = 3 \quad \text{--- (ii)}$$

Multiplying equation (i) by 3 and subtracting equation (ii) from the result:

$$\begin{array}{r} 6x - 3y = 15 \\ \pm 4x - 3y = \pm 3 \\ \hline 2x = 12 \end{array}$$

$$x = \frac{12}{2} = 6$$

$$2x - y = 5$$

$$2(6) - y = 5$$

$$-y = 5 - 12$$

$$-y = -7$$

$$y = 7$$

Hence, the sides are

$$x - y + 7 = 6 - 7 + 7 = 6 \text{ cm each}$$

Examples 5:

The sum of digits of a 2-digit number is 8. When the number with the same digits is reversed and subtracted from the original numbers, the difference is 18. What is the number?

Solution

Let unit place digit = x

Let ten's place digits = y

$$x + y = 8 \quad \text{--- (i)}$$

$$\text{The number} = x + 10y$$

$$\text{The reversed number} = 10x + y$$

As per conditions

$$x + 10y - (10x + y) = 18$$

$$x + 10y - 10x - y = 18$$

$$-9x + 9y = 18$$

$$\Rightarrow -x + y = 2$$

$$x + y = 8$$

$$-x + y = 2$$

$$2y = 10$$

$$y = \frac{10}{2}$$

$$y = 5$$

For x ,

$$x + y = 8$$

$$x + 5 = 8$$

$$x = 8 - 5$$

$$x = 3$$

In the number 63, 6 is ten's place digit and 3 is unit place digit. While making the number $3 + 6(10) = 3 + 60 = 63$

The number

$$= x + 10y$$

$$= 3 + 10 \times 5$$

$$= 3 + 50 = 53$$

Exercise 8.6

- The sum of two numbers is 15 and their difference is 3. Find the numbers.
- There is a certain pair of numbers such that the sum of twice the first number and thrice the second number is 61. However, when we subtract five times the second number from four times the first number, the answer is 45. Find the numbers.
- Zara bought five pencils and two sharpeners for Rs 62 while Madeeha bought eight pencils and two sharpeners for Rs 92. Find the price of a pencil and a sharpener.
- Ahmad bought 6 chairs and 2 tables for Rs 6300 while Ali bought 2 chairs and one table of the same quality from the same shop for Rs 2600. Find the price of a chair and a table.
- A shopkeeper has prepared bags of potatoes and carrots. The bag 'A' contains 5 kg potatoes and 2 kg carrots while the bag B contains 7 kg potatoes and 3 kg carrots. If the cost of the packet A is Rs 850 and that of the packet B is Rs 1200. Find the price of 1 kg of potatoes and 1 kg of carrots.
- The present age of a man is four times that of his son. In five years. The means age will be three times that of his son. Find the present age of the son and the father.

8.7 Simple Linear Inequalities

An inequality is a statement that one mathematical quantity is less than (or greater than) or less than or equal to (or greater than or equal to) another quantity. Thus, if a and b are real numbers, we can compare their positions on the real line by using the relations of less than, greater than, less than or equal to, and greater than or equal to, denoted by inequality symbols $<$, $>$, \leq and \geq respectively. The following table describes both algebraic and geometric interpretations of the inequality symbols.

Algebraic Statement	Equivalent Statement	Geometric Statement
$a < b$	a is less than b	a lies to the left of b .
$a > b$	a is greater than b	a lies to the right of b .
$a \leq b$	a is less than or equal to b	a coincides with b or lies to the left
$a \geq b$	a is greater than or equal to b	a coincides with b or lies to the right

Example 1:

Solve: $x + 1 < 4$.

Solution:

To solve this inequality, follow the same steps that are used in solving linear equations. Here 1 is on the same side as the variable. So, we need to isolate the variable. Subtract 1 from both sides:

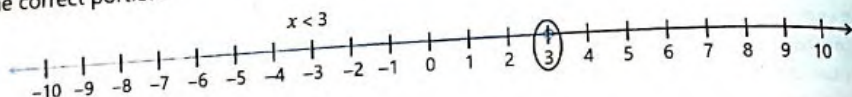
$$x + 1 - 1 < 4 - 1$$

$$x < 3$$

So, $x < 3$ is the solution for this inequality. We can say that the solution of this inequality are all the numbers less than 3.

We can also show the solution of an inequality on a number line.

Let's represent $x < 3$ on a number line. Draw number line and mark the points on it. Then look for the portion of number line that represents the solution. Here $x < 3$ means numbers less than 3 are the solutions. So, colour the correct portion.



Example 2:

Solve: $x - 2 > 4$.

Solution:

First isolate the variable.

Add 2 to both sides:

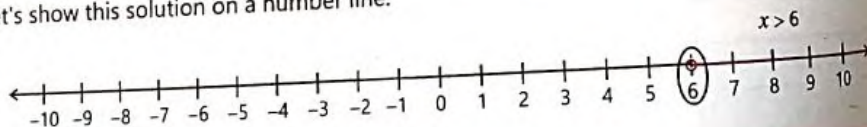
$$x - 2 + 2 > 4 + 2$$

$$x > 6$$

So, $x > 6$ is the solution for this inequality. We can say that the solution of this inequality is

"all the numbers greater than 6".

Let's show this solution on a number line.



Explain the students we can find the solution of an inequality by isolating the variables involved towards one side of the inequality.

NOT FOR SALE

Example 3:

Solve: $2x > 6$.

Solution:

First isolate the variable.

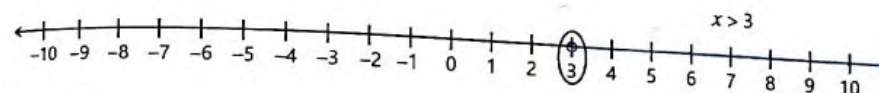
Divide both sides of the inequality by 2:

$$\frac{2x}{2} > \frac{6}{2}$$

$$x > 3$$

So, $x > 3$ is the solution for this inequality. We can say that the solution of this inequality is "all the numbers greater than 3".

Let's show this solution on a number line.



Example 4:

Solve: $4x < 8$.

Solution:

First isolate the variable.

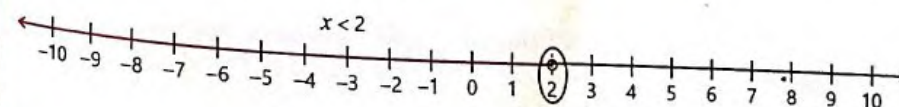
Divide both sides of the inequality by 4:

$$\frac{4x}{4} < \frac{8}{4}$$

$$x < 2$$

So, $x < 2$ is the solution for this inequality. We can say that the solution of this inequality is "all the numbers smaller than 2".

Let's show this solution on a number line.



Quick Check

Solve and represent the solution using a number line.

a) $3x - 2 > 7$

b) $-5x > 10$

c) $x + 2 < -4$

NOT FOR SALE

Example 5:

Solve: $-3x + 1 < 7$.

Solution:

First isolate the variable.

Subtract 1 from both sides

$$-3x + 1 - 1 < 7 - 1$$

$$-3x < 6$$

Divide both sides by -3 .

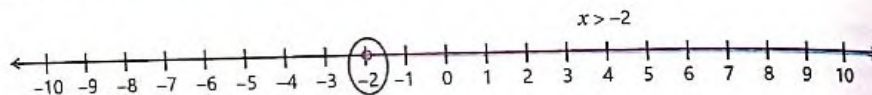
$$\frac{-3x}{-3} > \frac{6}{-3}$$

(as we divided by a negative number, the equality sign will be reversed)

$$x > -2$$

So, $x > -2$ is the solution for this inequality. We can say that the solution of this inequality is "all the numbers greater than -2 ".

Let's show this solution on a number line.



Exercise 8.7

1 Solve the following linear inequalities. Also represent the solution on a number line.

- a) $-3x - 2 > 8$ b) $4x < -8$ c) $6x - 2 > 12$ d) $7x + 3 < 2$
 e) $8x > 16$ f) $2(x + 2) > 10$ g) $5x - 2 < 8$ h) $-5x > 15$



Tell them that like equations, adding and subtracting the same number on both side of the inequality does not change the inequality. Similarly multiplying and dividing by a positive number does not change the inequality. But when we multiply or divide the inequality by a negative number, the equality sign will be reversed.

Note it down

Multiplying or dividing an inequality by a negative value changes the direction of the inequality.

8.9 Gradient of a Straight Line

Gradient is simply the measure of steepness. When we climb up a hilly road, or driving uphill, it is much more effort taking than driving or walking on a straight road or a downhill. Mathematically, we define gradient (also called slope) as "the rise over run". The vertical change between two points is called the rise, and the horizontal change is called the run.

Observe the figures below. In the right-side image, the hill is a lot steeper than the left one. Obviously its much difficult to ride the bicycle there.

Previous Knowledge Check

- What is Cartesian coordinate plane?
- Draw a graph of linear equation $2x + 3y = 5$
- Write two linear equations in one and two variables.



The gradient is positive when the line moves upward from left to right.



The gradient is 0 when the line is horizontal.



The gradient is negative when it moves downwards from left to right.



The gradient is undefined and can't be calculated when the line is vertical.



Tell students about the gradient of the straight line and how to find the value of gradient. Use various real-life examples to explain the concept. Practically move a line to change the gradients and ask the students how does it effect on x and y coordinates. Then plot a graph and ask students to find the value of gradient from the graph.

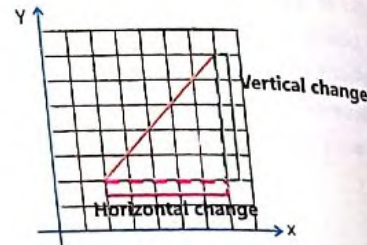
8.9.1 Calculating the Gradient of a Straight line

To calculate the gradient of a straight line we simply calculate the ratio of vertical change to the horizontal change.

Note it down

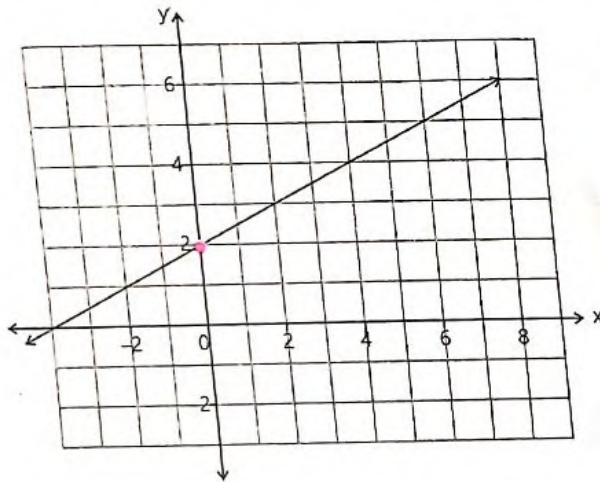
The gradient of a straight line can be calculated by drawing a right-angled triangle between any two points lying on the line.

$$\text{Gradient of a straight line} = \frac{\text{Vertical change}}{\text{Horizontal change}}$$



Example 1:

Look at the straight line and find its gradient.



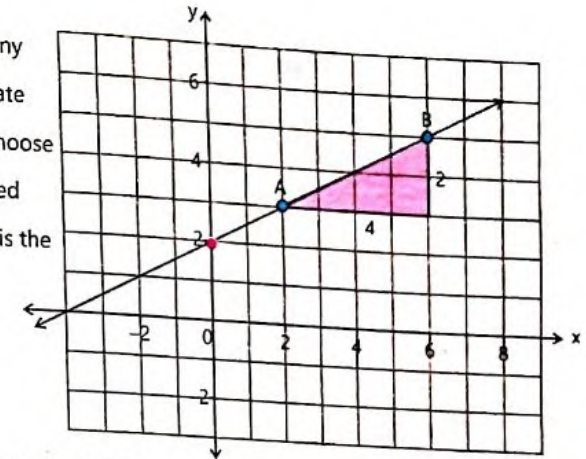
TEACHING POINT Explain them that gradient will be 0 when the line is horizontal as it is the ratio of change in y to change in x. Practically show them a few examples of positive, negative, 0 and unidentified gradient.

NOT FOR SALE

Solution:

On a straight line, the value of gradient (slope) is always constant.

So, to find the gradient, choose any two points on the line and calculate vertical and horizontal rise. We choose point A and B. Draw a right-angled triangle, so that the straight line is the hypotenuse of the triangle.



Calculate the vertical change and horizontal change.

Therefore:

$$\begin{aligned} \text{Gradient of a straight line} &= \frac{\text{Vertical change}}{\text{Horizontal change}} \\ &= \frac{2}{4} \\ &= \frac{1}{2} \end{aligned}$$

Quick Check

Draw a line for the equation $\frac{3}{2}x - y = -1$ and find its gradient.

So, the gradient of the given line is $\frac{1}{2}$

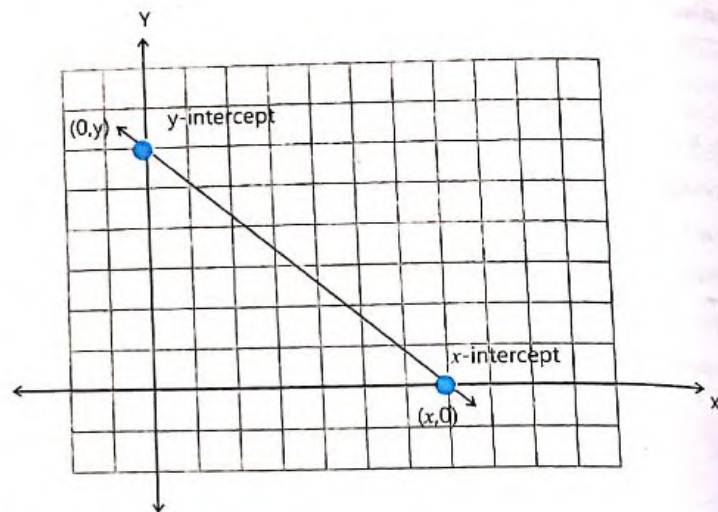
8.9.2 x and y intercepts

The points where a graph of an equation intersects the coordinate axes are called **intercepts**. The x-coordinate a of the point (a,0) at which the graph intersects x-axis is called x-intercept, and the y-coordinate b of the point (0,b) at which the graph intersects y-axis is called y-intercept.

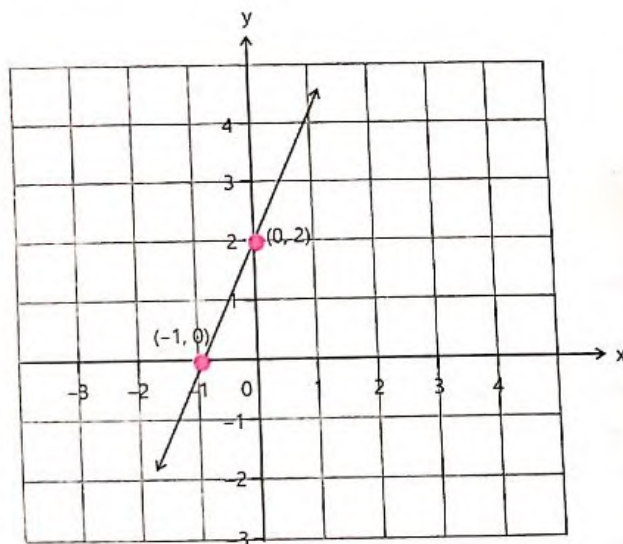


Tell the students that gradient on a straight line remains constant throughout so they can choose any two points on the line and make a right-angle triangle to find the change in x and change in y.

NOT FOR SALE



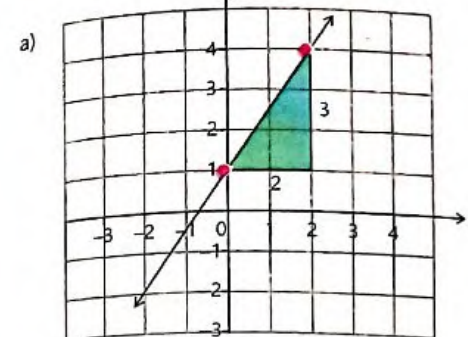
Observe the graph of equation $2x - y = -2$.



Here the x intercept is -1 and the y-intercept is 2.

Example 1:

Interpret the gradients of the following lines. Also find the x and y-intercepts.



Solution:

In (a), the gradient is positive as the line is going upward from left to right.

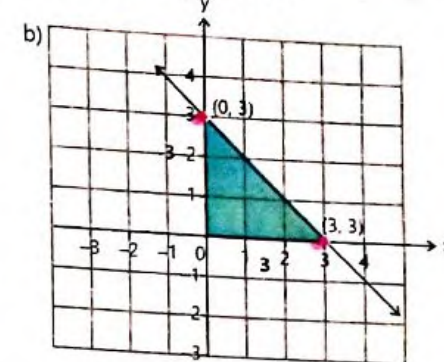
$$\text{Gradient} = \frac{\text{Vertical change}}{\text{Horizontal change}} = \frac{3}{2}$$

The line is crossing the y-axis at (0, 1), so the y-intercept is 1.

In (b), the gradient is negative as the line is going downward from left to right

$$\text{Gradient} = \frac{\text{Vertical change}}{\text{Horizontal change}} = \frac{-3}{3} = -1$$

The line is crossing the y-axis at (0, 3), so the y-intercept is 3.



What is the x-intercepts for these graphs?

Example 2:

Find the x and y intercepts of the graph of $y = x - 4$

Solution:

We have $y = x - 4$

To find x-intercept, put $y = 0$

$$y = x - 4 \Rightarrow 0 = x - 4 \text{ or } x = 4$$

To find y-intercept, put $x = 0$

$$y = x - 4 \Rightarrow y = 0 - 4 \text{ or } y = -4$$

Plot the graph for $y = x - 4$ and verify the intercepts graphically

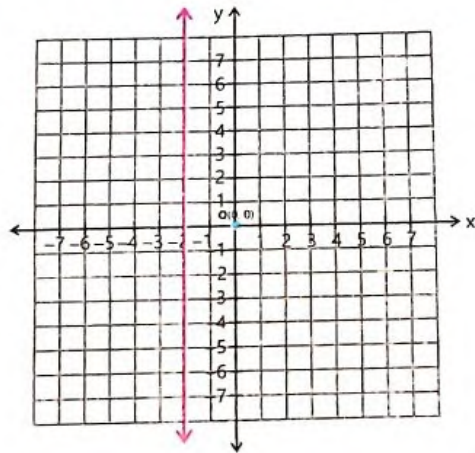


Make the student clear about x and y intercepts. Tell them that x and y intercepts are actually the coordinates x and y and not the points where they meet on the x and y axis. For example if the line intersects y axis at (0, -3), the y-intercept will be -3 and not (0, -3).

8.9.3 Equation of horizontal and Vertical Lines

The equation of a vertical line is in the form $x = a$. In a vertical line, the x -intercept is constant. The vertical line goes through a on the x -axis.

This graph is showing an equation of a vertical line $x=2$



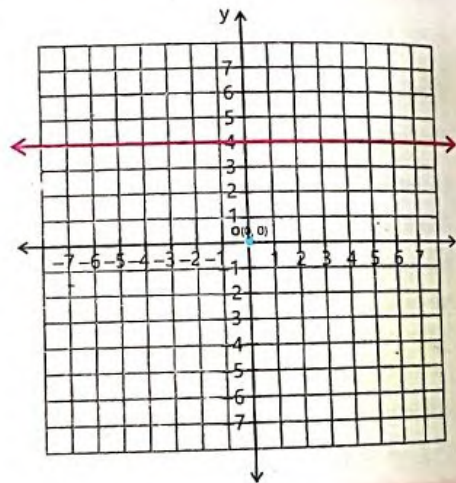
Note it down

$x = a$, represents the equation of a vertical line where a is a constant.

Note it down

The general form of equation of a straight line is $ax + by + c = 0$. If $a = 0$ then the line is horizontal, and if $b = 0$ then the line is vertical.

The equation of a horizontal line is in the form $y = b$. In a horizontal line, the y -intercept is constant. The horizontal line goes through b on the y -axis. This graph is showing an equation of a horizontal line $y=4$



Note it down

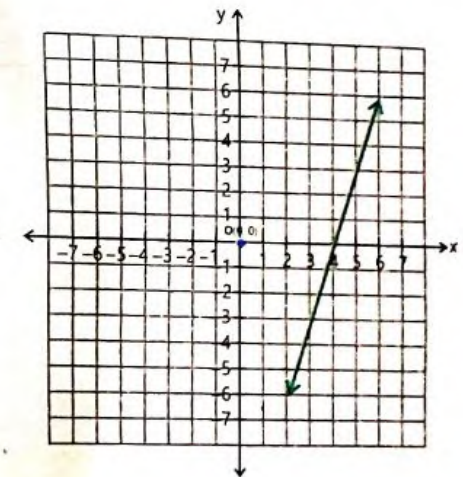
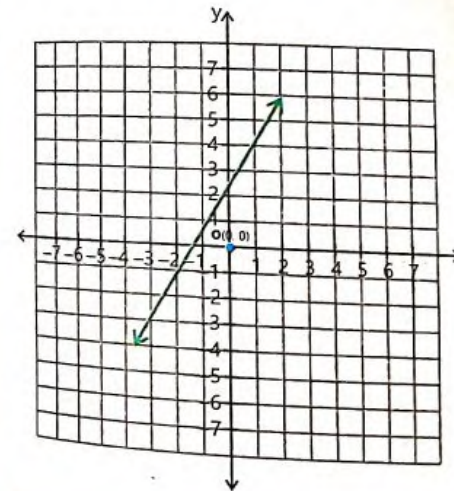
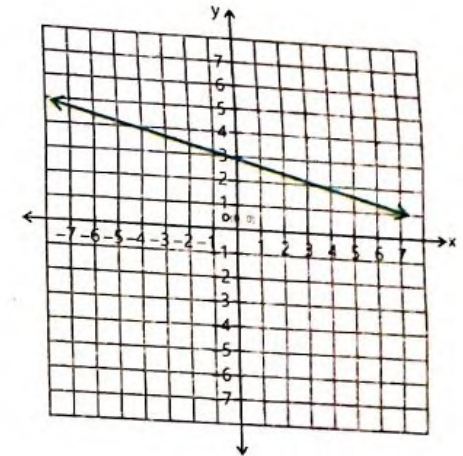
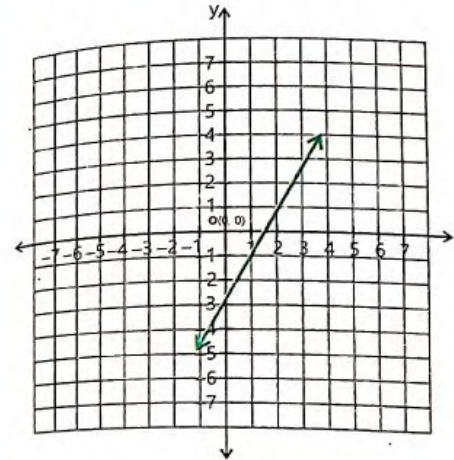
$y = b$, represents the equation of a horizontal line where b is a constant.



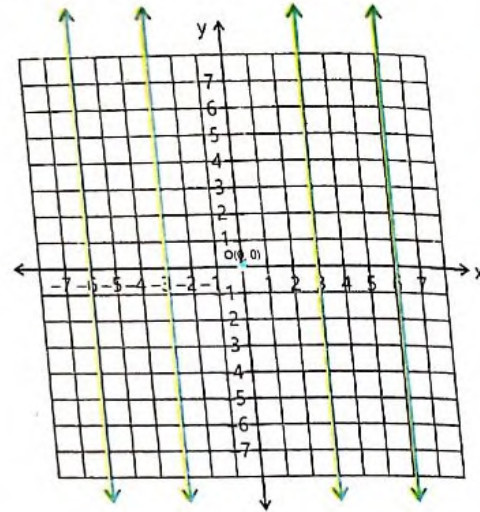
Have them recall the equation of a horizontal and vertical line. Tell them that when x is constant and y is changing, we get an equation of a vertical line and when y is constant and x is changing, then we get an equation of horizontal line.

Exercise 8.8

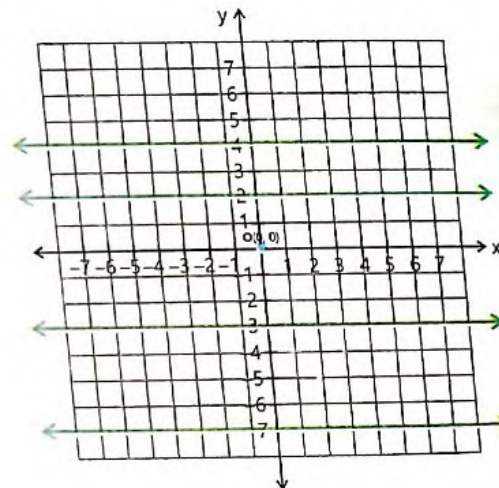
1 Find and interpret the gradient of the following straight lines. Also describe the y -intercept.



2 Write the equation for the following vertical lines.



3 Write the equation for the following horizontal lines.

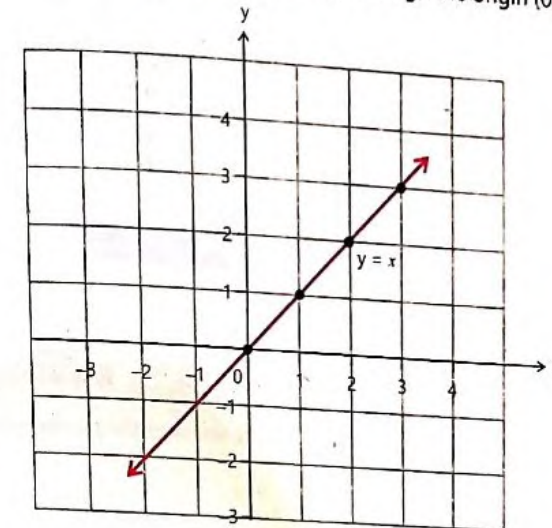


8.10 Graphs of equation of the form $y = mx$

Observe the line $y = x$. Here the y coordinate is equal to the x-coordinate. Let's find a few ordered pairs for the equation which satisfy the equation. Use Trial and Error method. Put values of x and find the corresponding values of y or vice versa.

x	0	1	2	3
y	0	1	2	3

We can see that the graph is a straight line passing through the origin (0, 0)



Again, consider $y = \frac{1}{2}x$. Here the y coordinate is equal to half the x-coordinate. Let's find a few ordered pairs for the equation which satisfy the equation. Use Trial and Error method. Put values of x and find the corresponding values of y or vice versa.

Note it down

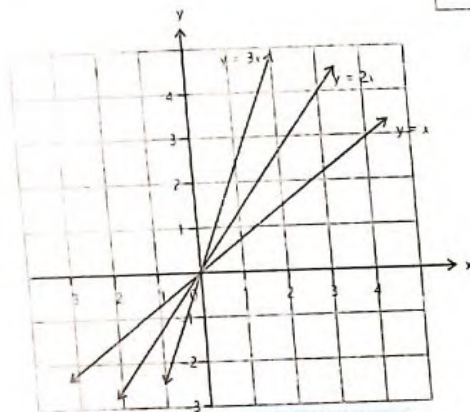
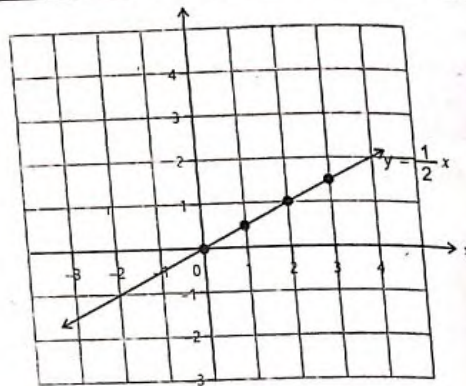
$y = mx$ represents the equation of a straight line with gradient m passing through the origin.



Tell the students that we can put various values of x to get the corresponding values of Y and similarly we can put many values of Y to get the corresponding values of x. Tell them that try to put smaller numbers like 0, -1, 1, -2, 2 etc.; when finding the ordered pairs satisfying an equation.

x	0	1	2	3
y	0	$\frac{1}{2}$	1	$\frac{3}{2}$

We can see that the graph is a straight line passing through the origin (0, 0). Similarly, for equations of the type $y = mx$ we will get a graph of a straight line that passes through the origin. In such equations "m" shows the gradient.



Graph of Equations of the form $y = mx$.

Gradient (Slope)

Use the formula:

$$\text{Gradient} = \frac{\text{Vertical change}}{\text{Horizontal change}}$$

And check the gradients of the lines:

- $y = x$
- $y = \frac{1}{2}x$
- $y = 5x$

8.10.1 Graphs of equation of the form $y = mx + c$

We know that the standard form of linear equation in two variables is $ax + by = c$. We have also learnt how to draw a graph of this type of equation. Let's learn about the graphs of straight line when the equation is in the form of $y = mx + c$.

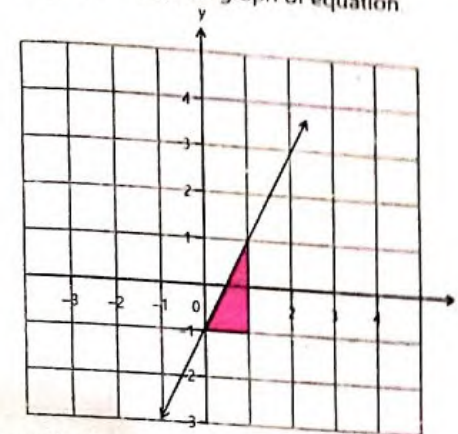
TEACHING TIP Explain the students that $y = mx$ is another form of the equation of straight line where the straight line always passes through the origin. Ask the students to draw various lines using the equation $y = mx$ and putting various values of m in it. Then ask them to interpret their results.

NO FOR SALE

Now observe an equation: $y = 2x - 1$.
Let's find some ordered pairs satisfying this equation by putting a few values get the corresponding values of y or x .

x	0	$\frac{1}{2}$	1
y	-1	0	1

plot these ordered pairs to get the required graph of equation



Choose any two points on the line and draw a right triangle and calculate the gradient

$$\text{Gradient} = \frac{\text{Vertical change}}{\text{Horizontal change}} = \frac{2}{1} = 2$$

Now find the y-intercept. When $x = 0$ the value of y is -1 . So this line cuts the y -axis at $y = -1$. So, the y intercept is -1 .

Observe the equation again:

$$y = 2x + (-1)$$

We can see that in this form, 2 is representing the gradient(slope) and -1 is representing the y -intercept of the straight line.

In general, when the equation of a straight line is in the form $y = mx + c$, m represents the gradient (or slope) and c represents the y -intercept of the line.

$$\begin{aligned} y &= mx + c \\ m &= \text{gradient} \\ c &= y \text{ intercept} \end{aligned}$$

NO FOR SALE

Example 1:

Identify the gradient and y intercept without plotting. Then plot the graph for $y = 2x + 3$ and verify the value of gradient and the y intercept.

Solution:

To find the gradient and y intercept, compare the given equation with the standard slope intercept form of the equation i.e. $y = mx + c$

$$y = 2x + 3$$

By comparison, we can see that:

$$m = 2 \text{ and } y \text{ intercept} = 3$$

Now let's draw a graph for this equation.

For this, let's find some ordered pairs satisfying this equation by putting a few values of x or y and get the corresponding values of y or x .

x	0	$-\frac{3}{2}$	1
$y = 2x + 3$	3	0	5

Plot these ordered pairs to get the required graph of equation

Quick Check

Complete the table

Equation	Gradient	y intercept
$y = 3x + 2$		
$y = 5x - 2$		
$y = -2x + 4$		



Tell the students that $y = mx + c$ is another form of the equation of a straight line and this form is called slope intercept form. In slope intercept form, m represents the slope or gradient of the line and c represents the y intercept of the line which can be calculated by putting $x=0$ in the equation

Note it down

The equation $y = mx + c$ is known as "the slope intercept form" of a straight line, where m represents the slope of the line, and c represents the y coordinate of the y intercept

The slope intercept form of a linear equation helps us easily identify the gradient (i.e. how steep a line is) and the y intercept (i.e. where it crosses the y axis).

To verify the gradient and y-intercepts, choose any two points on the line and draw a right triangle and calculate the gradient: e.g (0,1) (-1.5,3) etc

$$\text{Gradient} = \frac{\text{Vertical change}}{\text{Horizontal change}} = \frac{3}{1.5} = 2$$

Now find the y intercept. When $x = 0$ the value of y is 3. So this line cuts the y-axis at $y = 3$. So, the y intercept is 3. Hence verified.

Exercise 8.9

1 Find the value of x or y and complete the tables.

x	0		-1	
$y = 2x - 2$		0		2

x	0		2	1
$y = \frac{1}{2x+3}$		0		

x	0			-1
$y = -3x - 5$		0	-2	

2 Plot the graphs for the following equations of straight lines.

- a) $y = x$ b) $y = 3x$ c) $y = \frac{1}{2}x$ d) $y = \frac{5}{2}x$
 e) $y = -x$ f) $y = -2x$ g) $y = -3x$

3 Plot the graphs for the following equations of straight lines.

- a) $y = 2x + 1$ b) $y = 4x - 2$ c) $y = 3x - 1$ d) $y = x + 2$
 e) $y = \frac{1}{2}x - \frac{5}{2}$ f) $y = 4x - 3$

- 4 Identify the gradient and y-intercept for the following equations without plotting the graph. Then plot the graphs and verify the value of gradient and the y-intercept (if the equation is not in the slope intercept form, first write it in slope intercept form and then solve).

a) $x - y = 5$ b) $2x - y = -4$ c) $-4x - y = -2$ d) $y = 3x + 3$
 e) $y + x = 2$ f) $y = -2x - 3$

- 5 Find the slope intercept form of equation of the straight lines described below.

a) gradient 2, y-intercept -1 b) gradient -2, y-intercept 2
 c) gradient 2, y-intercept -1 d) gradient $-\frac{5}{2}$, y-intercept -2
 e) gradient $\frac{1}{2}$, passing through the origin
 f) gradient 3, passing through (0, 0) g) gradient -3, y-intercept 1

Think Higher

Without plotting graph, how can you tell if the point (6, -4) is on the graph of the equation $5y = 2x - 32$? Justify your answer.

Summary

- The gradient of a straight line can be calculated by drawing a right-angled triangle between any two points lying on the line.
- $x = a$, represents the equation of a vertical line is where a is a constant.
- The general form of equation of a straight line is $ax + by + c = 0$. If $a = 0$ then the line is horizontal, and if $b = 0$ then the line is vertical.
- $y = mx$ represents the equation of a straight line with gradient m passing through the origin.
- A solution for a single equation is any point that lies on the line for that equation. A solution for a system of equations is any point that lies on each line in the system.
- The general form of simultaneous linear equations is: $ax + by = c$, $a_1x + b_1y = c_1$, where x and y are variables, a , b , a_1 , b_1 are non-zero coefficients and c , c_1 are the constants.

Vocabulary

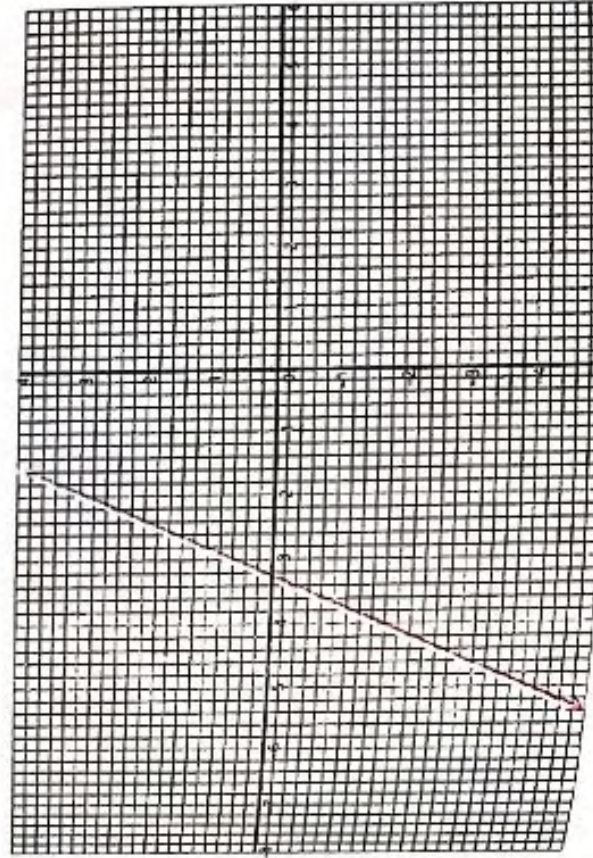
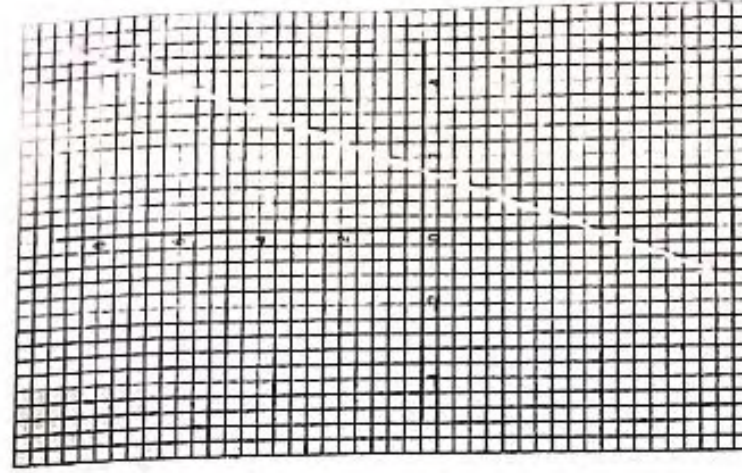
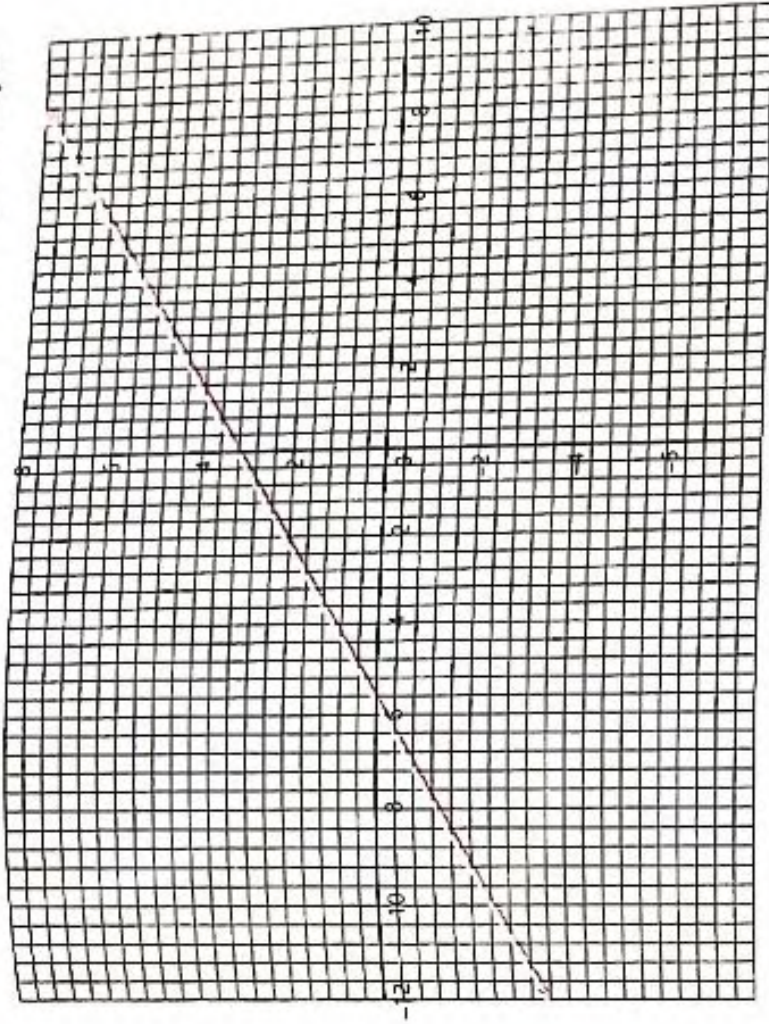
- Linear equation
- Simultaneous linear equation
- Intercept
- Gradient
- Linear inequality
- Straight line

Review Exercise

- 1 Choose the correct option.

- a) The measure of steepness is known as:
 i) Angle ii) Gradient iii) run iv) rise
- b) The gradient is positive when:
 i) The line moves downwards from left to right
 ii) The line moves downwards from right to left
 iii) The line moves upward from left to right
 iv) The line moves upward from right to left
- c) If the horizontal change is 3 and the gradient is 2 then vertical change is
 i) 2 ii) 3 iii) 5 iv) 6
- d) If a straight line is crossing the y-axis at (0, -2), the y-intercept is:
 i) 0 ii) 2 iii) -2 iv) -1
- e) To find x-intercept of a line, we put:
 i) $x = 1$ ii) $y = 0$ iii) $x = 0$ iv) $y = 1$
- f) In $ax + by + c = 0$, the line will be a horizontal line if:
 i) $a = 0$ ii) $b = 0$ iii) $a = 1$ iv) $b = 1$
- g) Which of these line passes through the origin?
 i) $y = 2x + 6$ ii) $y = 3x$ iii) $y = 3x - 4$ iv) $y = 4x - 1$
- h) The equation of horizontal line is:
 i) $y = c$ ii) $x = c$ iii) $x = y$ iv) $y = mx$
- i) The equation of a straight line is:
 i) $x = y^2$ ii) $y = mx + c$ iii) $y^2 = \frac{x}{2} + c$ iv) $x^2 = cy$
- j) The gradient of the line $y = x$ is:
 i) 1 ii) $\frac{1}{2}$ iii) $\frac{1}{3}$ iv) -6
- k) A number y is $\frac{5}{4}$ of another number x is shown by:
 i) $y = \frac{5}{4} + x$ ii) $y = \frac{5}{4} - x$ iii) $y = \frac{5}{4}x$ iv) $y = \frac{5}{4} \div x$

12 Find and interpret the gradient of these lines. Also find the y-intercept.



NOT FOR SALE

Math Project



Material Required:

- Equation cards
- Boxes with labels
- Graphing paper

Procedure:

- Work in pairs.
- Teacher will prepare multiple equation cards each having 2 simultaneous linear equations (including the ones having no solution, infinite solutions and unique solution).
- Each pair will graph the equations and check if they have a unique solution or not.
- Then they will put the card in the relevant box.
- The pair with quick and accurate solutions wins.



One
Solution



No
Solutions



Many
Solutions

NOT FOR SALE

207

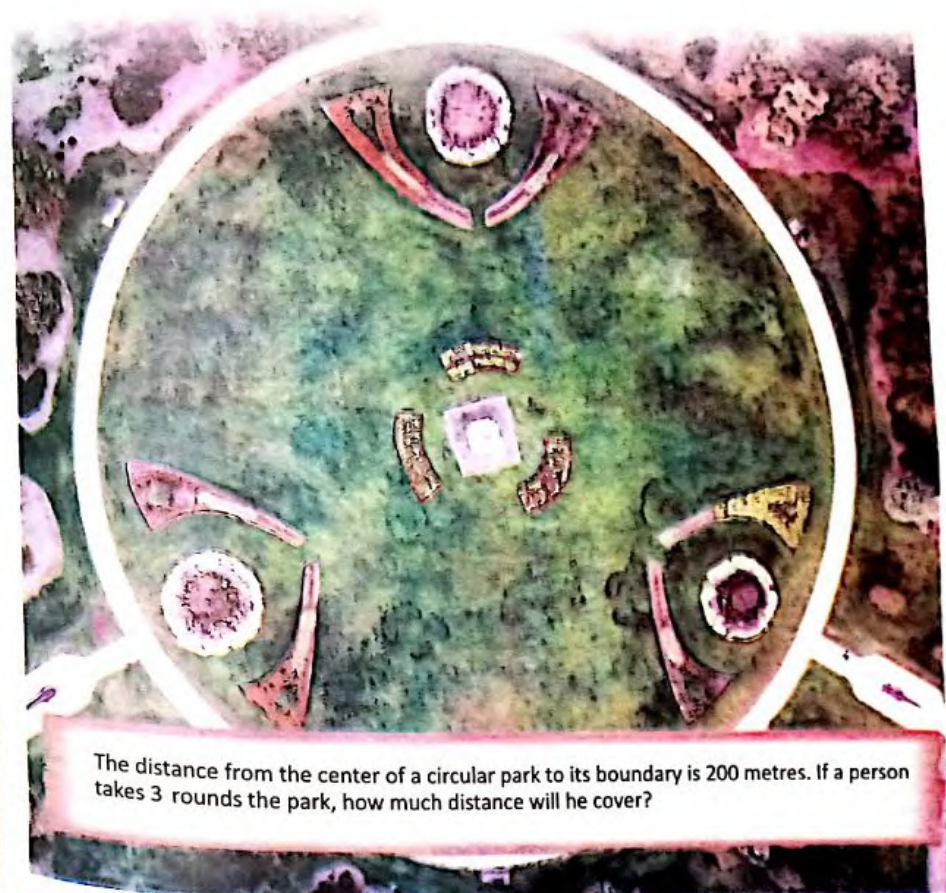
Unit 9

Surface, Area and Volume

Student Learning Outcomes

After completing this unit, students will be able to:

- State the Pythagoras theorem and use it to solve right angled triangles.
- Solve real life word problems using Pythagoras theorem.
- Calculate the arc length and the area of the sector of a circle.
- Calculate the surface area and volume of the pyramid, sphere, hemisphere and cone.
- Solve real life word problems involving the surface area and volume pyramid, sphere, hemisphere and cone.



The distance from the center of a circular park to its boundary is 200 metres. If a person takes 3 rounds the park, how much distance will he cover?

208

NOT FOR SALE

Introduction

In previous grades, we have learnt how to calculate the circumference and area of a circle, surface area and volume of cube, cuboids, right prisms, and cylinders. Now we will learn about the surface area and volume of some other solids like sphere, hemispheres, cones, pyramids etc. We will also learn about Pythagoras theorem and its application in solving right-angle related problems.

9.1 Pythagoras Theorem

Pythagoras theorem plays an important role in the geometry. This theorem explains the relation between the length of the right angled triangle. The statement of the Pythagoras theorem is stated as:

In a right angle triangle the square of the length of the hypotenuse is equal to the sum of the squares of the lengths of the other two sides.

For example, ABC is a right angled triangle and \overline{AC} is the hypotenuse. According to the Pythagoras theorem,

$$(\overline{AC})^2 = (\overline{AB})^2 + (\overline{BC})^2$$

9.1.1 Informal Proof of Pythagoras Theorem:

We shall consider the following steps to prove the Pythagoras theorem in an informal way.

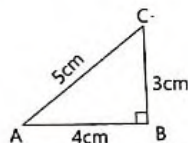
- Draw a right angle triangle

ABC in which:

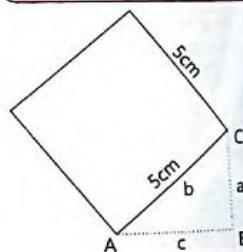
$$\overline{AC} \text{ (hypotenuse)} = 5\text{cm}$$

$$\overline{AB} \text{ (base)} = 4\text{cm}$$

$$\overline{BC} \text{ (altitude)} = 3\text{cm}$$



- Draw a square with length of a side equal to the length of the hypotenuse (i.e. $\overline{AC} = 5\text{cm}$).



- Similarly, draw squares on the other sides of $\triangle ABC$.

As we observed that:

$$\text{Area of square with length } 5\text{cm} = 5\text{cm} \times 5\text{cm} = 25\text{cm}^2$$

Math History

A Greek mathematician **Pythagoras** discovered the Pythagoras theorem 2500 years ago.



NOT FOR SALE

Area of square with length 4cm = $4\text{cm} \times 4\text{cm} = 16\text{cm}^2$
Area of square with length 3cm = $3\text{cm} \times 3\text{cm} = 9\text{cm}^2$
It is clear from the above result that:

$$25\text{cm}^2 = 16\text{cm}^2 + 9\text{cm}^2$$

$$(\overline{AC})^2 = (\overline{AB})^2 + (\overline{BC})^2$$

$$(\text{Hypotenuse})^2 = (\text{Base})^2 + (\text{Altitude})^2$$

With the help of the Pythagoras theorem, we can find the length of any side of a right angled triangle when we have the length of other two sides.

Example 1:

Find the length of \overline{YZ} in the given figure.

Solution:

According to the Pythagoras theorem,

$$(\overline{XZ})^2 = (\overline{XY})^2 + (\overline{YZ})^2$$

$$(13\text{cm})^2 = (5\text{cm})^2 + (x)^2$$

$$169\text{cm}^2 = 25\text{cm}^2 + x^2$$

$$\Rightarrow x^2 = 169\text{cm}^2 - 25\text{cm}^2 = 144\text{cm}^2$$

Taking square root on both sides, we get

$$\sqrt{x^2} = \sqrt{144\text{cm}^2}$$

$$\Rightarrow x = 12\text{cm}$$

$$\text{Hence, } \overline{YZ} = 12\text{cm}$$

With the help of the Pythagoras theorem, we can find the length of any side of a right angled triangle when we have the length of other two sides..

Example 2:

Find the length of \overline{BC} in the given figure.

Solution:

According to the Pythagoras theorem:

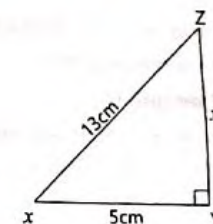
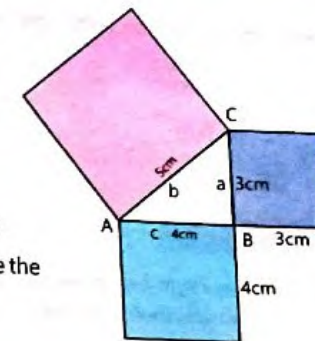
$$(\overline{AC})^2 = (\overline{AB})^2 + (\overline{BC})^2$$

$$(25\text{cm})^2 = (24\text{cm})^2 + (x)^2$$

$$625\text{cm}^2 = 576\text{cm}^2 + x^2$$

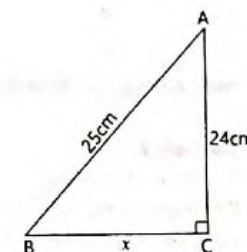
$$\Rightarrow x^2 = 625\text{cm}^2 - 576\text{cm}^2$$

$$\Rightarrow x^2 = 49\text{cm}^2$$



Quick Check

Draw a right angled $\triangle PQR$ whose hypotenuse $\overline{PR} = 5\text{cm}$, $\overline{PQ} = 3\text{cm}$, $\overline{QR} = 4\text{cm}$ and $\angle Q = 90^\circ$. Prove Pythagoras theorem is also true for this triangle.



NOT FOR SALE

Taking square-root on both sides, we get

$$\sqrt{x^2} = \sqrt{49\text{cm}^2}$$

$$x = 7\text{cm}$$

$$\overline{BC} = 7\text{cm}$$

Quick Check

Construct a right-angled triangle in which the length of the hypotenuse is 10cm and the length of other two sides are 8cm and 6cm. Verify the Pythagoras theorem by constructing squares on its sides.

9.1.2 Practical Application of Pythagoras Theorem

Now, we shall explain the practical application of the Pythagoras theorem through the following examples:

Example 1:

A ladder is 10m long and reaches the top of a wall. If the distance between the wall and the foot of the ladder is 6m, find the height of the wall.

Solution:

Let \overline{AC} be the ladder, \overline{BC} be the height of the wall and \overline{AB} be the distance between the foot of the ladder and the wall.

According to the Pythagoras theorem,

$$(\overline{AC})^2 = (\overline{AB})^2 + (\overline{BC})^2$$

$$(10\text{m})^2 = (6\text{m})^2 + (x)^2$$

$$100\text{m}^2 = 36\text{m}^2 + x^2$$

$$100\text{m}^2 - 36\text{m}^2 = x^2$$

$$\Rightarrow x^2 = 64\text{m}^2$$

Taking square root on both sides

$$\sqrt{x^2} = \sqrt{64}$$

$$\Rightarrow x = 8\text{m}$$

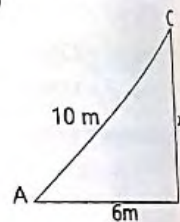
Hence, height of the wall = 8m

Example 2:

A ladder touches the top of a wall whose height is 12m. Find the length of the ladder if the distance between the foot of the ladder and wall is 5m.

Note it down

By applying Pythagoras theorem we can find the measurement of an unknown side when the lengths of the other two sides are known.



Note it down

Always draw rough figure before solving the given problem.

Note it down

An electric pole breaks at point C and its top touches the ground at the distance 3m as shown in the figure. If height of broken electric pole from the ground is 4m, find the total height of the pole.

Solution:

Let x be the length of the ladder. According to the Pythagoras theorem,

$$(\overline{PR})^2 = (\overline{PQ})^2 + (\overline{QR})^2$$

$$(x)^2 = (5\text{m})^2 + (12\text{m})^2$$

$$x^2 = 25\text{m}^2 + 144\text{m}^2$$

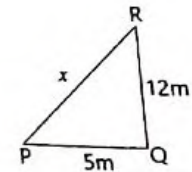
$$\Rightarrow x^2 = 169\text{m}^2$$

Taking square root on both sides

$$\sqrt{x^2} = \sqrt{169\text{m}^2}$$

$$x = 13\text{m}$$

The length of the ladder is 13m.



Example 3:

The hypotenuse of a right-angled triangular field is 30m. If the length of its one side is 24m, then find the length of its other side.

Solution:

Let the length of the unknown side be x m

Length of hypotenuse = 30 m

Length of base = 24 m

Length of altitude = x m

According to the Pythagoras theorem,

$$(\text{Base})^2 + (\text{Altitude})^2 = (\text{Hypotenuse})^2$$

$$(24)^2 + (x)^2 = (30)^2$$

$$576 + x^2 = 900$$

$$x^2 = 900 - 576 = 324$$

Taking square-root on both sides,

$$\sqrt{x^2} = \sqrt{324}$$

$$x = 18$$

Length of the unknown side = 18m.

Example 4:

Find the length of the diagonal of a square-shaped room whose each side is 10 metres long. (Give your answer correct to one decimal point)

Solution:

As room is square-shaped, therefore each angle is 90° . So $\triangle ABC$ is a right-angled triangle with $\angle B$ as 90° angle. According to the Pythagoras theorem

$$(\overline{AC})^2 = (\overline{AB})^2 + (\overline{BC})^2 \quad (i)$$

Let $\overline{AC} = x$ m eq(i) becomes,

$$x^2 = 10^2 + 10^2$$

$$x^2 = 100 + 100 = 200$$

Taking square-root on both sides,

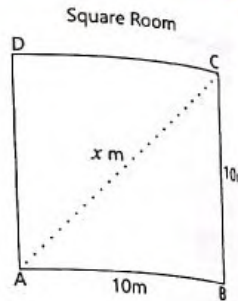
$$\sqrt{x^2} = \sqrt{200}$$

$$\Rightarrow x = 10\sqrt{2}$$

$$\Rightarrow x = 10 \times 1.414$$

$$\Rightarrow x = 14.14$$

Length of the diagonal of the room = 14.1 m



Example 9.1

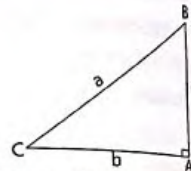
$\triangle ABC$ is a right-angled triangle with $\angle A = 90^\circ$. Find the value of the unknowns in the following parts.

Note: In $\triangle ABC$, the side opposite to the $\angle A$ will be denoted by a , the side opposite the $\angle B$ will be denoted by b and the side opposite the $\angle C$ will be denoted by c .

i) $a = 6\text{cm}$, $c = 4\text{cm}$

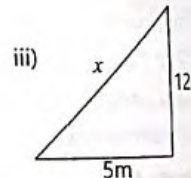
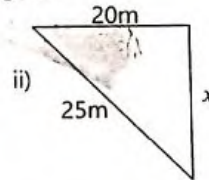
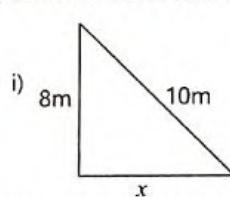
ii) $b = 10\text{cm}$, $c = 15\text{cm}$

iii) $c = 12\text{cm}$, $a = 8\text{cm}$

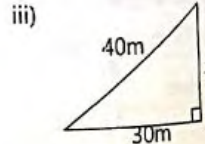
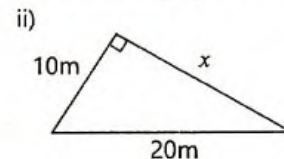
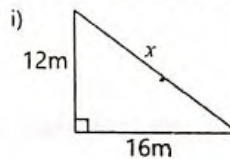


Exercise 9.1

1 Find the value of x in each figure.



2 Find the value of x in each of the following figures:



3 In $\triangle XYZ$, $\angle Y = 90^\circ$. Find the length of the missing side in each of the following:

i) $\overline{XZ} = 15\text{cm}$, $\overline{YZ} = 10\text{cm}$

ii) $\overline{XY} = 16\text{cm}$, $\overline{YZ} = 8\text{cm}$

iii) $\overline{XY} = 2.5\text{cm}$, $\overline{XZ} = 5\text{cm}$

4 In Triangle ABC , $\angle B = 90^\circ$ and b is the length of the hypotenuse, then find the length of the missing side:

i) $a = 12\text{cm}$, $c = 9\text{cm}$

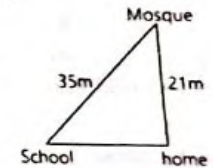
ii) $b = 24$, $a = 7\text{cm}$

iii) $a = 2\text{cm}$, $c = 1.5\text{cm}$

iv) $b = \frac{15}{25}\text{cm}$, $c = \frac{12}{25}\text{cm}$

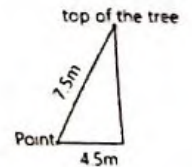
5 In a right angled triangle, the length of its altitude is 15m and its base is 8m. Find the length of its hypotenuse.

6 A 3.4m long ladder touches the top of a wall. Find the height of the wall if the distance between the foot of the ladder and the wall is 2.4m.



7 Observe the given figure and find the distance between the school and home.

8 The distance between the top of a tree and a point on the ground is 7.5m. Find the height of the tree if the distance between the point of the ground and the tree is 4.5m.

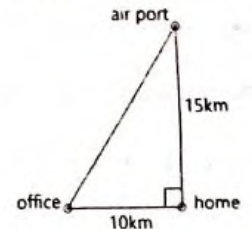


9 The length of the hypotenuse of a right angled isosceles triangle is 32 cm^2 . Find the length of its side.

10 A ladder which is 9m long touches the top of the wall when its foot is 3m away from the wall. Find the height of the wall.

11 The distance between the top of a hill and a point on the ground is 400m. The distance between the point on the ground and the hill is 300m. Find the height of the hill.

12 See the figure shown here and find the distance between the office and the airport.



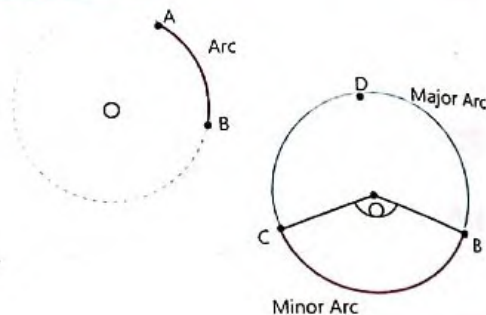
13 The altitude of a right-angled triangle is 12cm and its hypotenuse is 18cm long. Find the length of the base of the triangle.

9.2 Parts of Circles

Arcs and Sectors of a Circle

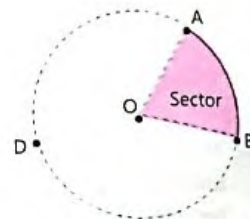
A part or portion of the circumference of a circle is called an arc. Look at the circle. The arc \widehat{AB} has two end points A and B. It is denoted by \widehat{AB} .

An arc which is smaller than half of the circle is called a minor arc. An arc which is more than half of the circle is called a major arc. Look at the circle. \widehat{BC} is the minor arc and the arc \widehat{BDC} is the major arc as it is more than half of the circle.



Sector of a Circle

A sector is the part of a circle enclosed by two radii and an arc. In the figure, AB is the arc, OA and OB are two radii. The area enclosed is representing a sector of the circle. The symbol sector is used to denote a sector.



Look at the circle in the figure. The shaded region enclosed by the two radii OA and OB and the minor arc AB is the minor sector of the circle. The unshaded region enclosed by the two radii OA and OB and the major arc ADB is the major sector of the circle.

Arc length of a Circle

We know that the circumference of a circle having radius r and central angle 360° is $2\pi r$ (where r is the radius of the circle the value of π is 3.1415).

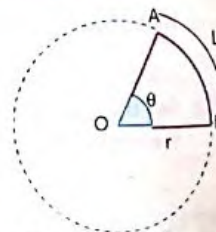
Note it down
When the central angle formed by the two radii is 90° , the sector is called a quadrant.

Let's find its arc length formula using unitary method:

The arc length (circumference) of circle with central angle $360^\circ = 2\pi r$

The arc length of circle with central angle $1^\circ = \frac{1}{360^\circ} \times 2\pi r$

So, the arc length of circle with central angle $\theta = \theta \times \frac{1}{360^\circ} \times 2\pi r$ (where θ is the angle subtended at the center, given in degrees and r is the radius of the circle).



Note it down

Central Angle of an angle formed by two radii having the center of circle as its vertex.

Note it down

The Greek letter θ (theta) is used in math as a variable to represent a measured angle.

Example 1:

A circular park has a radius of 18 m. A portion of the park is constructed in shape of a sector making an angle of 45° at center. Find the length of the curved path of this sector. Use $\pi = 3.14$.

Solution:

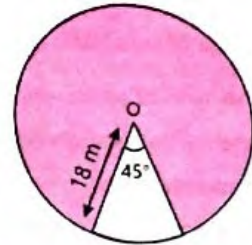
Here,

$$r = 18 \text{ m}$$

$$\theta = 45^\circ$$

$$\begin{aligned} \text{The arc length of circle} &= \frac{\theta}{360^\circ} \times 2\pi r \\ &= \frac{45^\circ}{360^\circ} \times 2 \times 3.14 \times 18 \\ &= 14.13 \text{ m} \end{aligned}$$

So, the curved path is 14.13 m long.



Example 2: Find the arc length if the radius of the circle is 4 cm and the central angle is 30° .

Here,

$$r = 4 \text{ cm}$$

$$\theta = 30^\circ$$

$$\begin{aligned} \text{The arc length of circle} &= \frac{\theta}{360^\circ} \times 2\pi r \\ &= \frac{30^\circ}{360^\circ} \times 2 \times 3.14 \times 4 \\ &= 2.09 \text{ cm} \end{aligned}$$

So, the curved path is 2.09 cm.

Area of a sector of a circle

Look at the circle. Here is the radius and θ is the central angle. Let's find the area of the shaded region (sector) enclosed by the two radii OA and OB and the minor arc \widehat{AB} .

We know that a complete circle has a central angle equal to 360° and area of a circle with central angle 360° is equal to πr^2 (where r is the radius of the circle the value of π is 3.1415).

Using unitary method:

Area of circle with central angle $360^\circ = \pi r^2$

Area of sector with central angle $1^\circ = \frac{1}{360^\circ} \times \pi r^2$

So, Area of sector with central angle $\theta = \frac{\theta}{360^\circ} \times \pi r^2$ (where θ is the angle subtended at the central, given in degrees and r is the radius of the circle).

Example 1:

A wiper of a windshield of car is 82 cm long. The angle within which it moves while wiping is 120° . Find the area it wipes.

Solution:

Here,

$$r = 82 \text{ cm}$$

$$\theta = 120^\circ$$

$$\begin{aligned} \text{So, Area of sector} &= \frac{\theta}{360^\circ} \times \pi r^2 \\ &= \frac{120^\circ}{360^\circ} \times 3.14 \times (82)^2 \\ &= 7037.7 \text{ cm}^2 \end{aligned}$$

So, the area it wipes is 7037.7 cm^2

Example 2:

Find the area of a wooden sector of a circular hall whose diameter is 40 metre and the central angle is 80° .

Solution:

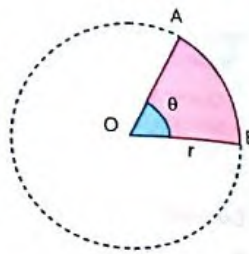
Here,

$$r = 40 \text{ cm}$$

$$\theta = 80^\circ$$

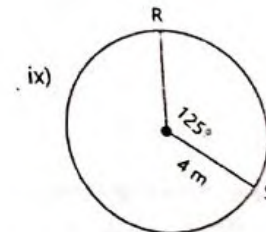
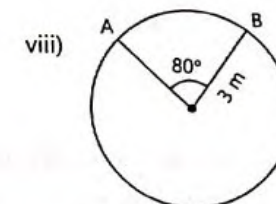
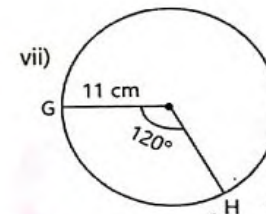
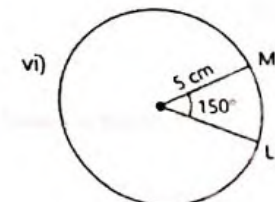
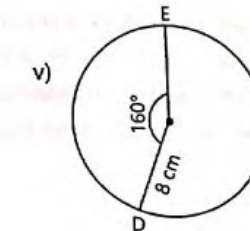
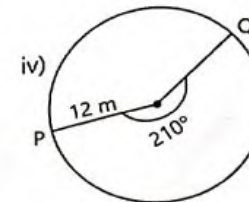
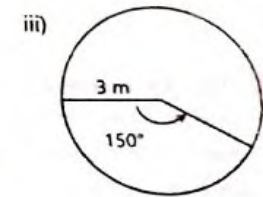
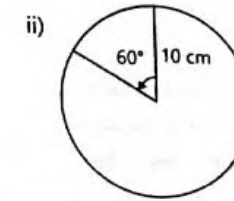
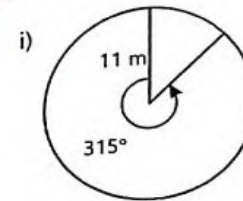
$$\begin{aligned} \text{So, Area of sector with central angle } 80^\circ &= \frac{\theta}{360^\circ} \times \pi r^2 \\ &= \frac{80^\circ}{360^\circ} \times 3.1415 \times (40)^2 \\ &= 1116.98 \text{ m}^2 \end{aligned}$$

So, the area of the wooden sector of hall is 1116.98 m^2 .



Exercise 9.2

1 Find the area and arc length of the following circles.



2 Find the length and area of the following measurements of a circle.

i) $r = 3 \text{ cm}$
 $\theta = 80^\circ$

ii) $r = 12 \text{ m}$
 $\theta = 120^\circ$

iii) $r = 10 \text{ cm}$
 $\theta = 55^\circ$

iv) $r = 3 \text{ cm}$
 $\theta = 80^\circ$

v) $r = 12 \text{ m}$
 $\theta = 120^\circ$

vi) $r = 10 \text{ cm}$
 $\theta = 55^\circ$

vii) $r = 20 \text{ cm}$
 $\theta = 110^\circ$

viii) $r = 15 \text{ m}$
 $\theta = 120^\circ$

ix) $r = 56 \text{ cm}$
 $\theta = 100^\circ$

x) $r = 21 \text{ cm}$
 $\theta = 150^\circ$

xi) $r = 6 \text{ m}$
 $\theta = 75^\circ$

xii) $r = 43 \text{ cm}$
 $\theta = 90^\circ$

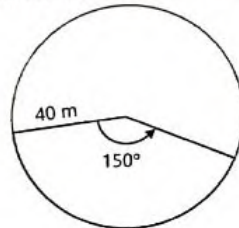
- 3 Find the area of a sector of a circular field having diameter 120 metre and the central angle 180° .



- 4 Find the area of a piece of pizza having radius 8 cm and the central angle 45° . If the pizza has 8 such identical pieces, what is the total area of the pizza.



- 5 A pendulum-like swing at an amusement park moving in a circular arc at the end of a 40 metre arm. Find the distance travelled by a rider at the end of the swing's arm if the angle made between the two extreme points is 150° .



- 6 A circular ground has been divided into three parts for different type of activities. Find the area of each part if the diameter of this ground is 220 metre.



9.3 Sphere

We have observed round solid objects in our daily life. For examples, a cricket ball, football, round piece of marble etc.



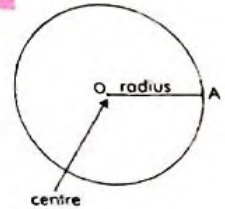
Note it down

Solid sphere is not empty from inside e.g. a cricket ball. A sphere that is hollow from inside is called hollow sphere e.g. an ordinary ball or football.

A sphere is a solid round figure which is generated by the rotation of circle on fixed point.

The distance between all the points of outer surface and the fixed point of a sphere is equal. This fixed point is called centre of a sphere. The distance between centre and outer surface points of a sphere is called radius of a sphere.

A sphere is shown in the figure. Here, O is the centre and \overline{OA} represents the radius of the sphere.



9.3.1 Surface Area and Volume of Sphere

Surface Area of Sphere:

Archimedes discovered that the surface area of a sphere is equal to the curved surface area of the cylinder having the same radius as the sphere and its height is equal to the diameter of the sphere.

Let, r be the radius of both sphere and cylinder, as shown in the figure. Here, h is equal to $2r$.

Now, as we know that,

$$\begin{aligned}\text{Curved surface area of the cylinder} &= 2\pi rh \\ &= 2\pi r(2r) \quad (\because h = 2r) \\ &= 4\pi r^2\end{aligned}$$

According to the Archimedes' statement,

$$\text{Surface area of the sphere} = 4\pi r^2$$

Volume of Sphere:

Put the sphere into the cylinder and fill the cylinder with water. Now, remove the sphere from the cylinder. We will observe that the remaining water in the cylinder will be equal to one third of water which is already measured.

Thus, the volume of a sphere is equal to the two-thirds volume of a cylinder having the same radius as the sphere and its height is equal to the cylinder of the sphere. It is also Archimedes' statement for volume of a sphere.

So, volume of a sphere = volume of a cylinder

$$\begin{aligned}&= \pi r^2 h \\ &= \pi r^2 (2r) \quad (\because h = 2r) \\ \text{Volume of a sphere} &= \frac{4}{3}\pi r^3\end{aligned}$$

Math History

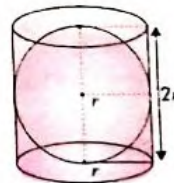
Archimedes (287BC – 212BC), a Greek mathematician, discovered the relationship between the surface area of sphere and cylinder



Sphere and a cylinder of the same radius

Note it down

When we divide a sphere into two equal parts, each part is called a hemisphere.



Surface area and volume of Hemisphere

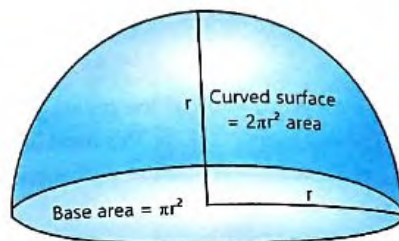
Hemisphere is half of the sphere, so:

The curved surface area of hemisphere

$$= \frac{1}{2} (\text{total surface area of the sphere})$$

$$= \frac{1}{2} (4\pi r^2)$$

$$= 2\pi r^2$$



The total surface area of the hemisphere = Curved surface area + base area

As the base of the hemisphere is circular in shape, use the area of the circle.

Base area of hemisphere = πr^2

Therefore,

The total surface area of the hemisphere = Curved surface area + base area

$$= 2\pi r^2 + \pi r^2$$

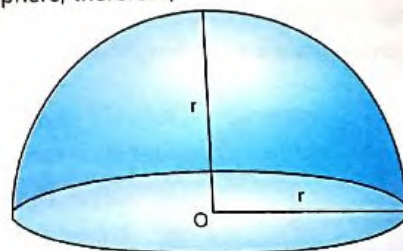
$$= 3\pi r^2$$

The volume of a hemisphere is half the volume of a sphere, therefore,

Volume of hemisphere = $\frac{1}{2}$ (volume of sphere)

$$= \frac{1}{2} \times \frac{4}{3} \pi r^3$$

$$= \frac{2}{3} \pi r^3$$



We can solve the problems related to the surface area and volume of the sphere using the formula.

Example 1:

A solid sphere has a radius of 3.5m. Calculate the surface area and volume of the sphere.

$$(\pi = \frac{22}{7})$$

Solution:

Surface area of the sphere = $4\pi r^2$

$$= 4 \times (\frac{22}{7}) (3.5)^2$$

$$= 4 \times \frac{22}{7} \times 3.5 \times 3.5 = 154\text{m}^2$$

Volume of the sphere = $\frac{4}{3} \pi r^3$

$$= \frac{4}{3} (\frac{22}{7}) (3.5)^3$$

$$= \frac{4}{3} \times \frac{22}{7} \times 3.5 \times 3.5 \times 3.5$$

$$= 179.66\text{m}^3$$

Example 2:

Find the radius of sphere with surface area of 616m^2 . ($\pi = \frac{22}{7}$)

Solution:

Surface area of sphere = $4\pi r^2$

$$616\text{m}^2 = 4\pi r^2$$

$$4\pi r^2 = 616\text{m}^2$$

$$r^2 = \frac{616}{4\pi}$$

$$= \frac{616}{4} \times \frac{7}{22}$$

$$\Rightarrow r^2 = 49\text{m}^2$$

$$r = 7\text{m} \text{ (taking square root on both sides)}$$

The radius of the given sphere is 7m.

Quick Check

- calculate the surface area and volume of the spheres for the following radii:
 - 21cm
 - 3.5m
 - 4.2m
- Find radius of the sphere with a surface area of:
 - 54cm²
 - 6.16m²
 - 385m²

9.3.2 Practical Application of Surface Area and Volume of a Sphere

We shall explain the use of the surface area and volume of a sphere in the following examples:

Example 1:

The radius of a spherical water tank is 2.5m.

- Calculate the cost of painting water tank at a rate of Rs. 175 per m². ($\pi = 3.14$)
- Find the capacity of the water tank.

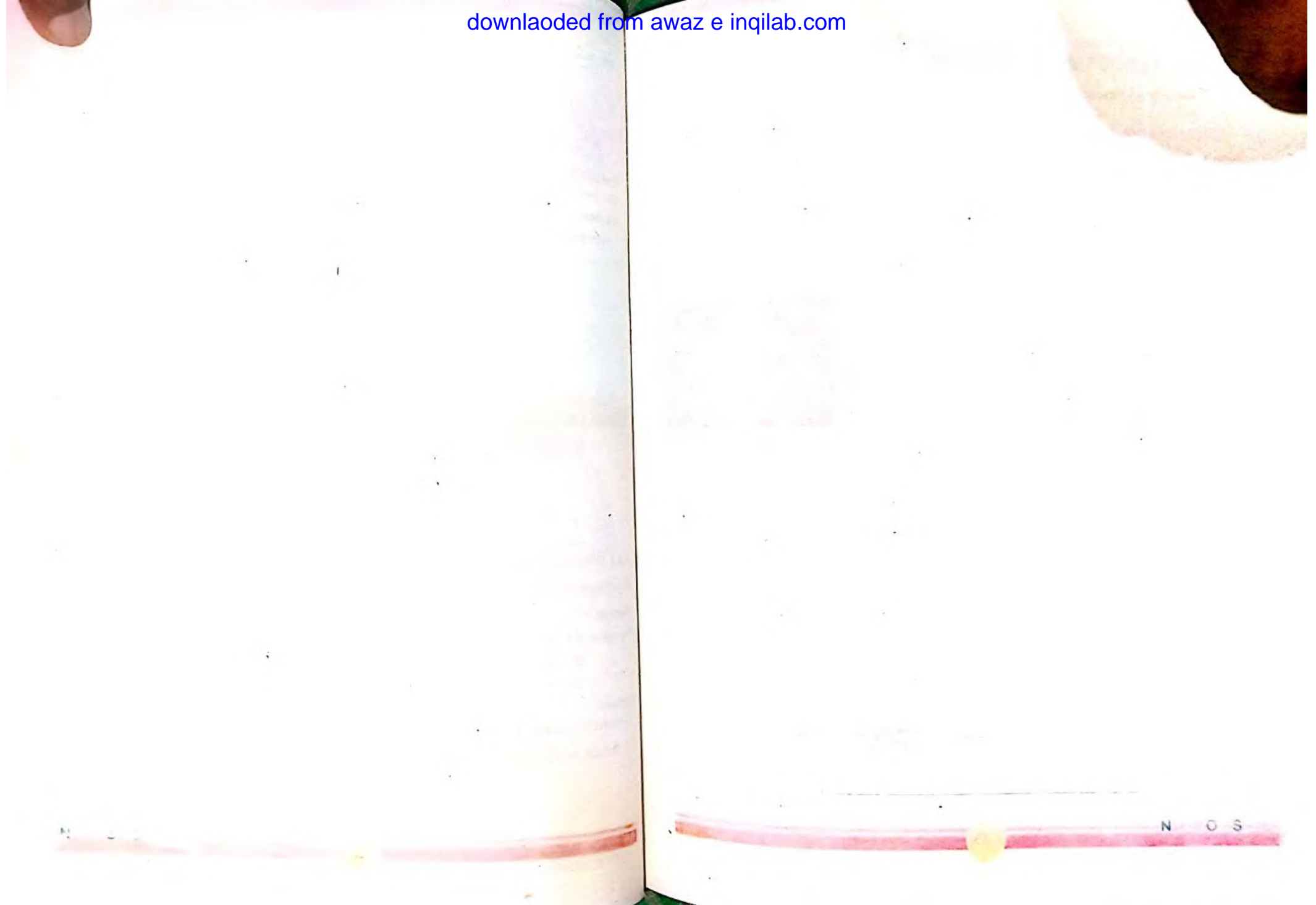
Solution:

The radius of the water tank (r) = 2.5m

i. Surface area of the water tank = $4\pi r^2$

$$= 4 (3.14) (2.5)^2$$

$$= 12.56 \times 2.5 \times 2.5$$



Each lateral face is a triangle with height h and base length " b ".

So, the area of each such lateral triangular face is:

$$\begin{aligned}\text{Area of each lateral (triangular) face} &= \frac{1}{2} \times \text{base} \times \text{slant height} \\ &= \frac{1}{2} \times b \times h\end{aligned}$$

$$\begin{aligned}\text{Total surface area of lateral (triangular) faces} &= 4 \times \left(\frac{1}{2} \times b \times h\right) \\ &= 2 \times b \times h = 2bh\end{aligned}$$

$$\begin{aligned}\text{Base (square) area of pyramid} &= \text{length} \times \text{length} \\ &= b \times b \\ &= b^2\end{aligned}$$

$$\begin{aligned}\text{So, the total surface area of pyramid} &= \text{Lateral Surface Area} + \text{Base Area} \\ &= 2bh + b^2\end{aligned}$$

Example 1:

Students are making a square pyramid shaped wooden model for their project. If the length of the square base is 50 cm and the slant height is 72 cm, find how much wooden sheet they used to cover all the sides.

Solution:

Here base length = $b = 50$ cm

Slant Height = $h = 72$ cm

$$\begin{aligned}\text{The total surface area of pyramid} &= \text{Lateral Surface Area} + \text{Base Area} \\ &= 2bh + b^2 \\ &= 2 \times 50 \text{ cm} \times 72 \text{ cm} + (50 \text{ cm})^2 \\ &= 9700 \text{ cm}^2\end{aligned}$$

So, they use 9700 cm² wooden sheet to cover all the sides.

Example 2:

Find the lateral surface area of a square pyramid if the length of the base is 8 metres and the slant height is 11 metres.

Solution:

Here;

$$b = 8 \text{ m}$$

$$h = 11 \text{ m}$$

$$\begin{aligned}\text{Total lateral surface area} &= 2bh \\ &= 2 \times 8 \text{ m} \times 11 \text{ m} \\ &= 176 \text{ m}^2\end{aligned}$$



Example 3:

Find the slant height of a square pyramid if its base length is 10 metres and total surface area is 1000 metres squares.

Solution:

Here;

$$h = ?$$

$$b = 10 \text{ m}$$

$$\text{Total surface area} = 1000 \text{ m}^2$$

$$\text{Total lateral surface area} = 2bh + b^2$$

$$1000 \text{ m}^2 = 2 \times 10 \text{ m} \times h + (10 \text{ m})^2$$

$$1000 \text{ m}^2 = 20 \text{ m} \times h + 100 \text{ m}^2$$

Solving for h , we get:

$$\begin{aligned}1000 \text{ m}^2 - 100 \text{ m}^2 &= 20 \text{ m} \times h \\ h &= \frac{900 \text{ m}^2}{20 \text{ m}} \\ &= 45 \text{ m}\end{aligned}$$

So, the slant height of the pyramid is 45 m.

9.4.2 Volume of Square Pyramid

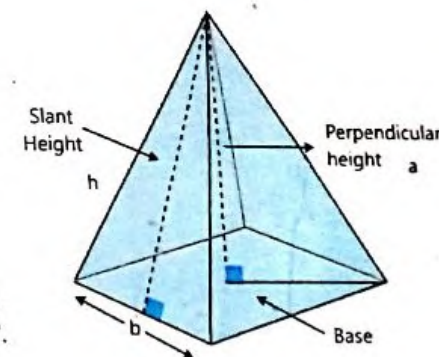
The volume is the space occupied by any object. The volume of a square pyramid means the space occupied by its square base and the four triangular lateral surfaces.

Let us consider a square pyramid whose base's length or length of one side of the square is ' b '. The slant height (lateral face height) ' l '. The altitude or height of the pyramid ' a '.

Each lateral face is a triangle with height h and base length " b ".

So, the area of this square pyramid can be calculated using the following formula:

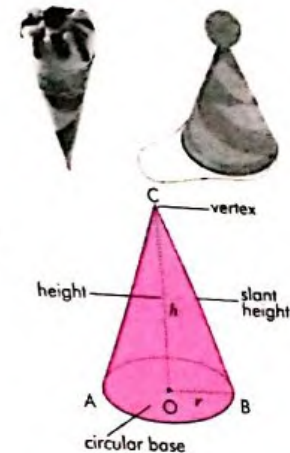
$$\begin{aligned}\text{Volume of a square pyramid} &= \frac{1}{3} \times \text{base area} \times \text{perpendicular height (altitude)} \\ &= \frac{1}{3} \times b^2 \times a = \frac{1}{3} \times b^2 a\end{aligned}$$



9.5 Cone

We are familiar with the term 'cone'. We have also observed cone shaped objects in our daily life. For example, ice-cream cone, Joker's cap, etc.

Cone is a solid region. A right circular cone of radius r with centre O is shown in the figure. The right circular cone has a flat circular shaped end which is called the base of the cone.



Note it down

The vertex of a cone is the farthest point from the base of the cone.

This cone has also a corner C which is called the vertex of the cone. The height of the cone is the distance between the base and the vertex of the cone. Here, \overline{OC} represents the height of the cone. The distance \overline{AC} is known as the slant height of the right circular cone.

We have noticed that in the given figure of cone, $\triangle COB$ is a right angled triangle. Because of this, the given cone is called a right circular cone. We can find the length of the slant height of the cone using the Pythagoras theorem.

Assume that slant height = l , then

$$(\overline{BC})^2 = (\overline{OB})^2 + (\overline{OC})^2$$

$$l^2 = r^2 + h^2$$

$$l = \sqrt{r^2 + h^2}$$

$$\text{Slant height} = \sqrt{r^2 + h^2}$$

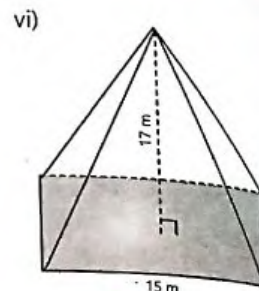
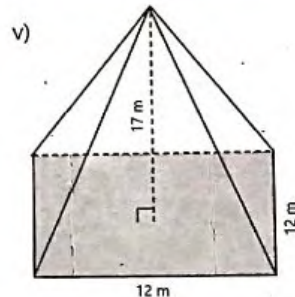
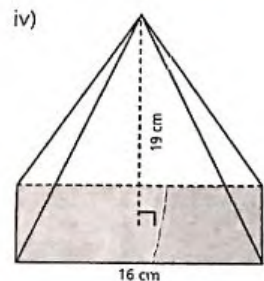
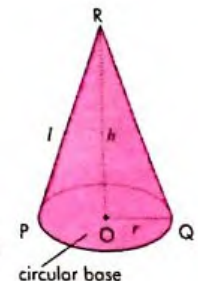
Surface Area of a Right Circular Cone:

A right circular cone with circular base of radius r is shown in the figure. h be the height and l be the slant height of the cone.

Total surface area of the right circular cone =

Area of the curved surface + Area of the base(i)

$$\begin{aligned} \text{Area of the curved surface} &= \frac{1}{2} (\text{perimeter of the base}) \times \text{slant height of the right circular cone} \\ &= \frac{1}{2} \times (2\pi r)l \\ &= \pi rl \text{ (ii)} \end{aligned}$$



4. An oil bottle is in the shape of a square pyramid. What is the capacity if its base length is 22 cm and the perpendicular height (altitude) is 30 cm.

5. A hall is in square pyramid shape. Its base length is 30 metre, the slant height is 42 metre and the altitude is 40 metres.

- a) What is the cost of painting its outer triangular walls if the rate of painting is Rs 975 per square metre?
b) What is the capacity of this building?

6. For comparing, a square pyramid shaped tent is prepared. How much fabric is used for making the four triangular surfaces if the base of this square pyramid is 4 metres and the slanting height is 6 metres. Also find its capacity if the altitude is 5 metres.



7. Find the surface area and volume of a pyramid shaped marble decoration piece if its base length is 20 cm slant height is 23 cm and the altitude is 23 cm. Also find the cost of polishing its outer surface if the rate of polishing is Rs 320 per square centimetre.



Solution:

Height of a conical flask (h) = 6cm

Now, first we find radius (r) and slant height (l) of the conical flask.

Area of the base of the conical flask = πr^2

$$50 \text{ cm}^2 = \pi r^2$$

$$\Rightarrow (3.14)r^2 = 50 \text{ cm}^2$$

$$r^2 = \frac{50}{3.14} \text{ cm}^2$$

$$r^2 = 16 \text{ cm}^2$$

$$r^2 = 16 \text{ cm}^2$$

$$r = 4 \text{ cm}$$

$$\text{Slant height of the conical flask } (l) = \sqrt{r^2 + h^2}$$

$$= \sqrt{(4)^2 + (6)^2}$$

$$= \sqrt{16 + 36}$$

$$= \sqrt{52}$$

$$l = 7.21 \text{ cm}$$

$$\text{i. Total surface area of conical flask} = \pi r (l + r)$$

$$= 3.14 (4) (7.21 + 4)$$

$$= 3.14 (4) (11.21)$$

$$= 140.79 \text{ cm}^2$$

$$\text{ii. Volume of the conical flask} = \frac{1}{3} \pi r^2 h$$

$$= \frac{1}{3} (3.14) (4)^2 (6)$$

$$= (3.14) (4) (4) (2)$$

$$= 100.48 \text{ cm}^3$$

Exercise 9.5

1 Find the curved surface area and total surface area of cones for the following measurements: ($\pi = 3.14$)

- i) $r = 5 \text{ cm}, l = 8 \text{ cm}$ ii) $r = 8.2 \text{ cm}, l = 20 \text{ cm}$
 ii) $r = 3 \text{ m}, h = 4 \text{ m}$ iv) $r = 6 \text{ cm}, h = 8 \text{ cm}$

2 Calculate the total surface area and volume for the following measurements: ($\pi = 3.14$)

- i) $r = 2.5 \text{ cm}, h = 12 \text{ cm}$ ii) $r = 3.2 \text{ cm}, h = 8.5 \text{ cm}$
 ii) $r = 1.25 \text{ m}, h = 3 \text{ m}$ iv) $r = 5 \text{ cm}, l = 13 \text{ cm}$

- 3** The radius of the conical roof of a the minar of a mosque is 1.25m and its slant height is 5m. Find the cost of the metal sheet to cover the roof at a rate of Rs 250 per m^2 .
- 4** The slant height of a conical funnel is 11 cm. How much metal plate is required to cover a conical funnel, if the diameter of its base is 8.2 cm. ($\pi = \frac{22}{7}$)
- 5** The radius of a cone is 6 cm and its slant height is 10 cm.
 i) How much paper is required to cover the cone?
 ii) Find the cost of the cone at a rate of Rs. 0.09 per cm^2 .
- 6** A conical vessel has a radius 5 cm and its slant height is 12 cm.
 i) Find its curved surface area.
 ii) Calculate its total surface area.
 iii) Find the capacity of the conical vessel.

Summary

- By applying Pythagoras theorem we can find the measurement of an unknown side when the lengths of the other two sides are known.
- Always draw rough figure before solving the given problem.
- When the central angle formed by the two radii is 90° , the sector is called a quadrant.
- Central Angle of an angle formed by two radii having the center of circle as its vertex.
- The Greek letter θ (theta) is used in math as a variable to represent a measured angle.
- The faces of solid other than the base is called the lateral faces of the solid. They meet at a common vertex.
- The vertex of a cone is the farthest point from the base of the cone.

Vocabulary

- Pythagoras theorem
- Arc length
- Sector
- Pyramid
- Sphere
- Hemisphere
- Cone

Unit 10 Geometry

Student Learning Outcomes

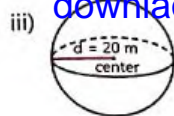
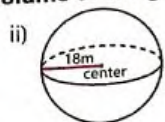
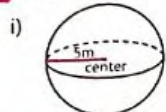
After completing this unit, students will be able to:

- Rotate an object and find the centre of rotation by construction.
- Enlarge a figure (with the given scale factor) and find the centre and scale factor of enlargement.
- Describe chord, arcs, major and minor arc, semi-circle, segment of a circle, sector, central angle, secant, tangent and concentric circles.
- Construct a triangle when: -three sides (SSS) -two sides and included angle (SAS) -two angles and included side - a right-angled triangle when hypotenuse and one side (HS) are given
- Construct different types of quadrilaterals (square, rectangle, parallelogram, trapezium, rhombus and kite).
- Draw angle and line bisectors to divide angles and sides of triangles and quadrilaterals
- Identify congruent and similar figures (in your surroundings), apply properties of two figures to be congruent or similar and apply postulates for congruence between triangles.

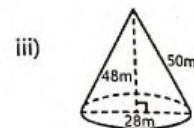
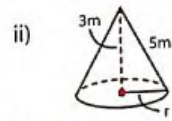
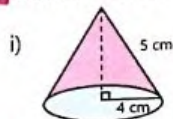


Can you identify the center of rotation in a Ferris wheel?

Find the surface area and volume of the given spheres.



Find the surface area and volume of cones.



Find the surface area and volume of hemispheres:

i) Radius 6 cm

ii) diameter 24 m

Spherical water tank whose radius is 3.5 m.

i) Find the cost of painting water tank at the rate of Rs 135 per m^2 .

ii) Find the capacity of water tank. ($\pi = \frac{22}{7}$)

The height of a conical flask is 10 cm and its area of the circular base 78.50 cm^2 .

i) Find the curved surface area of the conical flask.

ii) Calculate its total surface area.

iii) Find the capacity of the conical flask.

Jamal wants to make a tent. For this he wants to bought a high-quality fabric. Find the surface area of the tent including the floor of the tent below. Find the cost of the cloth if per square metre cloth is in Rs 450. Also find the volume of the tent.



Math Project

Material Required:

- Cardboard
- Scissors
- Pins
- Markers

Procedure:

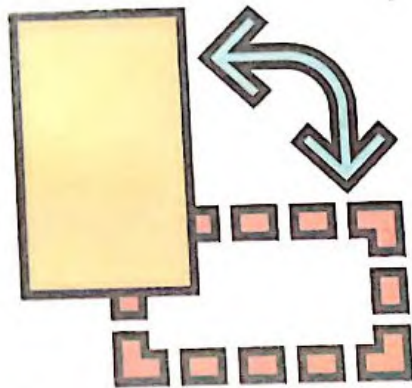
- Work in pairs.
- Use card board to cut circular pieces and two arcs which will act as radius.
- Take print out of a protractor or draw it (not necessarily to the scale) on the circular piece. It will be showing the central angle as the arms of the angle (radii) move.
- Mention the radius for each circle.
- One member from each pair will show an angle to the other and the other member will calculate the sector area or arc length accordingly

Introduction

In this we will learn about rotation of an object and find center and scale factor of enlargement of a figure. We will also learn to construct different types of quadrilateral and identify congruent and similar figures.

10.1 Rotation of objects

Rotation is a type of a transformation. Rotation means turning a shape around a fixed point (centre of rotation). It can be clockwise or anticlockwise. For rotation, we have to specify the angle of rotation and rotation point. When no direction is specified, it can be assumed that a rotation is counterclockwise (unless otherwise mentioned.)



Previous Knowledge Check

Draw a pentagon on a coordinate plane. Rotate it through 90° counterclockwise about the origin.

10.1.1 Rotating an object about Origin

Step I: Identify the coordinates of the vertices of the given shape.

Step II: Figure out the coordinate of the points in which it will be reflected.

Step III: Mark the coordinates which will be the vertices of the reflected object/image.

Join the vertices. The object/image thus formed will be the reflected object/image for the original object/image.

Note it down

- a) Rotating 90° counterclockwise or 270° clockwise about the origin:
 $P(x, y) \rightarrow P'(-y, x)$. i.e. Change the sign of y and switch x, y
- b) Rotating 180° clockwise or 180° counterclockwise about the origin:
 $P(x, y) \rightarrow P'(-x, -y)$. i.e. Just change the sign of x and y.
- c) Rotating 90° clockwise or 270° counterclockwise about the origin:

Example 1:

Rotate the given image ABCD through 90° counterclockwise about the origin.

Solution

To rotate the rectangle ABCD about the origin 90° counterclockwise we would follow the rule $(x, y) \rightarrow (-y, x)$, where the y-value of the original point becomes the new x-value with opposite sign and the x-value of the original point becomes the new y-value.

Let's apply the rule to the vertices to create the new rectangle A'B'C'D':

A (2, 6) becomes A' (-6, 2)

B (7, 6) becomes B' (-6, 7)

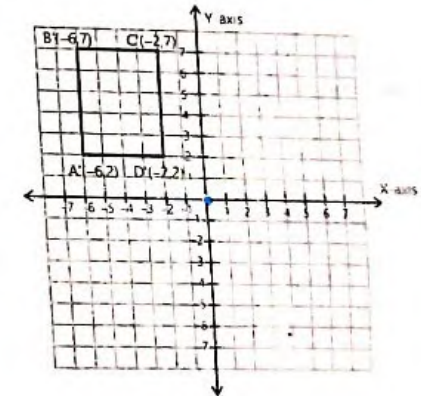
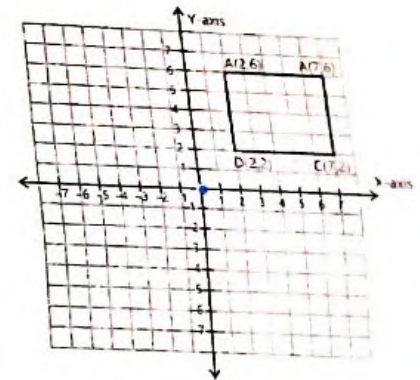
C (7, 2) becomes C' (-2, 7)

D (2, 2) becomes D' (-2, 2)

Mark the coordinates which will be the vertices of the reflected object/image.

Join the vertices.

The rectangle A'B'C'D' is the required rotated image of rectangle ABCD.



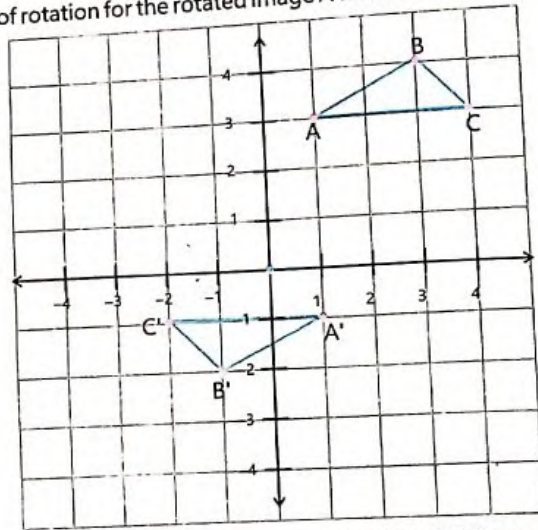
10.1.2 Centre of Rotation

The point around which a shape rotates around is called Centre of rotation. For example, the center of a wheel, a fan, Ferris wheel, spinner etc.

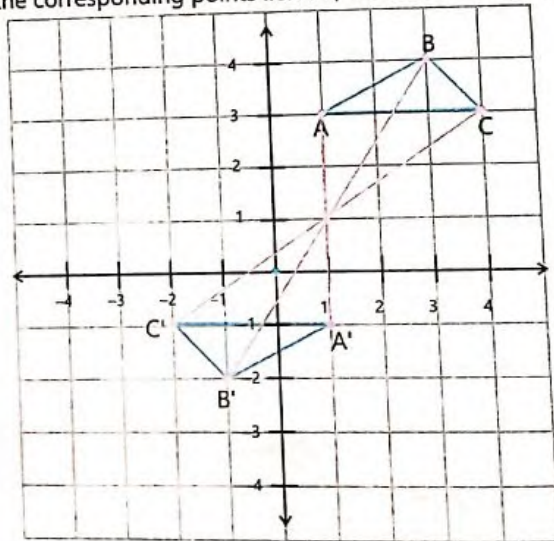


10.1.3 Finding the Center of Rotation

Let's find the center of rotation for the rotated image $A'B'C'$ of triangle ABC .

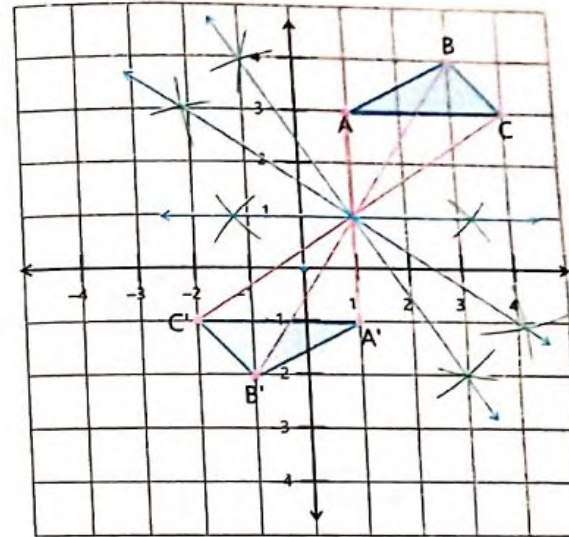


Draw a line joining the corresponding points i.e. AA' , BB' and CC' respectively.



TEACHING POINT Explain the centre of rotation of objects by drawing an object on the Cartesian coordinate plane then rotate the object and then finding the angle of rotation. Instruct students to work in pairs and draw shape of your own choice on Cartesian plane and then rotate and find the centre of rotation.

Construct the perpendicular bisector of these segments AA' , BB' and CC' .

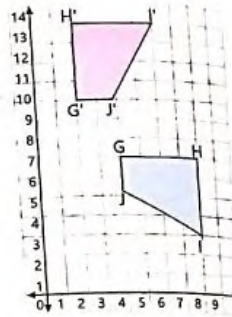
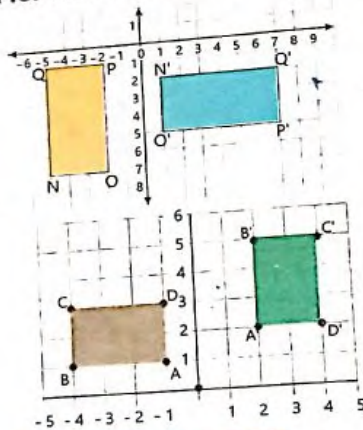
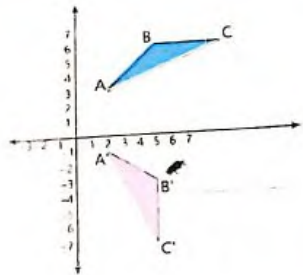


3. The point $X(1,1)$ where the three perpendicular bisectors meet is the center of rotation.

Exercise 10.1

- 1 Draw a Square on coordinate plane. Then rotate it through 90° clockwise about the origin.
- 2 Draw a triangle on the coordinate plane. Then rotate it through 180° clockwise about the origin.
- 3 What will be the values of x and y for P' if an image having point $P(2, -9)$ is rotated:
 - a) 90° clockwise
 - b) 180° counterclockwise
 - c) 270° clockwise
- 4 What will be the values of x and y for P' if an image having point $P(-5, -3)$ is rotated:
 - a) 90° counterclockwise
 - b) 180° clockwise
 - c) 270° counter clockwise

4 Find the center of rotation for the following figures.



10.2 Enlargement

We know that a transformation is a way of changing the size or position of a shape or object. Enlargement is a type of transformation where the shape of the objects remains the same but the size increases.

If a shape is enlarged, it is similar to the original shape. Similar shapes are the same shape but not the same size.

For enlargement of a figure, we need the center of enlargement and the scale factor for the enlargement. The scale factor describes the size of an enlargement. For example, a scale factor of 3 means that the new shape is three times the size of the original one.

10.2.1 Enlargement of a Shape

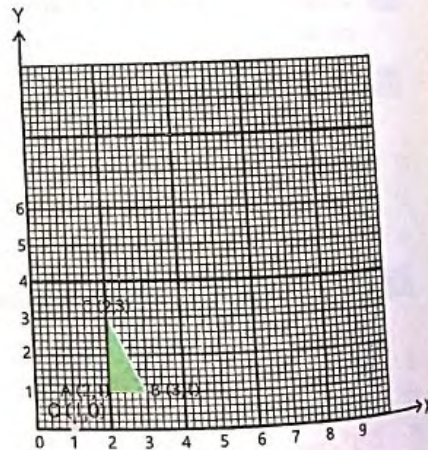
Let's learn how to enlarge a shape.

Example 1:

Enlarge the given triangle by a scale factor of 3 about the centre of enlargement (1, 0).

Solution:

Step 1: Identify and note the coordinates of the shape to be enlarged.



Here, A(2, 1), B(3,1) and C(2,3) are the coordinates of the vertices of the given triangle. The center of enlargement C is given i.e. C(1,0).

Step 2: Count and note the distance of each vertex from the center of enlargement.

First consider Vertex A. Let's find the coordinates for A' of the enlarged shape.

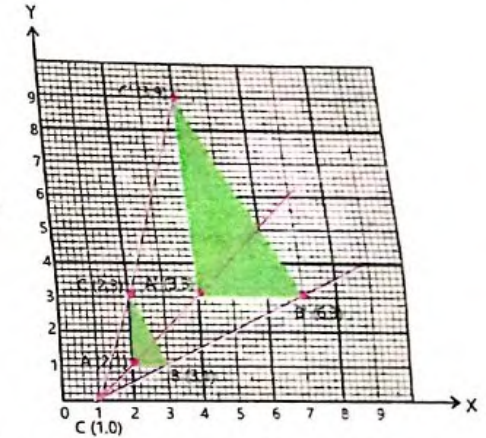
Distance of C from A is: (2-1, 1-0) or (1,1).

As scale factor is 3 here, multiply the coordinates of this distance by 3.

Distance of C from A' is: (1 x 3, 1 x 3) or (3, 3).

So A' will be at the distance of (3, 3) from C.

Count and mark point A'. Similarly find the coordinates for B' of the enlarged shape.



Distance of C from B is: (3-1, 1-0) or (2,1). As scale factor is 3 here, multiply the coordinates of this distance by 3.

Distance of C from B' is: (2 x 3, 1 x 3) or (6, 3).

So, B' will be at the distance of (6, 3) from C.

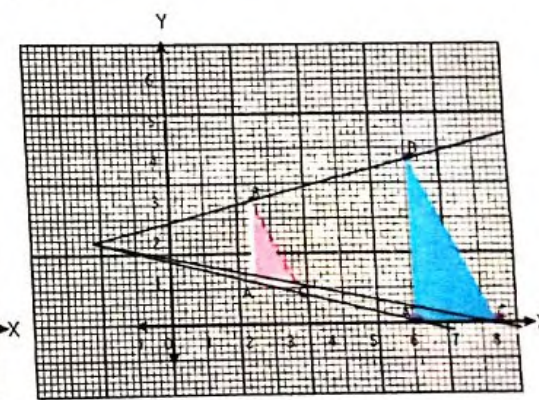
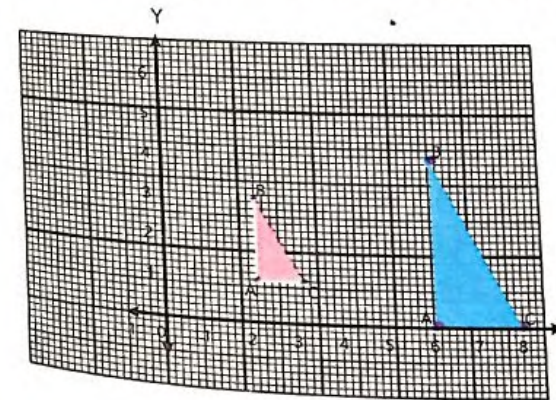
Count and mark point B'.

Similarly, C' will be at a distance of (3, 9) from C. Mark it and join the vertices.

A'B'C' is the required enlarged figure.

10.2.1 Finding the scale factor and center of enlargement

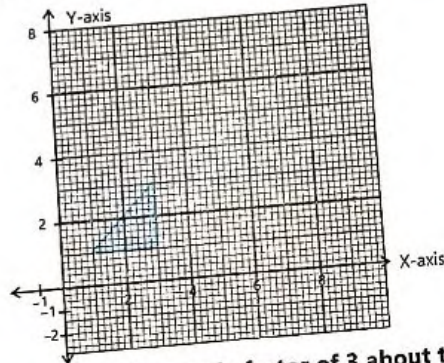
Look at the shape ABC and its enlarged shape A'B'C'.



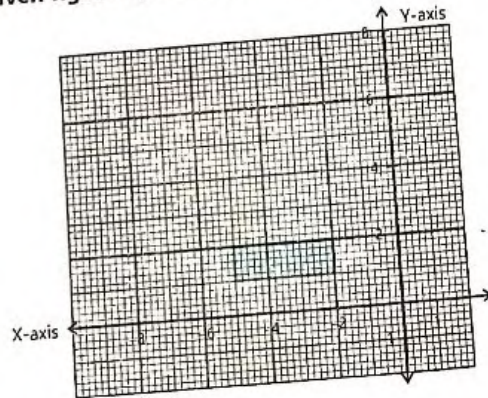
To find its centre of enlargement, join its vertices by the corresponding vertices of the original shape. The point where three vertices meet is the center of enlargement. So, Point C is the center of enlargement. Similarly, we can see that AC is one unit long and the corresponding side A'C' of enlarged figure is 2 units long. So, here the scale factor enlargement is 2.

Exercise 10.2

- 1 Enlarge the given figure by a scale factor of 2 about the centre of enlargement (1, -1).



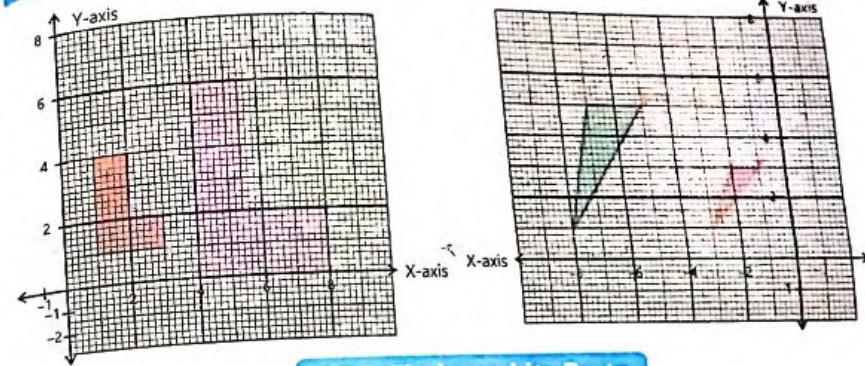
- 2 Enlarge the given figure by a scale factor of 3 about the centre of enlargement (1, 1).



TEACHING POINT Explain to students how we can enlarge the figure by using a scale and how we find the centre. Instruct them to draw a figure of your own choice and then enlarge that figure and find the centre and scale factor of the enlargement.

NOT FOR SALE

- 3 Finding the scale factor and center of enlargement of these shapes.

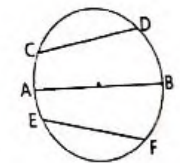
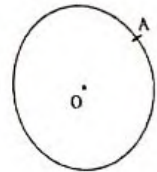


10.3 Circle and its Parts

All points that are equidistant from a fixed point form a circle. The fixed point is called the centre of the circle and the fixed distance is called the radius of the circle. O is the centre of the circle and the distance OA is the radius of the circle, where A is a point on the circumference of the circle.

Chord

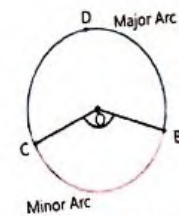
A line segment with end points on the circle is called chord. In fig. 10.6, AB, CD, and EF are chords.



Arc

A part or portion of the circumference of a circle is called an arc. Look at the circle. The arc AB has two end points A and B. It is denoted by \widehat{AB} .

An arc which is smaller than half of the circle is called a minor arc. An arc which is more than half of the circle is called a major arc. Look at the circle. \widehat{BC} is the minor arc and the arc \widehat{BDC} is the major arc as it is more than half of the circle.



Quick Check

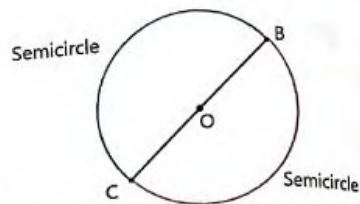
Draw a circle and label its minor and major arcs.

NOT FOR SALE

Semicircle

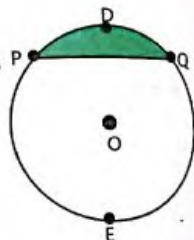
The diameter of the circle divides the circle into two equal parts. Both equal parts of the circle are called semicircles.

Look at the circle, the diameter \overline{BC} divides the circle into two equal parts. Each part is called a semicircle.



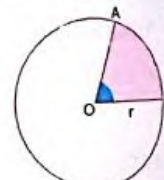
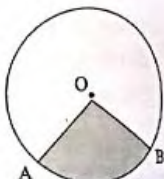
Segment of a Circle

A segment is a region of a circle which is separated by a chord. When a circle is cut into unequal parts by a chord, the smaller part of the circle is called the minor segment and the larger part of the circle is called the major segment. The symbol \frown is used to show a segment. Look at the circle in the figure. The shaded region enclosed by the chord PQ and the minor arc PDQ is the minor segment of the circle. The unshaded region enclosed by the chord PQ and the major arc PEQ is the major segment of the circle.



Sector of a Circle

If we join points A and B of the circle with centre O, the circle is divided into two regions. One is a shaded region and the other is an unshaded region. Both regions are called sectors. The smaller region is called the minor sector and the larger region is called the major sector.



An angle formed by two radii having the center of circle as its vertex is called central angle. Here $\angle AOB$ is the central angle.

Interior of a Circle

The points lying inside the circumference of a circle form the interior of the circle. In the figure, the interior of the circle is shaded.



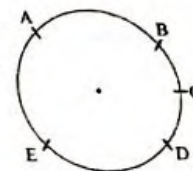
Exterior of a Circle

The points which lie outside the circle form the exterior of the circle. In the figure the exterior of the circle is shaded.



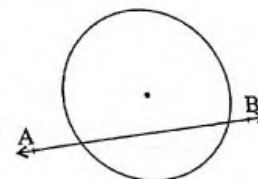
Concyclic Points

Points lying on the circumference of the same circle are called concyclic points. In the figure, A, B, C, D, E are all concyclic points.



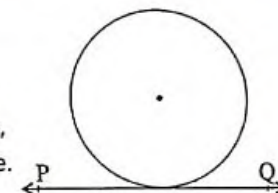
Secant to a Circle

If a line intersects a circle at two distinct points, then this line is called secant to the circle. In the figure, the line AB is secant to the circle.



Tangent to a Circle

If a line touches a circle at only one point, then this line is called tangent to the circle. In the figure, the line PQ is tangent to the circle.

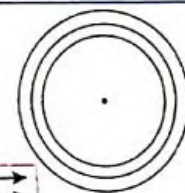


Note it down

- The points on the circumference of the circle are neither included in the interior nor in the exterior of the circle.
- If points A and B on the circumference of the circle divide it into two equal arcs then each arc is called a semi-circle.

Concentric Circles

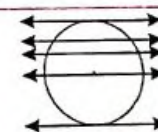
Two or more circles with the same centre but different radii are called concentric circles. In the figure, all circles are concentric circles.



Quick Check

Observe the given figure and sort out the following.

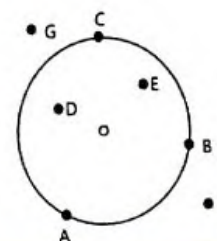
- Secants
- tangents



Exercise 10.3

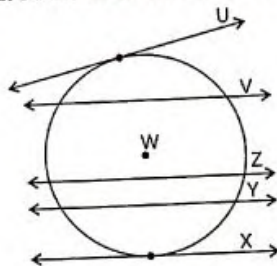
1 Sort out points from the given figure.

- interior points
- exterior points
- concyclic points



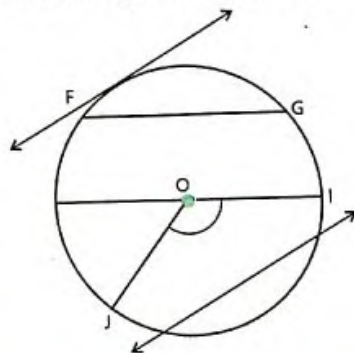
Draw a circle on the board with the help of board geometry and then label its parts and explain about each part of the circle. Then ask them to draw circle in their notebooks and label its parts and discuss with each other about its parts.

2 Observe the given figure and write the name of secants and tangents.



3 Match the labels with the correct part of the circle.

Minor Arc
Secant
Segment
Tangent



Central Angle
Sector
Chord
Major Arc

10.4 Triangles

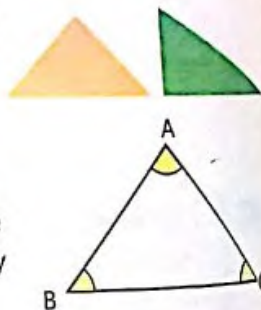
A geometrical shape that has 3 sides and three angles is called a triangle. The word triangle is made up of two words tri and angle. Tri means three so, it's a figure that has three angles. Look at the triangle ABC.

The line segments \overline{AB} , \overline{BC} and \overline{AC} are known as three sides of the triangle ABC. $\angle ABC$, $\angle ACB$, $\angle BAC$ are the three angles. The sum of the angles of a triangle is 180° . A triangle is denoted by the symbol Δ . So, this is a ΔABC .

Types of Triangles

Triangles are classified in two ways:

- With respect to their sides (equilateral triangle, isosceles triangle, scalene triangle).
- With respect to their angles (acute angled triangle, obtuse angled triangle, right angled triangle).



NOT FOR SALE

Construction of Triangles

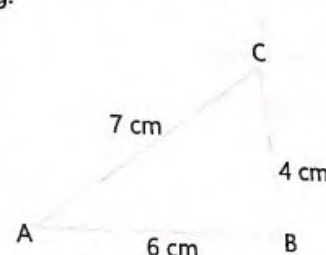
A triangle has six elements, its three sides and three angles. When we construct a triangle we do not need all six elements at a time. Before learning to construct triangles let's learn about an important property of triangles which is called triangle inequality property.

Triangle Inequality Property

Let's draw a triangle ABC, where $\overline{AB} = 6$ cm and $\overline{BC} = 4$ cm and $\overline{CA} = 7$ cm.

We can verify the following:

- $m\overline{AB} + m\overline{BC} > m\overline{CA}$
- $6 + 4 > 7$ or $10 > 7$
- $m\overline{BC} + m\overline{CA} > m\overline{AB}$
- $4 + 7 > 6$ or $11 > 6$
- $m\overline{CA} + m\overline{AB} > m\overline{BC}$
- $7 + 6 > 4$ or $13 > 4$



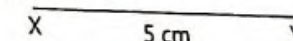
Is it possible to draw a triangle ABC in which $\overline{AB} = 4$ cm, $\overline{BC} = 2$ cm, $\overline{AC} = 1$ cm?

Hence, in any triangle, the sum of the length of any two sides is always greater than the length of the third side.

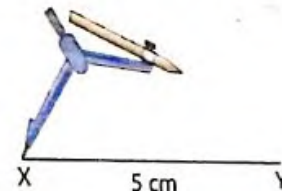
Construction of Triangles when three sides are given (SSS)

Construct triangle XYZ in which $\overline{XY} = 5$ cm, $\overline{YZ} = 3.5$ cm, $\overline{XZ} = 2$ cm.

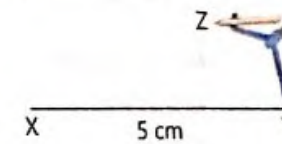
Step I: Draw a line segment $\overline{XY} = 5$ cm.



Step II: Place the pointer of the compass at point X and draw an arc of length 3.5 cm.

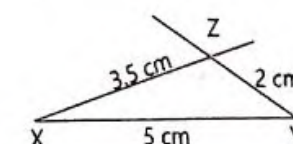


Step III: Place the pointer of the compass at point Y and draw an arc of radius 2 cm that cuts the previous arc at point Z.



Step IV: Join Z to X and Y.

So, ΔXYZ is the required triangle.



Make two groups of the students and distribute flash cards of different types of triangles and ask them to sort them according to their sides and angles.

NOT FOR SALE

Construction of Triangles when two sides and one angle is given (SAS)

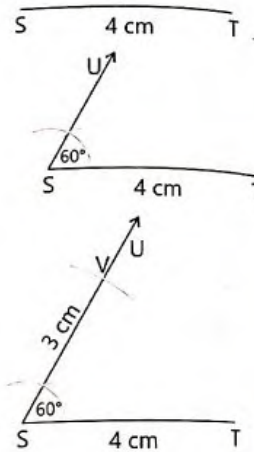
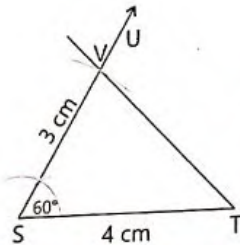
Construct triangle $\triangle STV$ in which $\overline{ST} = 4$ cm, $\overline{SV} = 3$ cm, $\angle S = 60^\circ$.

Step I: Draw a line segment \overline{ST} of 4 cm.

Step II: Draw an arc of angle 60° at point S.

Step III: Place the pointer of the compass at point S and draw an arc of length 3 cm which cuts the arm of the angle \overrightarrow{SU} at point V.

Step IV: Using ruler join V to T. So, $\triangle STV$ is the required triangle.



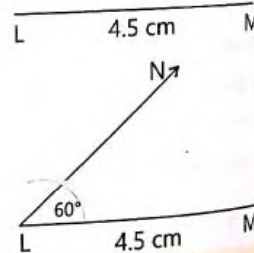
Quick Check
Draw a triangle ABC if $\overline{AB} = 5$ cm, $\overline{BC} = 4$ cm and $\angle B = 75^\circ$.

Construction of triangles when two angles and one side is given (ASA)

Construct triangle LMO in which $\overline{LM} = 4.5$ cm, $\angle L = 60^\circ$ and $\angle M = 75^\circ$.

Step I: Draw a line segment \overline{LM} of length 4.5 cm with the help of a ruler.

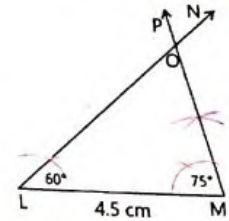
Step II: Draw an angle of 60° at point L.



TEACHING POINT Explain to the students how to construct a triangle with the help of pair of compasses when three sides are given. Write some examples on the board and ask them to construct these triangles in their notebook.

NOT FOR SALE

Step III: Place the pointer of the compass at point M and draw $\angle LMP = 75^\circ$.
 \overrightarrow{LN} and \overrightarrow{MP} cut to each other at point O. So, $\triangle LMO$ is the required triangle.



Construction of triangles when Hypotenuse and one side of a right angled triangle is given (HS)

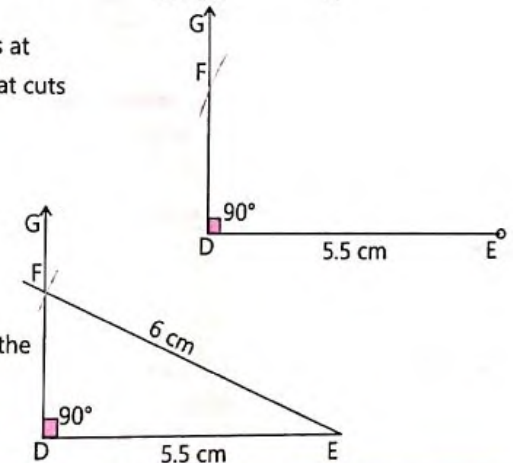
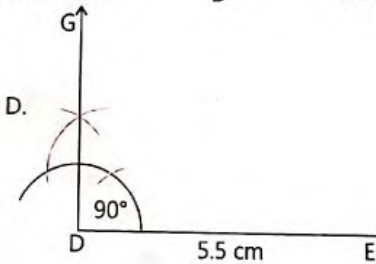
Construct triangle EDF in which $\overline{DE} = 5.5$ cm and \overline{EF} (hypotenuse) = 6 cm.

Step I: Draw a line segment \overline{DE} of length 5.5 cm.

Step II: Draw an angle of 90° at point D.

Step III: Place the pointer of the compass at point E and draw an arc of length 6 cm that cuts arm \overrightarrow{DG} at point F.

Step IV: Join the point E to F. So, $\triangle EDF$ is the required triangle.



Note it down

In a right angle triangle, the side opposite to the right angle is called the hypotenuse.

TEACHING POINT Explain to the students how to construct a right angled triangle with the help of a pair of compasses when one side and hypotenuse is given. Write some examples on the board and ask them to construct these triangles in their notebook.

NOT FOR SALE

Exercise 10.4

1 Draw the following triangles using a compass and ruler. (if possible)

- a) $\overline{AB} = 4$ cm, $\overline{AC} = 9$ cm, $\overline{BC} = 6$ cm
 b) $\overline{JK} = 2$ cm, $\overline{KL} = 8$ cm, $\overline{JL} = 5$ cm
 c) $\overline{XY} = 3$ cm, $\overline{YZ} = 6$ cm, $\overline{XZ} = 7$ cm
 d) $\overline{LN} = 7$ cm, $\overline{LM} = 7$ cm, $\overline{MN} = 8$ cm
 e) $\overline{DE} = 6$ cm, $\overline{DF} = 8$ cm, $\overline{EF} = 5$ cm
 f) $\overline{PQ} = 9$ cm, $\overline{QR} = 4$ cm, $\overline{PR} = 4$ cm

2 Draw the following triangles using a compass and ruler.

- a) $\overline{GH} = 7$ cm, $\overline{HI} = 5$ cm, $\angle G = 45^\circ$
 b) $\overline{XY} = 4$ cm, $\overline{YZ} = 6$ cm, $\angle Y = 30^\circ$
 c) $\angle M = 30^\circ$, $\overline{LM} = 8$ cm, $\overline{MN} = 8$ cm
 d) $\overline{DE} = 9$ cm, $\angle E = 105^\circ$, $\overline{EF} = 7$ cm
 e) $\angle P = 15^\circ$, $\overline{QR} = 4$ cm, $\overline{PR} = 3$ cm

3 Draw the following triangles using a compass and ruler.

- a) $\overline{AB} = 3$ cm, $\angle A = 45^\circ$, $\angle B = 30^\circ$
 b) $\angle X = 45^\circ$, $\angle Y = 75^\circ$, $\overline{XZ} = 7$ cm
 c) $\angle N = 70^\circ$, $\angle L = 60^\circ$, $\angle N = 105^\circ$
 d) $\angle E = 15^\circ$, $\angle F = 105^\circ$, $\overline{EF} = 5$ cm
 e) $\angle R = 15^\circ$, $\overline{QR} = 4$ cm, $\angle Q = 30^\circ$

4 Draw the following right angle triangles using a compass and ruler.

- a) $\overline{AB} = 4$ cm, \overline{AC} (Hypotenuse) = 5 cm
 b) $\overline{XY} = 2$ cm, \overline{YZ} (Hypotenuse) = 4 cm
 c) $\overline{LN} = 7$ cm, \overline{LM} (Hypotenuse) = 9 cm
 d) $\overline{DE} = 4$ cm, \overline{DF} (Hypotenuse) = 6 cm
 e) $\overline{PQ} = 5$ cm, \overline{QR} (Hypotenuse) = 7 cm

10.5 Construction of Square and Rectangle

10.5.1 Construction of Square

In the previous class, you learnt to draw a square when the length of its side was given. Now you will learn the following cases of construction of a square.

Construction of Square When its Diagonal is Given

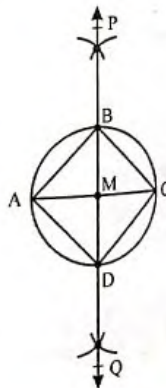
The length of the diagonal of a square is 3.5 cm. Construct the square.

Solution:

Steps of Construction:

- Draw $\overline{AC} = 3.5$ cm.
- Draw \overline{PQ} the right bisector of \overline{AC} which cuts \overline{AC} at point M.
- With M as centre, draw a circle with radius \overline{MA} which cuts \overline{PQ} at points B and D.
- Join A with B, B with C, C with D and D with A.

Here, ABCD is the required square.



Construction of square when difference of diagonal and side is given

The method is explained in the following example.

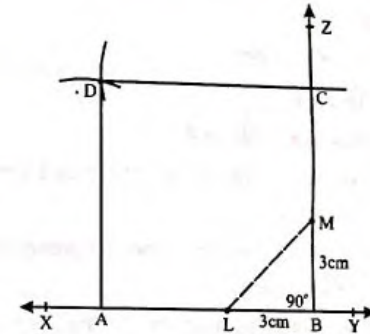
Example:

Construct a square ABCD when the difference of its diagonal and side is 3 cm.

Solution:

Steps of Construction:

- Draw a line \overline{XY} .
- Chose a point B on \overline{XY} and draw $\angle XBZ = 90^\circ$.
- On \overline{XY} , mark a point L such that $\overline{LB} = 3$ cm.
- On \overline{BZ} , mark a point M such that $\overline{BM} = 3$ cm where 3 cm is the difference between the diagonal and the side of the required square.
- Join \overline{LM} and mark a point A on \overline{XY} , such that $\overline{LA} = \overline{LM}$.



- Taking \overline{AB} as one side, complete the square ABCD.

Here, ABCD is the required square in which $\overline{AC} - \overline{AB} = 3$ cm.

Construction of Square when Sum of Diagonal and Side is Given

The method is explained as follows,

Example:

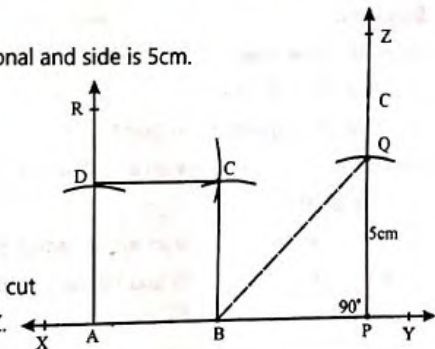
Construct a square ABCD when the sum of its diagonal and side is 5 cm.

Solution:

Steps of construction:

- Draw a line \overline{XY} .
- Mark a point P on \overline{XY} .
- Draw $\angle XPZ = 90^\circ$.
- As the sum of the diagonal and a side is 5 cm, so cut off $\overline{PB} = 5$ cm on \overline{PX} and cut off $\overline{PQ} = 5$ cm on \overline{PZ} .
- Mark a point A on \overline{XY} , such that $\overline{PA} = \overline{BQ}$.
- On side \overline{AB} , construct a square ABCD

Here, ABCD is the required square in which $\overline{AC} + \overline{AB} = 5$ cm.



10.5.2 Construction of Rectangle

In this section, you will learn to construct a rectangle in different given situations

Case-I

Construction of a Rectangle when its Two Sides are Given:

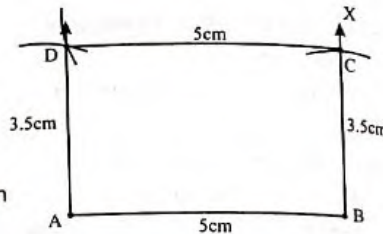
Example 1:

Construct a rectangle ABCD, when $\overline{AB} = 5\text{cm}$ and $\overline{BC} = 3.5\text{cm}$.

Solution:

Steps of construction:

- Draw $\overline{AB} = 5\text{cm}$.
- Draw an angle of 90° at B.
- With B as centre, draw an arc of radius 3.5cm which cuts BX at point C.
- Using the same opening of the compasses draw an arc with A as centre.
- With C as centre, draw an arc of radius 5cm which cuts the previous arc at point D.
- Join A with D and C with D. So, ABCD is the required rectangle.



Case-II

Construction of a Rectangle When the Diagonal and a Side are Given:

Example 2:

Construct a rectangle PQRS, when $\overline{PQ} = 5.5\text{cm}$ and the length of its diagonal is 7cm .

Solution:

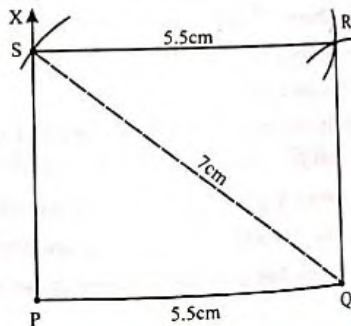
Steps of Construction:

- Draw $\overline{PQ} = 5.5\text{cm}$.
- Draw an angle of 90° at point P.
- With Q as centre, draw an arc of radius 7cm which cuts PX at S.
- With S as centre, draw an arc of radius 5.5cm .
- Measure the length \overline{PS} and taking point Q as centre, draw an arc of radius equal to \overline{PS} . Mark the point of intersection of these two arcs as R.
- Join R with S and join R with Q. Here PQRS is the required rectangle.

Quick Check

Construct the rectangle PQRS for the following measurements:

- $\overline{PQ} = 4.8\text{cm}$, $\overline{QR} = 3.2\text{cm}$
- $\overline{PQ} = 6.2\text{cm}$, $\overline{QR} = 4.5\text{cm}$
- $\overline{PQ} = 6.5\text{cm}$, $\overline{PS} = 4.4\text{cm}$



10.5.3 Construction of Rhombus

A parallelogram in which all four sides are equal in measurement and none of the angles is 90° is called a **rhombus**. We shall learn the following cases of construction of rhombus:

i) One side and a base angle are given.

ii) One side and a diagonal are given.

Case-I

When One Side and a Base Angle are Given.

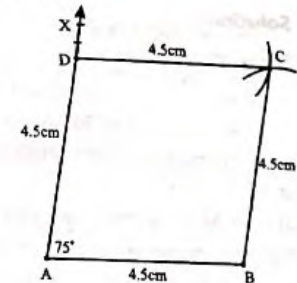
Example 1:

Construct a rhombus whose one side is 4.5cm and a base angle is 75° .

Solution:

- Draw $\overline{AB} = 4.5\text{cm}$.
- Draw $\angle BAX = 75^\circ$.
- With A as centre, draw an arc of radius 4.5cm which cuts AX at point D.
- With the same opening of compasses, with centre B and D, draw arcs which cut each other at point C.
- Join C with D and C with B.

ABCD is the required rhombus.



Case II

When One Side and Diagonal are Given:

Example 2:

Construct a rhombus JKLM in which $\overline{JK} = 5\text{cm}$ and $\overline{JL} = 6.4\text{cm}$.

Solution:

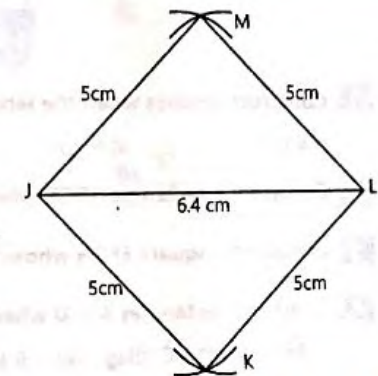
Steps of Construction:

- Draw $\overline{JL} = 6.4\text{cm}$.
- With J as centre, draw arcs of radius 5cm above and below \overline{JL} .
- With L as centre and with the same opening as in (ii), draw arcs above and below \overline{JL} .
- Mark the point of intersection of the two pairs of arcs as K and M.
- Join J with K, K with L, L with M and M with J. Hence, the rhombus JKLM is complete.

Quick Check

Construct a rhombus for the following measurements:

- PQRS in which $\overline{PQ} = 4.6\text{cm}$ and $\angle Q = 45^\circ$
- EFGH in which $\overline{EF} = 5.2\text{cm}$ and $\angle E = 60^\circ$
- ABCD when $\overline{AB} = 5\text{cm}$ and diagonal $\overline{AC} = 6.8\text{cm}$



10.5.4 Construction of a Parallelogram

In the previous class, we learnt to construct a parallelogram when its two adjacent sides and their included angle were given. Now we will learn to construct a parallelogram when the lengths of its two diagonals and their included angle are known.

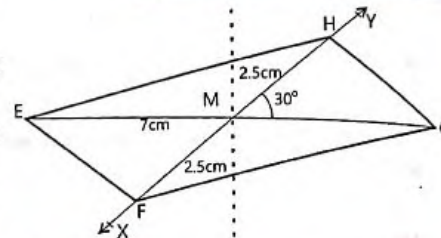
Example 1:

Construct a parallelogram EFGH when the lengths of its diagonals are 7cm and 5cm and the included angle of the diagonals is 30° .

Solution:

Steps of Construction

- Draw $\overline{EG} = 7\text{cm}$ and bisect it at point M.
- Draw an angle of 30° at point M and produce the arm of this angle on both sides of M to form XY.
- With M as centre, draw arcs of radius 2.5cm (half of 5cm) which cut XY at points F and H.
- Join E with F, F with G, G with H and E with H. Parallelogram EFGH is complete now.



Note it down

A parallelogram is a quadrilateral in which opposite sides are parallel.

Note it down

- Diagonals of a parallelogram are unequal.
- Diagonals of a parallelogram bisect each other.
- Opposite sides of a parallelogram are equal.
- Opposite angles of a parallelogram are equal.

Quick Check

Construct a parallelogram when:

- Lengths of its diagonals are 6cm and 4.8cm and included angle of the diagonals is 45° .
- Lengths of its diagonals are 6.8cm and 5.2cm and included angle of the diagonals is 150° .

Exercise 10.5

1 Construct squares when the lengths of their diagonals are given below:

- 4.8cm
- 5.2cm
- 5.6cm

2 Construct a rectangle ABCD when $\overline{AB} = 4.4\text{cm}$ and $\overline{BC} = 3.2\text{cm}$.

3 Construct a square EFGH whose diagonal $\overline{EH} = 6\text{cm}$.

4 Construct rectangles ABCD when

- $\overline{AB} = 5.6\text{cm}$, \overline{AC} (diagonal) = 6.4cm
- $\overline{AB} = 5.2\text{cm}$, \overline{AC} (diagonal) = 7.2cm

NOT FOR SALE

5 Construct rhombus ABCD when the measurement of its side and included angle are given.

- $\overline{AB} = 5\text{cm}$, $\angle A = 60^\circ$
- $\overline{AB} = 5.8\text{cm}$, $\angle B = 135^\circ$

6 Construct rhombus ABCD when measure of its side and a diagonal are given:

- $\overline{AB} = 5.4\text{cm}$, \overline{AC} (diagonal) = 5cm.
- $\overline{AB} = 6\text{cm}$, \overline{BD} (diagonal) = 7.2cm

7 Construct parallelogram EFGH when \overline{EG} (diagonal) = 4.8cm, \overline{FH} (diagonal) = 6cm and the angle between them is 30° .

8 Construct a square ABCD when the difference of its diagonal and a side is 4cm.

9 Construct a square PQRS when the sum of its diagonal and a side is 6cm.

10.6 Construction of a Trapezium

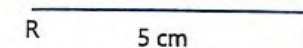
Draw trapezium RSTU such that

$\overline{RS} = 5\text{cm}$, $\overline{UT} = 3\text{cm}$, $\angle \text{URS} = 75^\circ$

$\overline{RU} = 1.5\text{cm}$ and $\overline{UT} \parallel \overline{RS}$.

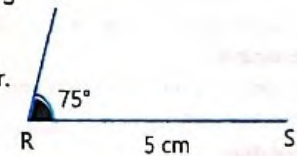
Step 1:

Draw line $\overline{RS} = 5\text{cm}$.



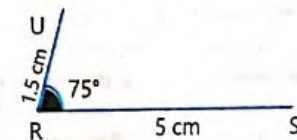
Step 2:

Draw an angle of 75° taking R as a vertex using a protractor.



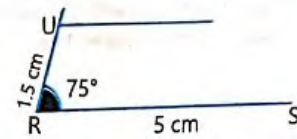
Step 3:

Mark point U such that $\overline{RU} = 1.5\text{cm}$.



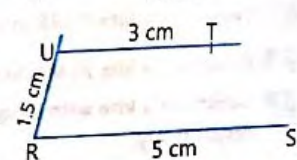
Step 4:

Use a set square and ruler to draw a line parallel to RS through U.



Step 5:

Make point T such that $\overline{UT} = 3\text{cm}$.

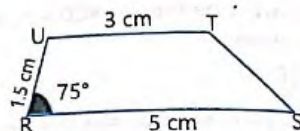


NOT FOR SALE

Step 6:

Join S to T.

RSTU is the required trapezium.



Exercise 10.6

- 1 Draw a trapezium WXYZ such that $WX \parallel YZ$, $ZY = 4.8$ cm, $WX = 7$ cm, $WZ = 2.6$ cm and $\angle XYZ = 112^\circ$
- 2 Draw a trapezium ABCD in which $AB \parallel DC$, $AB = 8.2$ cm, $BC = 6$ cm, $AD = 7.5$ cm and $\angle B = 75^\circ$
- 3 Draw a trapezium PQRS in which $PQ \parallel RS$, $PQ = 7.5$ cm, $QS = 5.9$ cm, $PR = 4.5$ cm and $\angle Q = 60^\circ$

10.7 Construction of a Kite

A quadrilateral with two pairs of equal adjacent sides with unequal diagonals is called a **kite**.

In the figure shown here, ABCD is a kite with $\overline{AB} = \overline{BC}$ and $\overline{AD} = \overline{CD}$. The diagonals are \overline{AC} and \overline{BD} .

Construction of kite when its Diagonal and Sides are Given:

Example 1:

Construct a kite ABCD when the length of its longer diagonal is 5.4cm and the length of its two sides are 3.6cm and 5cm.

Solution:

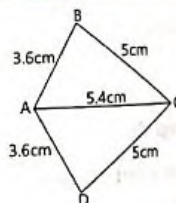
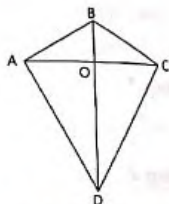
Steps of Construction.

- i) Draw $\overline{AC} = 5.4$ cm.
- ii) With A as centre, draw arcs of radius 3.6cm above and below \overline{AC} .
- iii) With C as centre, draw arcs of radius 5cm above and below \overline{AC} which intersect the previous arcs at points B and D.
- iv) Join A with B, B with C, C with D and D with A.

ABCD is the required kite.

Note it down

The longer diagonal of a kite bisects the shorter diagonal perpendicularly.

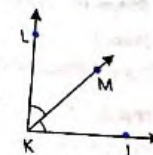


Exercise 10.7

- 1 Construct a kite PQRS in which $\overline{PQ} = 3$ cm, $\overline{QR} = 5$ cm and \overline{PR} (diagonal) = 7cm.
- 2 Construct a kite KLMN in which $\overline{KL} = 2.5$ cm, $\overline{LM} = 4.5$ cm and \overline{KM} (diagonal) = 5.5cm.
- 3 Construct a kite with length of diagonal 4 and length of sides are 5 cm and 6.2 cm respectively

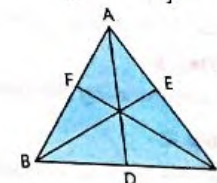
10.8 Angle and Line Bisectors

The angle bisector of an angle divides the angle into two equal angles. In the figure given, KM is the angle bisector of $\angle LKJ$ which is dividing $\angle LKJ$ into two equal angles $\angle LKM$ and $\angle MKJ$.



Angle and Line Bisectors

In a triangle, the angle bisector of an angle divides the angle into two equal angles.



Constructing Angle Bisectors of a Triangle

Let's construct the angle bisector for a triangle ABC.

Steps of Construction:

Step 1:

Draw a triangle ABC.

Step 2:

Take B as the center and draw an arc ED of any appropriate radius using compass.

Step 3:

Without changing the radius, mark two arcs by taking E and D as center. The two arcs meet at point F.

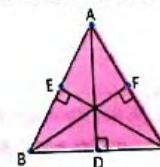
Step 4:

Join B to F and extend it. This ray BF is the required angle bisector of angle ABC. Similarly bisect the other two angles.

The bisectors meet at point O. This point O is called its **incenter**.

Perpendicular Bisector of Sides of a Triangle

The perpendicular bisector of a side of a triangle is a line perpendicular to the side and passing through its midpoint.



Drawing Line Bisectors of a Triangle

Let's construct the line bisectors of sides of a triangle.

Note it down

A line segment from the vertex to the opposite side such that it bisects the angle at the vertex is called as angle bisector.

Steps of Construction:

Step 1:

Draw a triangle ABC.

Step 2:

Take A as the center and draw two arcs on both sides of AB by taking radius a little more than half of segment AB.

Step 3:

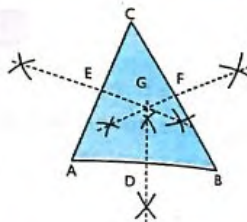
Similarly, using the same radius take B as center and draw two arcs on both sides of AC.

Step 4:

Join the point of intersection that cuts AB at D. This line is the perpendicular bisector of AB.

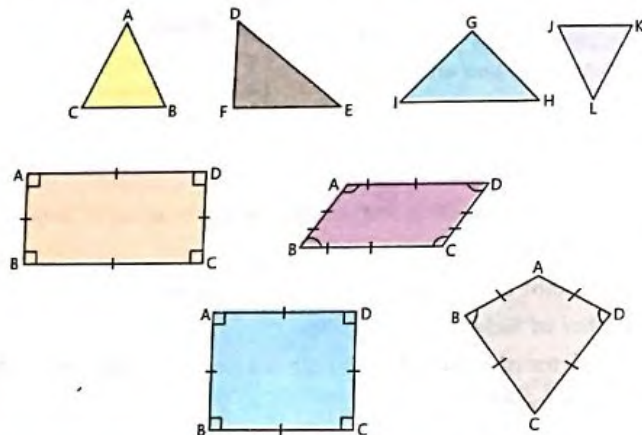
Similarly, draw line bisectors of side AC and BC. The point G where the three bisectors of sides of triangle meet is called the **circumcenter**.

In the same way we can construct angle or perpendicular line bisectors of any shape.



Exercise 10.8

Draw angle bisectors for these shapes. Also draw perpendicular side bisectors.



Ask the students to draw various types of quadrilaterals and draw angle and line bisectors for their angles and sides by following the steps given above.

10.9 Congruent and Similar Figures

Congruent Figures

We observe many objects in daily life that look exactly identical. For example, the blades of a fan, the two doors of the cupboard, two spoons from the cutlery set, etc. Look at the following objects.

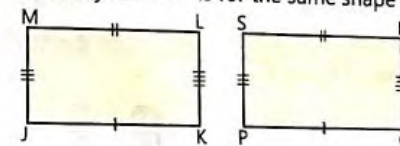
Note it down

Angles and sides of congruent figures are the same.



These objects have the same shape and size. The objects or figures that have the same shape and size are called congruent objects. The above pictures are showing congruent objects. Congruent comes from the Latin word "congruere", which means "to agree" or "correspond with". The symbol for congruence is " \cong ". The symbol " \sim " is for the same shape and " $=$ " for the same size.

The matching sides are called corresponding sides and the matching angles are called corresponding angles.



Now observe the following figures. They have the same shape and same size. Same size means the measurement of sides and angles are the same.

In the above figures, $JKLM \cong PQRS$. We read it as "JKLM is congruent to PQRS".

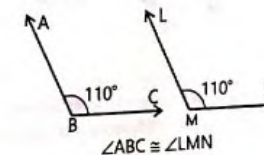
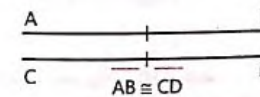
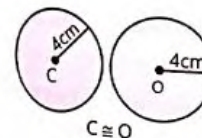
We can see that:

$$JK = PQ, KL = QR, LM = RS, JM = PS$$

So all the corresponding sides are equal. Hence the above figures are congruent figures.

Similarly:

- Two line segments of equal length are congruent.
- Two circles having equal radius or equal circumference are also congruent.
- Two angles are congruent if they have the same measurement.



NOT FOR SALE

NOT FOR SALE

Similar Figures

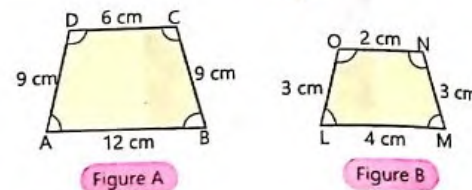
Look at the rugs. They look similar by their shape but their size is not the same. So we can say that the rugs are similar to each other but they are not congruent as they have a different size. The figures that have the same shape but not the same size are called similar figures. The symbol that is used for similar figures is '~'. It is read as "is similar to".

Properties of Similar Figures

Two figures are similar if:

1. Their corresponding angles are equal.
2. The lengths of their corresponding sides are in proportion (the ratios between the corresponding sides are equal).

Look at the following figures.



The corresponding angles are equal of the given figures:

$$\angle A = \angle L, \angle B = \angle M, \angle C = \angle N \text{ and } \angle D = \angle O$$

All sides of figure A are proportional to the corresponding sides of figure B.

$$\frac{AB}{LM} = \frac{BC}{MN} = \frac{CD}{NO} = \frac{DA}{OL} = 3$$

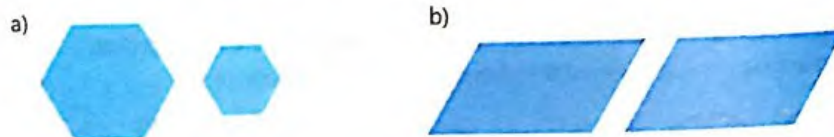
So, these figures are similar figures.

Note it down

All congruent figures are similar figures but all similar figures are not congruent. In similar figures, the ratios must compare to the correct corresponding lengths.

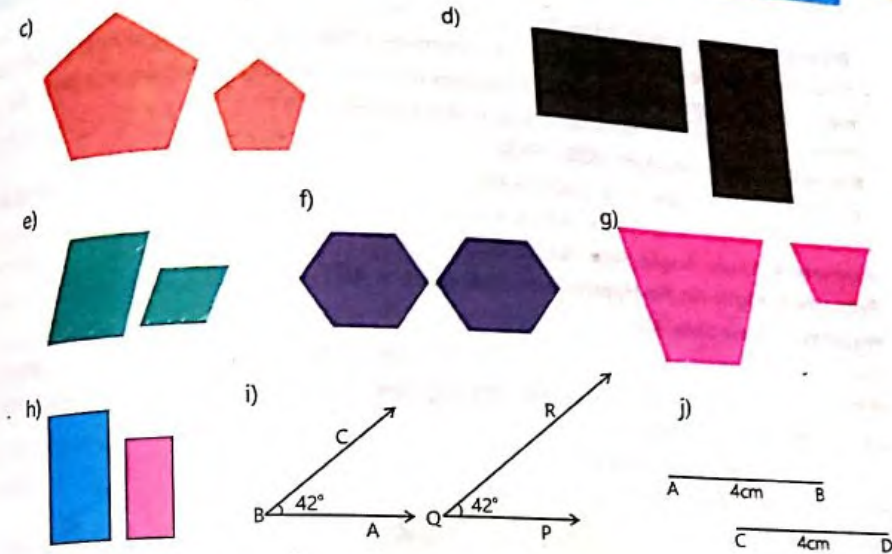
Exercise 10.9

- 1 Tick (✓) the pair of congruent figures and circle the pair of similar figures.

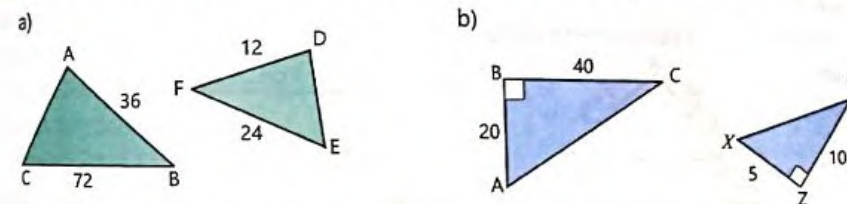


Explain to the students about congruent and similar figures. Show some flash cards and ask them to tell which figures are congruent and which are similar.

NOT FOR SALE



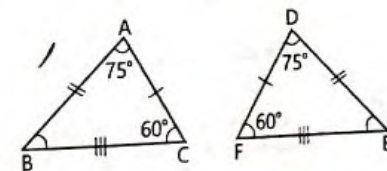
- 2 Apply properties of similar figures and check if the triangles are similar or not.



10.10 Congruent Triangles and their Properties

We know that two line segments are congruent when one of them is an exact copy of the other. Also two angles are congruent if one of them is a copy of the other. So the congruent figures exactly match with each other when placed over the other i.e. they cover each other exactly.

Look at the triangle ABC and DEF. These two triangles are congruent as they have the same shape and size. The same shape and size refers to the equality of all its 3 corresponding angles and all the corresponding sides.



NOT FOR SALE

Properties of Congruent Triangles

If two triangles are given and we need to determine if they are congruent or not, we can use the following properties of congruent triangles to check the congruence. So, it is not necessary to measure all three sides and all three angles.

Property 1: Side-Side-Side ($SSS \cong SSS$)

Property 2: Side-Angle-Side ($SAS \cong SAS$)

Property 3: Angle-Side-Angle ($ASA \cong ASA$)

Property 4: Angle-Angle-Side ($AAS \cong AAS$)

Property 5: Right-Angle-Hypotenuse-Side ($RHS \cong RHS$)

Property 1: Side-Side-Side ($SSS \cong SSS$)

This property states that "If the measure of three sides of a triangle are equal to the measure of the corresponding three sides of the other triangle, then the two triangles are congruent". Look at the $\triangle ABC$ and $\triangle WXY$.

The measure of all sides of the triangle ABC are equal to the corresponding sides of triangle WXY i.e.

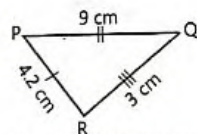
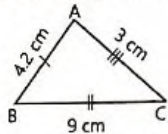
$$\overline{AB} = \overline{WX}, \overline{AC} = \overline{WY}, \overline{BC} = \overline{XY}$$

$$\therefore \triangle ABC \cong \triangle WXY.$$

Example 1:

Check whether $\triangle ABC$ is congruent to $\triangle RPQ$.

Solution:

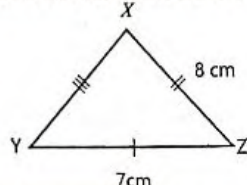
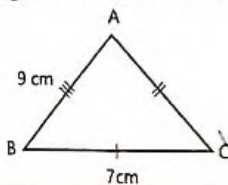


In the triangles ABC and RPQ, the three sides of the triangles ABC are equal to the three sides of triangle RPQ i.e.

$$\overline{AB} = \overline{RP} = 4.2 \text{ cm}, \overline{AC} = \overline{RQ} = 3 \text{ cm}, \overline{BC} = \overline{PQ} = 9 \text{ cm}. \text{ Hence } \triangle ABC \cong \triangle RPQ.$$

Example 2:

The given triangle ABC and XYZ are congruent. Find each unknown sides of the triangles.



Explain to the students about congruent triangles. Make sure they compare the correct corresponding sides and angles while looking for congruency in triangles.

NOT FOR SALE

Solution:

As the given triangles are congruent, so all three sides of triangle ABC are equal to corresponding sides of triangle XYZ.

$$\text{So, } \overline{AB} = \overline{XY} = 9 \text{ cm}$$

$$\text{Similarly } \overline{AC} = \overline{XZ} = 8 \text{ cm}$$

Example 3:

Are the triangles ABD and CBD congruent?

Solution:

Look at the triangles $\triangle ABD$ and $\triangle CBD$.

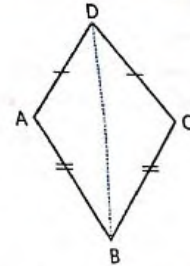
The three sides of the triangle ABD are equal to the three corresponding sides of the triangle CBD.

$$\overline{AB} = \overline{CB} \text{ (given)} \quad \overline{AD} = \overline{CD} \text{ (given)} \quad \text{and } \overline{BD} = \overline{BD} \text{ (common in both)}$$

As three sides are congruent so,
 $\triangle ABD \cong \triangle CBD$

Note it down

The lines |, || and ||| represent equal sides.



Property 2: Side-Angle-Side ($SAS \cong SAS$)

This property states that "If the measure of two sides and their included angle of a triangle are equal to the measure of the corresponding two sides and their included angle of the other triangle, then the two triangles are congruent."

Look at the given triangles:

The measure of two sides of triangle ABC are equal to the corresponding sides of triangle DEF.

$$\overline{AB} = \overline{DE}, \overline{BC} = \overline{EF}$$

Also, the included $\angle B$ of triangle ABC is equal to the corresponding included $\angle E$ of triangle DEF.

$$\angle B = \angle E$$

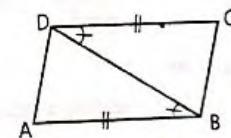
So, $\triangle ABC \cong \triangle DEF$.

Example 1:

Prove that $\triangle ABD$ and $\triangle CDB$ are congruent.

Solution:

The given triangles are congruent, if two sides and included angle of $\triangle ABD$ are equal to corresponding sides and included angle $\triangle CDB$.



NOT FOR SALE

$\overline{AB} = \overline{CD}$ (given) $\angle ABD = \angle CDB$ (given) and $\overline{DB} = \overline{DB}$ (common in both)
As two sides and one included angle are equal to the corresponding two sides and included angle so,
 $\triangle ABD \cong \triangle CDB$.

Property 3: Angle-Side-Angle (ASA \cong ASA)

This property states that "If two angles and their included side of a triangle are equal to the two corresponding angles and their included side of the other triangle, then the two triangles are congruent".

Look at triangle ABC and triangle PQR.

The two angles $\angle A$ and $\angle B$ and their included side \overline{AB} is equal to the corresponding angle $\angle P$ and $\angle Q$ and their included side \overline{PQ} of the other triangle PQR i.e. $\angle A = \angle P$, $\angle B = \angle Q$ and $\overline{AB} = \overline{PQ}$.

$\therefore \triangle ABC \cong \triangle PQR$

Example 1: Look at the pair of triangles and find out if these are congruent.

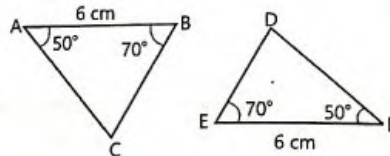
Solution:

In $\triangle ABC$ and $\triangle FED$

$\angle A = \angle F = 50^\circ$, $\overline{AB} = \overline{FE} = 6$ cm

$\angle B = \angle E = 70^\circ$.

So, according to ASA property, $\triangle ABC$ is congruent to $\triangle FED$.



Note it down

In congruent triangles if the triangles have congruent SSS and SAS then they also have fulfilled the property ASA.

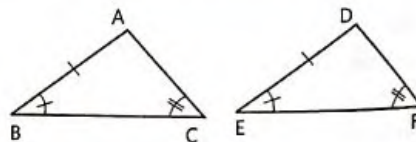
Look at triangle ABC and triangle DEF.

The two angles $\angle B$ and $\angle C$ and the non-included side \overline{AB} is equal to the corresponding angle $\angle E$ and $\angle F$ and one non-included side \overline{DE} of the other triangle DEF. i.e. $\angle B = \angle E$, $\angle C = \angle F$ and $\overline{AB} = \overline{DE}$.

$\therefore \triangle ABC \cong \triangle DEF$

Example 1:

Prove that the two given triangles are congruent.



Note it down

In two triangles if two angles are equal then the third angle is also equal as the sum of three angles of a triangle is 180° .

Solution:

From $\triangle ABC$ and $\triangle YXZ$ we can see that:

$\angle A = \angle Y$

$\angle C = \angle Z$

$\overline{AB} = \overline{YX}$

Therefore, both triangles are congruent by AAS property.

Property 5: Right-Angle-Hypotenuse-Side (RHS \cong RHS)

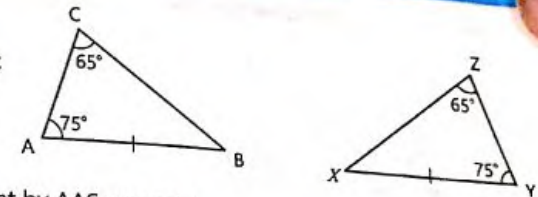
The property states that "In a right-angled triangle, if the hypotenuse and one side of a triangle is equal to the corresponding hypotenuse and the side of the other triangle, then the triangles are congruent".

Look at the triangles ABC and DEF.

The length of one side and the hypotenuse of triangle ABC are congruent to the corresponding side and the hypotenuse of triangle DEF i.e.

$\overline{AC} = \overline{DF}$ and $\overline{BC} = \overline{EF}$.

$\therefore \triangle ABC \cong \triangle DEF$



Example 1:

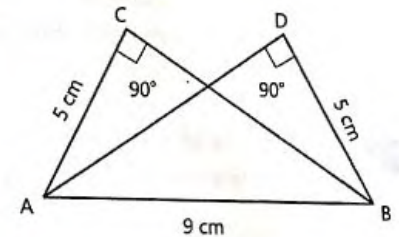
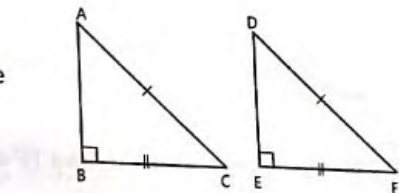
Find if the $\triangle ABC$ and $\triangle BAD$ are congruent triangles.

Solution:

The hypotenuse \overline{AB} is the common side of both triangles.

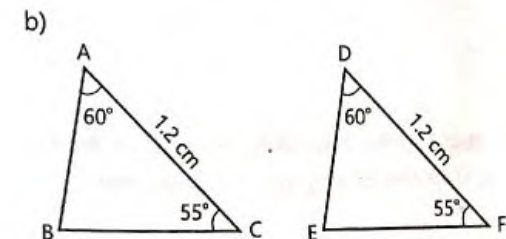
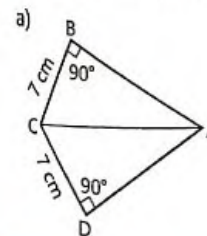
The length of the base is 5 cm of both triangles, so $\overline{AB} = \overline{AB} = 9$ cm, $\overline{AC} = \overline{BD} = 5$ cm

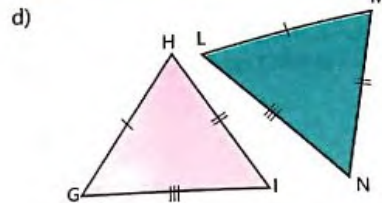
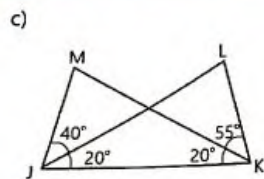
So, according to RHS property $\triangle ABC \cong \triangle BAD$.



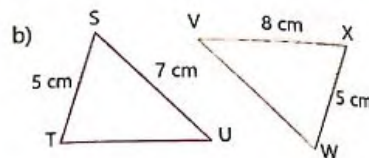
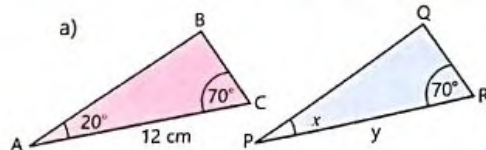
Exercise 10.10

1 Apply properties of congruent triangles and find if the following triangles are congruent or not.



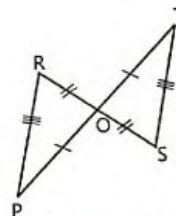


2 Find the unknown sides or angles of the given congruent triangles.



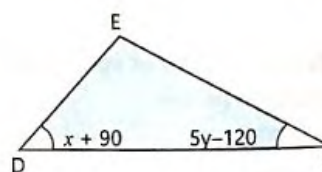
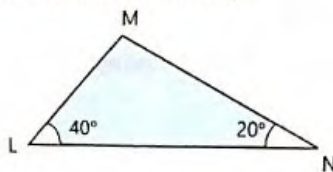
3 Look at the figure. \overline{RS} and \overline{TP} bisect each other.

- State the three pairs of equal sides in the two triangles POR and TOS.
- Which of the following statements are true?
 $\triangle POR \cong \triangle STO$ or $\triangle POR \cong \triangle TOS$

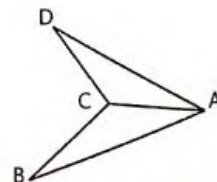


4 The measure of one angle of a triangle is 90° . The measure of its hypotenuse and base is 6 cm and 3 cm respectively. Draw a triangle congruent to this triangle.

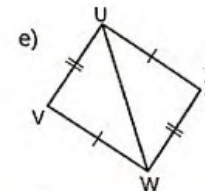
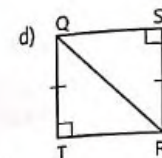
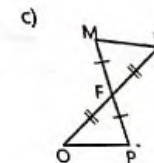
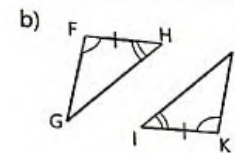
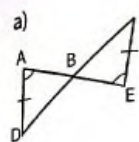
5 $\triangle LMN$ and $\triangle DEF$ are congruent by Angle-Side-Angle congruence property. Find the value of x and y .



6 In triangles ABC and ADC, $\overline{BC} = \overline{DC}$, $\overline{BA} = \overline{DA}$, prove that the triangles are congruent.



7 Prove the congruence of these triangles. Also state the property that is applied to prove the congruence.



Think Higher

Draw an isosceles triangle. Then prove that in an isosceles triangle the bisector of the vertex angle is the right bisector of the base.

Summary

- In a right angle triangle, the side opposite to the right angle is called the hypotenuse.
- The longer diagonal of a kite bisects the shorter diagonal perpendicularly.
- A line segment from the vertex to the opposite side such that it bisects the angle at the vertex is called as angle bisector.
- Angles and sides of congruent figures are the same.
- All congruent figures are similar figures but all similar figures are not congruent.
- In similar figures, the ratios must compare to the correct corresponding lengths.
- In congruent triangles if the triangles have congruent SSS and SAS then they also have fulfilled the property ASA.
- In two triangles if two angles are equal then the third angle is also equal as the sum of three angles of a triangle is 180° .

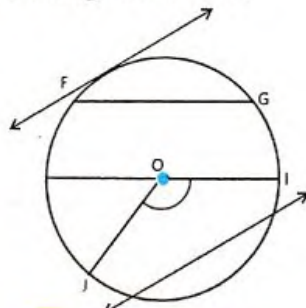
Review Exercise

1 Choose the correct option.

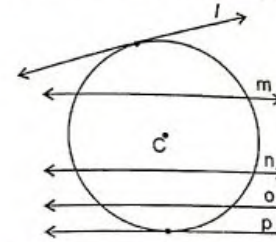
- The sum of the angles of the triangle is:
 - 360°
 - 220°
 - 180°
 - 90°
- A line which meets or touches a circle at only one point is called:
 - secant
 - tangent
 - sector
 - Arc
- A line which passes through a circle and intersects it at two points is called:
 - secant
 - tangent
 - sector
 - Arc
- Rhombus is a quadrilateral whose four sides are equal in length but none of its angles is:
 - acute
 - right
 - obtuse
 - reflex
- A _____ is a quadrilateral whose opposite sides are equal and angles between their adjacent sides are 90°.
 - rectangle
 - kite
 - rhombus
 - triangle
- Two or more circles with same centre but different _____ is called concentric circles.
 - tangent
 - secant
 - radii
 - Arc
- The sum of angles of the quadrilateral is:
 - 120°
 - 240°
 - 360°
 - 180°
- A trapezium has _____ pair/s of parallel sides.
 - 1
 - 2
 - 0
 - 4
- The objects or shape that have same shape and size is called:
 - congruent
 - similar
 - sector
 - equal
- The symbol for congruency is:
 - \neq
 - \cong
 - \ll
 - \approx

2 Define chord, secant, arc, sector and tangent of a circle.

3 Label the parts of the circle.

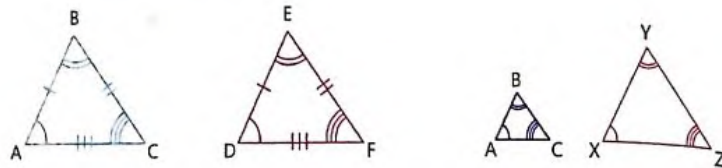


4 Observe the given figure and write the name of secants and tangents.

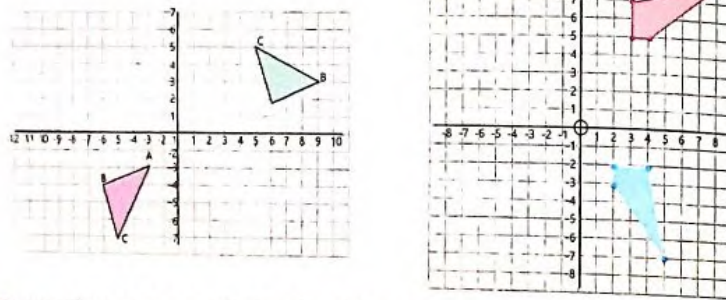


- Construct a triangle ABC when $\angle B = 60^\circ$, $\angle A = 45^\circ$ and the length of one side is 6 cm.
- Construct a triangle LMN when $\angle L = 45^\circ$ and the length of two sides are 4 cm and 3.8 cm.
- Construct a triangle GHI when its three side GH = 5 cm, IG = 6.9 cm and HI = 4 cm
- Draw right angle triangle when the length of its hypotenuse is 7.1 cm and its base is 5.7 cm. Draw its angle and side bisectors as well.
- Draw square ABCD with side AB = 4.5 cm.
- Draw parallelogram PQRS such that PQ = 5 cm, QR = 2 cm, $\angle SPQ = 75^\circ$
- Draw rhombus LMNO such that LM = 6 cm and $\angle LMN = 60^\circ$
- Draw trapezium STUV such that St = 7 cm, SV = 3 cm, VU = 4 cm, $\angle VST = 70^\circ$ and VU || ST.
- Construct a kite with length of diagonal 6 cm and length of sides are 7 cm and 8 cm respectively
- Draw an equilateral triangle and then draw angle and line bisector that divides the given angles and sides of the triangle.
- Draw parallelogram and then draw angles and line bisector to divide the angles and sides of the parallelogram.

16 Which pair of triangles is congruent?

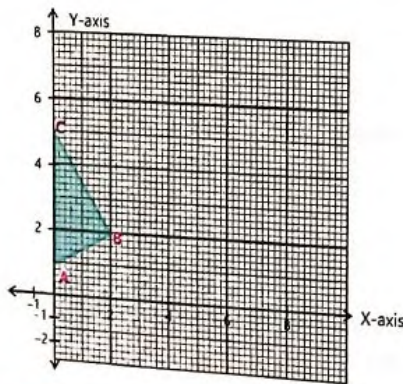


17 Find the centre of rotation.

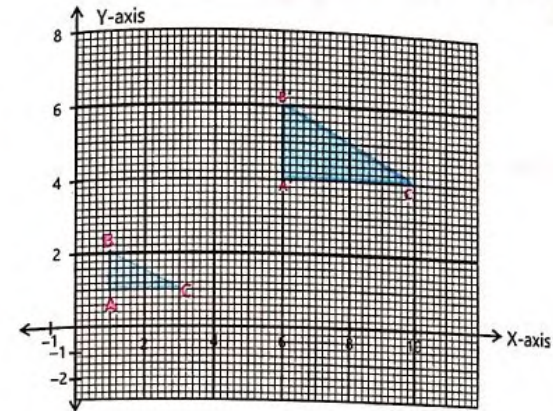


18 Draw a Square on coordinate plane. Then rotate it through 90° clockwise about the origin.

19 Draw an enlargement of the given shape using scale factor 3 and center of enlargement $(-2, 1)$



20 Finding the scale factor and center of enlargement of these shapes.



Math Project

Material Required:

- Shapes cutouts
- Print of coordinate plane
- Pencils
- Ruler

Procedure:

- Get students work in groups.
- Give each group print of coordinate plane and cutouts of various shapes.
- Instruct them to paste any shape in the coordinate plane.
- Then ask them to pin it using thumb pin and rotate it about that point.
- This will be the center of rotation.
- Ask them to use same shape with different locations and centers and then record their observations.
- Similarly, ask them to explore enlargement of objects by using cutouts of shapes on the coordinate plane.

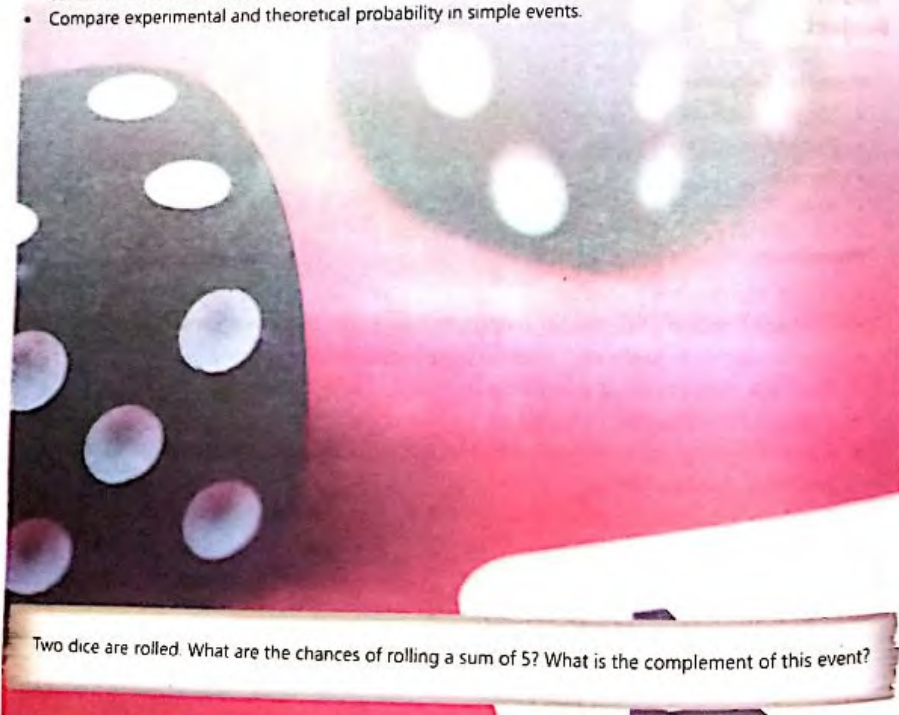
Unit 11

Data Management and Probability

Student Learning Outcomes

After completing this unit, students will be able to:

- Select and justify the most appropriate graph(s) for a given data set and draw simple conclusions based on the shape of the graph.
- Recognise the difference between discrete, continuous, grouped and ungrouped data.
- Calculate range, variance and standard deviation for ungrouped data and solve related real-world problems
- Construct frequency distribution tables, histograms (of equal widths) and frequency polygons and solve related real-world problems.
- Explain and compute the probability of: mutually exclusive, independent, simple combined and equally likely events (including real-world word problems).
- Perform probability experiments (for example tossing a coin, rolling a die, spinning a spinner etc. for certain number of times) to estimate probability of a simple event.
- Compare experimental and theoretical probability in simple events.



Two dice are rolled. What are the chances of rolling a sum of 5? What is the complement of this event?

NOT FOR SALE

Introduction

In previous classes we have learnt about histogram, bar graph, types of data, mean, median, mode of ungrouped data and frequency distribution table, probability of certain event, complement of an event etc. Now we will also learn about frequency distribution table, histograms, frequency polygon, probability of different events, perform probability experiment and comparing experimental and theoretical probability in simple event.

11.1 Data and its Representation

Statistics is related to the gathering of data, its presentation and investigation. The information collected in the form of numbers, words, figures, facts, etc. is called data. When we present data in a meaningful way it is known as data handling.

In the previous class we have learnt about data.

The information that is collected initially without any organisation or arrangement is called raw data. For example, the data collected about the favourite fruit of a group of 10 students. The data will be like "apple, banana, apple, orange, orange, orange, pear, grapes, grapes, and grapes". The process of collecting information is known as data collection.

Previous Knowledge Check

- Define mean, median and mode.
- What is the difference between bar graph and histogram?
- What are certain and impossible events?



11.1.1 Presentation of Data

After the collection of data is over, the next main task is to decide how to present that data in a meaningful and easy way to let us quickly look and understand it. Data presentation is very important, because if it is not presented properly we cannot get the required result or correct information and it may lead us to a wrong or inauthentic decision.

Ways to collect data

There are many ways to collect data. For example:

- Observation
- Interview
- Questionnaire
- Newspapers
- Magazines
- Internet

NOT FOR SALE

Types of Data

We can classify collected data in two types:

- a) Ungrouped data
- b) Grouped data

Grouped and Ungrouped Data

Ungrouped data is the raw data which has just been collected from any survey, experiment, or study. It not grouped in any form like categories, classes, intervals etc.

Grouped data is the refined form of the data collected that has been sorted out in bundles, categories, class intervals etc.

Suitable Presentation for Ungrouped Data

- Line graph
- Bar graph
- Pictograph
- Pie chart

Suitable Presentation for grouped Data

- Histogram
- Frequency polygon

Quantitative Data

Continuous Data

Continuous data is a data that can be measured. It has countless number of possible values within a particular range e.g. Height of students, weight of family members, Volume of water in a tank, temperature of a city.

Discrete Data

Discrete data is quantitative data that is countable. It has a limited number of possible values e.g. number of days, number of boys in grade 8, number of plants in a garden, number of crayons, Number of pages in a book etc.

11.1.2 Choosing an Appropriate Type of Graph/Chart

Data can be presented in various forms on the basis of the type of data collected. We have already learnt about a few ways to present data, for example through picture graphs, bar graphs, line graphs, etc. Choosing an appropriate and suitable chart mean the one that show what we really want it to show, and don't misinform the reader. It strictly depends on what type of data is being represented through it. In general,

- A picture graph is suitable when you have to show the data in an attractive and lively way. It is the most basic chart that we use from the beginning of our introduction to graphs.

NOT FOR SALE

- A **bar chart** can be the most appropriate choice when the data values are independent and no value is dependent on the other. Example data might include the favourite sport of a group of people, the marks of 5 students in Mathematics etc.
- A **multiple bar chart** is suitable when we have two or more than two categories of different kinds of data. For example, the data of men and woman who voted for three different candidates.
- A **line graph** is suitable when we want to show a data that is changing over time. For example the rate of dollar, the temperature of a city etc.
- A **pie chart** is suitable when we have to show and compare the parts or proportions of a whole. For example, the percentage of share among business partners.

Exercise 11.1

- 1 What do the terms discrete and continuous data mean?
- 2 Write 2 examples each of discrete and continuous data.
- 3 For each of the following, state if the data would be discrete or continuous:

- a) The number of animals in a zoo
- b) The length of a pencil
- c) The number of pages in a book
- d) The mass of a chair
- e) The time taken to cook a dish
- f) The number of books in a library
- g) The capacity of a pool
- h) The number of mathematics questions in an exercise
- i) The mass of water melon

11.2 Measure of Dispersion

Measures of dispersion describe the spread of the data. They include the following:

- a) Range
- b) Variance
- c) Standard Deviation

a) Range

A range is the most common and easily understandable measure of dispersion. It is defined as the difference between the greatest and the smallest values given in the data. It gives us



Discuss various types of graphs in class which the student is well familiar with and how to draw simple conclusions based on the shape of the graph. Then ask them to discuss which graph is suitable for various kinds of data values.

a decent indication of how dispersed the data is. It is the difference between two extreme observations in any given set of data. We denote range by R.

$$\text{Range} = R = x_{\max} - x_{\min}$$

Where x_{\max} and x_{\min} are the two extreme observations among the given data values.

x_{\max} = Greatest value in the data

x_{\min} = Smallest value in the data

Let's observe some examples regarding the calculation of range.

Example 1:

The following data shows the number of students who participated in Mathematics competition from grade 1-VIII. Find the range of the given data values.

8, 5, 6, 5, 9, 10, 7, 8

Solution:

In the give data, greatest value is 10 and the smallest value is 5.

$$x_{\max} = 10$$

$$x_{\min} = 5$$

$$\text{Range} = x_{\max} - x_{\min}$$

$$= 10 - 5$$

$$= 5$$

So, the range of this data is 5.

Example 2:

Find the range of the given data values.

Students	No. of questions solved in 1 min
Ibrahim	8
Nida	5
Ahad	6
Sehrish	5
Marwa	9
Arham	8
Sidra	7
Zaeem	6

NOT FOR SALE

Solution:

In the give data, greatest value is 9 and the smallest value is 5.

$$x_{\max} = 9 \quad x_{\min} = 5$$

$$\text{Range} = x_{\max} - x_{\min}$$

$$= 9 - 5$$

$$= 4$$

So, the range if this data is 4.

b) Variance

The variance is another measure of dispersion. Variance actually tell us about the spread of data i.e., how broadly the values in a group diverge. If one or more observation within the given data values diverges significantly from the mean, the variance value increases and vice versa. If all the values in a data are same, the value of variance will be zero. The value of variance is always positive. A small result of variance shows that the values in a data set are close to the mean of the data and a high variance indicates that the values of the given data have spreaded widely around the mean.

Variance is denoted by S^2 .

$$\text{Variance} = S^2 = \frac{\sum(x - \bar{x})^2}{n}$$

Where:

\bar{x} is the mean

n is the total number of data values

x stands for each data value.

Example 1:

The following data shows the obtained marks of 7 students out of 30 marks in monthly test.

10, 15, 16, 18, 20, 22, 25

Find the variance for this data.

Solution:

Step I:

Find the mean. Add up all the data values, then divide them by the number of values.

$$\text{Sum} = \sum x = 10 + 15 + 16 + 18 + 20 + 22 + 25 = 126$$

$$\text{Mean} = \bar{x} = \frac{\text{Sum of marks}}{\text{Number of students}} = \frac{\sum x}{n} = \frac{126}{7} = 18$$



NOT FOR SALE

Step II:

Find the difference of mean and each of the values.

Subtract the mean from each value to get the deviations from the mean.

Since $\bar{x} = 18$, take away 18 from each value.

x	$(x - \bar{x})$
10	$10 - 18 = -8$
15	$15 - 18 = -3$
16	$16 - 18 = -2$
18	$18 - 18 = 0$
20	$20 - 18 = 2$
22	$22 - 18 = 4$
25	$25 - 18 = 7$

Step III:

Take Square of each deviation from the mean. This will result in positive numbers. The find the sum of these squares.

Putting the values in the formula:

$$S^2 = \frac{\Sigma(x - \bar{x})^2}{n}$$

$$S^2 = \frac{146}{7}$$

$$S^2 = 20.85$$

This is the required variance for the given data values.

c) Standard Deviation

The standard deviation is another measure the dispersion of a data values (relative to its mean). It is the positive square root of the variance. We use the symbol σ or S to show standard deviation.

$$\text{Standard deviation} = S = \sqrt{\frac{\Sigma(x - \bar{x})^2}{n}}$$

Example 1:

Grade 8 students planted some plants during plantation campaign. The following data shows the number of plants planted by 10 groups.

42, 60, 28, 35, 45, 40, 56, 64, 60, 70

Find the standard deviation for this data.



Conduct a poster making competition among students for making a step by step poster for calculating various measures of dispersion (range, sd, variance etc.)

NOT FOR SALE

Solution:

Step I:

Find the mean. Add up all the data values, then divide them by the number of values.

$$\text{Sum} = \Sigma x = 42 + 60 + 28 + 35 + 45 + 40 + 56 + 64 + 60 + 70 = 500$$

$$\text{Mean} = \bar{x} = \frac{\text{Sum of marks}}{\text{Number of students}} = \frac{\Sigma x}{n} = \frac{500}{10} = 50$$

Step II:

Find the difference of mean and each of the values.

Subtract the mean from each value to get the deviations from the mean.

Since $\bar{x} = 50$, take away 50 from each value.

x	$(x - \bar{x})$
42	-8
60	10
28	-22
35	-15
45	-5
40	-10
56	6
64	14
60	10
70	20

Step III:

Take Square of each deviation from the mean. This will result in positive numbers. The find the sum of these squares.

Putting the values in the formula:

$$\text{Standard deviation} = S = \sqrt{\frac{\Sigma(x - \bar{x})^2}{n}}$$

$$S = \sqrt{\frac{1730}{10}}$$

$$S = \sqrt{173}$$

$$S = 13.15$$

So, the standard deviation of the given data is 13.15 cm (correct to 4 significant figures)

x	$(x - \bar{x})$	$(x - \bar{x})^2$
42	-8	64
60	10	100
28	-22	484
35	-15	225
45	-5	25
40	-10	100
56	6	36
64	14	196
60	10	100
70	20	400
		$\Sigma(x - \bar{x})^2 = 1730$

NOT FOR SALE

Exercise 11.2

- 1 Find the range, variance, and standard deviation for the following set of data.

- a) 35, 79, 44, 63, 92, 28, 20
b) 400, 450, 520, 380, 495, 575, 444
c) 6, 7, 10, 11, 14, 13, 16, 18, 25
d) 83, 23, 24, 71, 52, 62, 63
e) 63, 25, 43, 28, 72, 61, 45, 46, 13
f) 8, 9, 8, 11, 13, 15, 10, 18, 20

- 2 The following data shows the marks obtained by 11 students in mathematics terminal exams.

82, 83, 59, 94, 70, 76, 32, 80, 87, 94, 62

Calculate:

- a) Range
b) Variance
c) Standard Deviation
for this data.

- 3 The following data shows the age of 10 teachers in a school.

28, 31, 45, 35, 30, 50, 54, 38, 40, 36

Calculate:

- a) Range
b) Variance
c) Standard Deviation
for this data.

- 4 According to a library's weekly record, 42, 45, 58, 62, 70, 66 and 88 books were issued during the last 7 days. Find the range and standard deviation for this data.

- 5 The following data shows the number of pages of 8 books.

120, 225, 100, 154, 188, 204, 212, 170

Find the variance for this data. Also calculate the range.

- 6 The weekly income of 10 employees of a factory is;

Rs. 6000, Rs. 65230, Rs. 78600, Rs. 8000, Rs. 9350,

Rs. 9680, Rs. 1030, Rs. 1236, Rs. 14340, Rs. 15000.

Calculate the range, standard deviation, and variance for this data.



Make groups of students. Provide them with 3 dice. Ask them to roll 3 dice simultaneously several times and record the result. Then ask them to calculate range, variance and standard deviation of the data collected.

NOT FOR SALE

11.3 Frequency Distribution and Histogram

Frequency is a measure of the number of occurrences of a repeating event. It is denoted by f .

The following are the marks of 10 students in a class test out of maximum marks 15.
8, 9, 8, 11, 13, 15, 13, 13, 14, 14.

Here 8 is repeated two times so frequency of 8 is 2. Frequency of 13 is 3 because it is repeated 3 times. 14 occurs 2 times so its frequency is 2. Frequency of 9, 11 and 15 is 1 because they occur only once.

Math History

Frequency was introduced first time in France.

11.3.1 Frequency Distribution

If we represent the values and their respective frequencies in a table, it is called frequency distribution table.

In order to make frequency distribution table we arrange data in increasing order and for each observation we put a tally mark (I) opposite to the observation.

Tallies are marked in bunches of five.

If an observation repeats 2 times we put two tallies (II). In other words we put tally marks as many as the observation repeated.

For an observation repeated 5 times we put 4 tallies and a fifth tally orthogonally across the four tallies (||||). If an observation is repeated 12 times, we put it in bunches of

two fives and two separate tallies (|||| |||). We make

frequency for ungrouped and grouped both type of data. The

following examples illustrate the procedure of frequency distribution table.

Example 1:

The following are the marks obtained by 20 students in a mathematics test out of total marks 50.

48, 47, 45, 48, 35, 40, 30, 30, 48, 36, 35, 48, 47, 30, 45, 38, 30, 48, 36, 36.

Solution:

We construct a frequency distribution table, for this we make three columns.

Note it down

Sum of all frequencies is equal to total number of observations.

Quick Check

If data is arranged systematically in different groups, it is called grouped data.

NOT FOR SALE

Marks Obtained	Tally	Frequency (f)
30		4
35		2
36		3
38		1
40		1
45		2
47		2
48		5
Total		20

When observations are very large, we organise them into groups and obtain a grouped frequency distribution. In this case we write the number of observations falling in each group.

Example 2:

Construct the frequency distribution of 35 persons when their heights (in inches) are given below.

60, 72, 70, 69, 58, 59, 60, 63, 62, 60, 72, 72, 70, 71, 57, 65, 67, 65, 66, 53, 55, 65, 60, 72, 70, 69, 72, 72, 60, 57, 63, 65, 66, 72, 59

Solution:

Here, the smallest height is 53 inches and the greatest height is 72 inches. If we take

Size of class interval = 5

$$\text{Then number of intervals} = \frac{72 - 53}{5} = \frac{19}{5} = 3.8 = 4$$

Frequency Distribution Table

Frequency	Tally	Frequency (f)
53 — 57		4
58 — 62		9
63 — 67		9
68 — 72		13
Total		35

Note it down

The difference between the upper and lower class boundaries of the same class is called the size or width of the class.

11.3.2 Histogram

The word *histogram* comes from the Greek word *histos*, meaning pole, and *gram*, which means chart or graph. So, we can say that a "histogram" is a "pole chart". A histogram is used to represent data where the class intervals (or boundaries) are taken along x-axis and the corresponding frequencies are on the y-axis. The histogram is drawn in such a way that there is no gap between the bars while in a bar graph, there is proper equal spacing between the bars. A histogram is drawn by dividing the range of the given data into a smaller intervals or boundaries. Let's learn about drawing a histogram.

Example 1:

The data below shows the number of people of different ages who visited the library on the weekend.

12, 14, 15, 14, 14, 16, 19, 18, 15, 18, 20, 20, 20, 22, 25, 30, 32, 35, 39, 20, 24, 25, 30, 23, 30, 38, 32, 30, 25, 25



a) Construct the frequency distribution table for this data.

b) Construct the histogram for this data.

Solution:

a) The data is in mixed form. First arrange it in ascending order for convenience.

12, 14, 14, 14, 15, 15, 16, 18, 18, 19, 20, 20, 20, 20, 22, 23, 24, 25, 25, 25, 25, 30, 30, 30, 30, 32, 32, 35, 38, 39

Here the lowest data value is 12 and the highest data value is 39. So,

$$\text{Range} = \text{Highest data value} - \text{Lowest data value} = 39 - 12 = 27$$

Divide the data in 6 classes:

$$\text{Size of the class interval} = \text{Range} \div \text{No. of classes} = 27 \div 6 = 4.5 \approx 5.$$

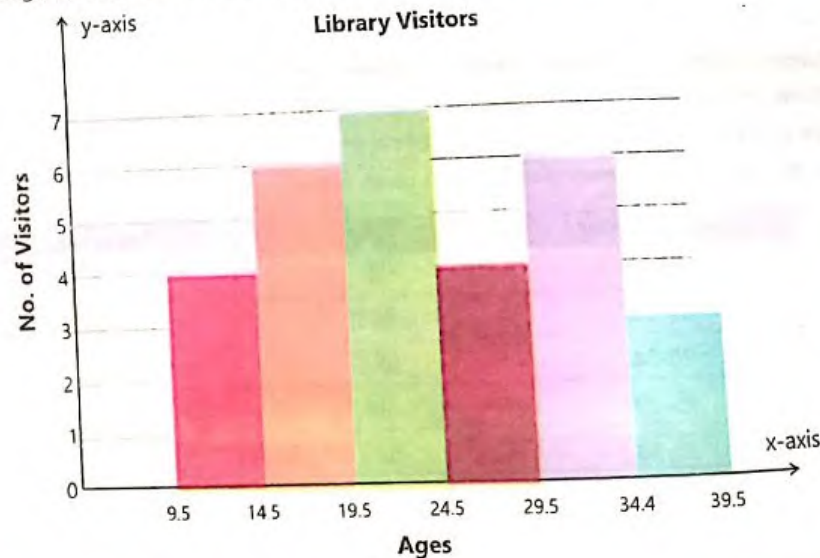
So, the required frequency distribution is given below.

Class Intervals	Tally	Frequency
10-14		4
15-19		6
20-24		7
25-29		4
30-34		6
35-39		3
		$\Sigma f = 30$

- b) To draw histogram, let's find the class boundaries for each class. For this, first subtract the upper-class limit of the first class from the lower-class limit of the second class. Divide this result by two. Subtract the result of division from the lower-class limit of each class. Similarly add the result of division to the upper-class limit of each class. In this way we will find the class boundaries.

Class Intervals	Class Boundaries	Frequency
10-14	9.5 - 14.5	4
15-19	14.5 - 19.5	6
20-24	19.5 - 24.5	7
25-29	24.5 - 29.5	4
30-34	29.5 - 34.5	6
35-39	34.4 - 39.5	3
		$\Sigma f = 30$

- Draw x and y axis.
- Label x-axis and y axis as ages and no. of visitors respectively
- On the horizontal axis, label the lower boundary of each interval.
- Draw a bar extending from the lower value of each interval to the lower value of the next interval.
- The height of each bar should be equal to its corresponding frequency.



11.3.3 Frequency Polygon

A Frequency Polygon is a multi-sided closed figure. It is constructed by plotting the class frequencies against their corresponding class marks (mid-points). Then the points obtained are joined through straight lines.

Example 1:

The following data shows the height of 50 people (in cm). Make a Frequency polygon for the data.

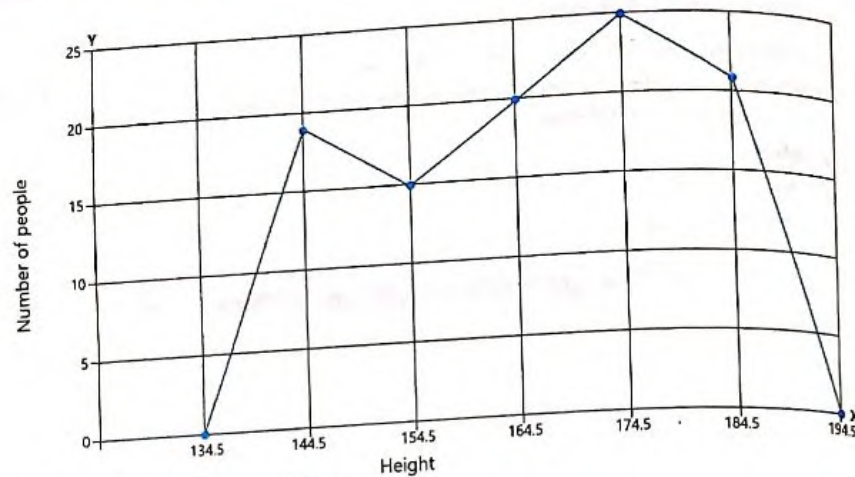
Height in (cm)	No. of People
140 - 149	18
150 - 159	15
160 - 169	20
170 - 179	25
180 - 189	22

Solution:

- First find the class boundaries of each interval.
- Add two extra classes, one before 140 and one after 189 of same width but with 0 frequency.
- Find the mid-point (or class marks) for each class.

Class Intervals	Class Boundaries	Mid-Point	Frequency
130 - 139	129.5 - 139.5	134.5	0
140 - 149	139.5 - 149.5	144.5	18
150 - 159	149.5 - 159.5	154.5	15
160 - 169	159.5 - 169.5	164.5	20
170 - 179	169.5 - 179.5	174.5	25
180 - 189	179.5 - 189.5	184.5	22
190 - 199	189.5 - 199.5	194.5	0

- Mark mid-points (class marks) along x-axis and frequency along y-axis using appropriate scale.
- Plot all the mid-points (class marks) against the corresponding frequencies.



This is the required frequency polygon.

Exercise 11.3

1. Make frequency distribution table of weights in kilogram of 20 students of class VIII.
30, 39, 36.2, 42, 38, 42, 45, 39, 38.2, 41.5, 34, 44.8, 36.2, 38, 41.5, 44.8, 35, 41.5, 39, 42.
2. Marks obtained by 25 students in a certain class test out of a total mark 30 are given below.
25, 19, 26, 15, 25, 28, 25, 27, 19, 28, 28, 19, 19, 15, 28, 19, 19, 25, 27, 25, 28.
a) Construct a frequency distribution table having equal class interval.
b) What is total number of students having highest marks.
3. Draw histogram for the following data values.
a) 228 640 772 264 937 514 763 770 908 864 950 328 306 574 233 265 163 431 807
130 532 553 576 639 675 738 205 535 368 140
b) 18, 25, 21, 20, 27, 18, 16, 17, 19, 19, 19, 23, 45, 12, 32, 30, 31, 36, 28, 25, 27, 30, 42, 40, 43, 44
4. The following data shows the number of people who visited the park daily during a month. Construct a histogram for this data.
42 64 36 62 41 47 87 69 65 55 83 79 34 68 25
28 46 51 74 27 29 44 59 96 40 20 50 88 86 93

NOT FOR SALE

289

5. The following data consists of number of pages in 30 books of a home library. Construct a frequency polygon for this data.
195, 206, 100, 98, 150, 210, 195, 106, 195, 168,
180, 212, 104, 195, 100, 216, 195, 209, 112, 99,
206, 116, 195, 100, 142, 100, 135, 98, 160, 155
6. Construct a frequency polygon for the given data.

Class Interval	Frequency
91-100	6
101-110	3
111-120	3
121-130	5
131-140	8

7. Construct a histogram and frequency polygon for the following data.

Class Interval	Frequency
4 - 6	4
7 - 9	8
10 - 12	6
13 - 15	12

8. Construct a frequency polygon for the given data.

Class Interval	Frequency
150 - 153	7
154 - 157	7
158 - 161	15
162 - 165	10
166 - 169	5
170 - 173	6

NOT FOR SALE

290

11.4 Probability

We know that probability means how likely something is to happen. It is a measure of the likelihood or possibility of an event.



Previous Knowledge Check

- Which of these events are impossible? Which ones are certain?
- A randomly chosen polygon is a circle.
 - A right-triangle have 90° angle.
 - A prime number has 3 factors.

The most commonly used phrase involving probability is predictions and forecasting about weather.

Similarly in investment, election results, sales forecasting etc. there is a wide-ranging use of "probability".

The environmental departments of countries also work extensively to predict the chance of how likely is that any natural disaster like earth quack, flood, storm etc. will hit the country or any specific area in the country. All these and many other events around us need predictions and calculations involving chance and probability.



11.4.1 Probability Line

The probability of any event is a value that is between 0 and 1.

- If the probability of an event is 0, the event is **impossible** i.e., the chance of happening of that event is 0%.
- If the probability of an event is 1, the event is **certain**, i.e., there is 100% chance that the event will happen.
- When there is 50% chance of an event, i.e., $\frac{1}{2}$, then the event is said to be **equally likely**.
- When the probability of an event is between $\frac{1}{2}$ and 1, we say that it is "**more likely**" that the event will happen.
- If the probability of an event is between 0 and $\frac{1}{2}$, we say that it is "**less likely**" that the event will happen.

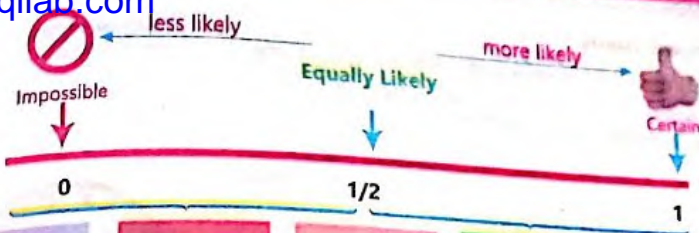
Note it down

The closer the probability to 0, the more is the chances that the event will not occur and the closer the probability to 1, the more area the chances that the event will occur.



Share the following link with the students to have them practice probability related vocabulary online: <https://www.transum.org/Maths/Activity/Probability/Discuss.asp>

NOT FOR SALE



1	5	14
3	7	15
9	11	

1	5	9
3	2	15
7	10	

2	5	11
1	8	14
6	3	

2	6	12
4	5	14
6	11	

2	6	12
4	8	14
16	10	

Probability line for desired outcome of: "Picking up an even number randomly"

11.4.2 Combined Events

Probabilities of Combined Events

When more than one event occurs, we say that a combined events have been occurred. The probability of combined events can be calculated by different methods depending upon the type of event.

Before learning about calculating the probability of combined events, let's learn about different types of events.

Mutually Exclusive Events

Suppose you are travelling.

Case 1: Is it possible that the vehicle's movement direction is towards east as well as towards west at the same time?

Case 2: Is it possible that the vehicle's movement direction is towards east and you are smiling?

The two events in case 1 cannot occur simultaneously.

In case 2, the events can occur simultaneously.

So, in case 1, the events are **mutually exclusive** as they cannot occur at the same time.

- When tossing a coin, the event of getting head and tail are mutually exclusive as we cannot get a head and tail at the same time.

When a die is rolled, the events "1" and "3" are mutually exclusive, as they both cannot be occurred at the same time.



NOT FOR SALE

Independent Events

If the occurrence or happening of one event doesn't affect the probability of another event, then these events are known as independent events.

- If we roll a die two times, the outcome of the first roll doesn't affect the outcome of the second roll, so these two events are independent event.
- If we toss a coin and spun the spinner, the outcome of tossing a coin doesn't affect the outcome of the spinning of the spinner.

So, all such events are independent events

Equally Likely Events

There are 4 red, 5 blue and 1 green colour pencils in a pouch. Ali picks a pencil randomly without looking. What is the probability that he will pick a blue pencil?

Here the number of favorable or desired outcomes is 5, as there are 5 blue pencils.

Numbers of blue pencils (desired outcome) = 5

Total number of possible outcomes = 10

$$\text{Probability of picking up blue pencil (desired outcome)} = \frac{5}{10} = \frac{1}{2}$$

This shows that the probability of the desired outcome i.e. blue pencil is $\frac{1}{2}$.

SO, there is a 50-50 chance for him to pick or not to pick the blue pencil as the number of un-desired outcome and desired outcomes is the same. Or we can say that it is as **equally likely** for him to pick a blue pencil as it is for not picking a blue one.

- A die is rolled. The chance of getting a number less than or equal to 3 is same as getting a number greater than or equal to 4. So, these two outcomes are also equally likely.
- In this spinner, the chance of stopping at 3 is not same as the chance of stopping at 5. So, the outcomes are not equally likely.

Probability of Combined Events

When two or more events are combined, their probability is calculated by associating the words "and" or "or."



Use various suitable examples from daily life and ask the students to identify different combined events.

Note it down

Two or more events that cannot occur simultaneously are called mutually exclusive events.



Note it down

A desired or favorable outcome is an outcome which we want to get from an experiment or the one we are looking for.



The word "and" means that we are talking about the probability of only those outcomes that satisfy more than one event at the same time.
The word "or" means that we are talking about the probability of any of the outcomes.

Example 1:

Suppose, A = the vehicle's movement is towards east
B = the vehicle's movement is towards west

Then we can say that both the events cannot occur at the same time.

So, "the probability of A and B together equals 0 (impossible)"

$P(A \text{ and } B) = 0$ as A and B can never occur together.

But also, we can say that "The probability of A or B equals the sum of their individual probabilities".

$$P(A \text{ or } B) = P(A) + P(B)$$

Consider the examples.

Example 2:

A die is rolled. Find whether the events are mutually exclusive or not. Then calculate the probability $P(A \text{ and } B)$ and $P(A \text{ or } B)$.

A = {an odd}

B = {2}

Solution:

The events A and B are mutually exclusive as these two events cannot happen at the same time. Either we can get an odd or we can get 2. There is no number which is both odd and 2.

The intersection of these two events A and B is the empty set. So, the probability of occurring both at a time is zero.

$$P(A \text{ and } B) = P(A \cap B) = 0 \text{ (as the two events cannot have any common element).}$$

$$P(A \text{ or } B) = P(A \cup B)$$

$$\begin{aligned} &= P(A) + P(B) = \frac{3}{6} + \frac{1}{6} \\ &= \frac{4}{6} \\ &= \frac{2}{3} \end{aligned}$$

Note it down

To describe the probability of a single event A occurring, we use the notation $P(A)$.

Note it down

For mutually exclusive events:
 $P(A \text{ or } B) = P(A \cup B) = P(A) + P(B)$
 $P(A \text{ and } B) = P(A \cap B) = 0$

Example 3:

Two fair dice are tossed and different events are recorded.

	1	2	3	4	5	6
1	(1, 1)	(1, 2)	(1, 3)	(1, 4)	(1, 5)	(1, 6)
2	(2, 1)	(2, 2)	(2, 3)	(2, 4)	(2, 5)	(2, 6)
3	(3, 1)	(3, 2)	(3, 3)	(3, 4)	(3, 5)	(3, 6)
4	(4, 1)	(4, 2)	(4, 3)	(4, 4)	(4, 5)	(4, 6)
5	(5, 1)	(5, 2)	(5, 3)	(5, 4)	(5, 5)	(5, 6)
6	(6, 1)	(6, 2)	(6, 3)	(6, 4)	(6, 5)	(6, 6)

Let the events A and B be as follows:

A = {the sum is 5} = {(1, 4), (2, 3), (3, 2), (4, 1)}

B = {both numbers are odd} = {(1, 1), (1, 3), (1, 5), (3, 1), (3, 3), (3, 5), (5, 1), (5, 3), (5, 5)}

Then:

- Are events A and B mutually exclusive?
- Find $P(A \cup B)$ and $P(A \cap B)$
 - A and B cannot occur at the same time because the sum of two odd numbers is always even and 5 is not even. So, A and B are mutually exclusive.
 - We know that for mutually exclusive events:

$P(A \text{ and } B) = P(A \cap B) = 0$ (as the two events cannot have any common element).

There are 36 total outcomes.

$$P(A \text{ or } B) = P(A \cup B) = \frac{P(A) + P(B)}{1} = \frac{\frac{4}{36} + \frac{9}{36}}{1} = \frac{13}{36}$$

Probability of an Independent Event

Consider the example. If A and B are two events where:

A = rolling a die

B = Tossing a coin

As we can see that the result of the first event doesn't affect the result of the second event, so these two are independent events. No matter what the outcome of rolling a die is, there will be no effect of it on tossing a coin.

Note it down

The probability of two independent events can be found by multiplying the probability of the first event by the probability of the second event.
 $P(A \text{ and } B) = P(A) \times P(B)$



Share the following link with the students to have them practice probability problems through online games: <https://www.transum.org/software/SW/Dice/>

NOT FOR SALE

We can find the probability of two independent events by using the formula:
 $P(A \text{ and } B) = P(A \cap B) = P(A) \times P(B)$

Example 4:

A person rolls a die and spins the spinner. What is the probability that he will get a 6 on the die and landing on a yellow?

Solution:

Let's say:

Event A = getting a 6 on the die

B = landing on a yellow

We can see that getting any outcome of the spinner does not depend on the outcome of the die. So, these events are independent.

$$P(A) = \frac{1}{6}$$

$$P(B) = \frac{2}{4} = \frac{1}{2}$$

$$P(A \text{ and } B) = P(A \cap B) = P(A) \times P(B) = \frac{1}{6} \times \frac{1}{2} = \frac{1}{12}$$

So, the probability of getting a 6 and landing on yellow is $\frac{1}{12}$.

Example 5:

Two dice are rolled. What is the probability of getting 3s on both?

Solution:

In this case, a combined event has occurred.

Let's find the probability of getting 3s.

$$\text{Probability of an event} = \frac{\text{Number of Favorable outcomes}}{\text{Total number of outcomes}}$$

$$\text{Probability of getting 3 (1st die)} = \frac{1}{6}$$

$$\text{Probability of getting 3 (2nd die)} = \frac{1}{6}$$

As they are independent events, So:

$$P(A \cap B) = P(A) \times P(B)$$

$$\text{Probability of getting two 3's} = \frac{1}{6} \times \frac{1}{6} = \frac{1}{36}$$

Example 6:

Ahad tossed a coin. Find the probability of getting a head?

Solution:

Sample space or total number of possible outcomes = H, T

Number of possible outcomes = 2



Quick Check

A die is rolled. What the probability of getting a 7? A die is rolled. What the probability of getting a number greater than 0?

NOT FOR SALE

Number of favorable (desired) outcomes = 1(H)

Probability of an event = $P(E) = \frac{\text{Number of favorable (desired) outcomes}}{\text{Total number of possible outcomes}}$

Probability of an event = $P(E) = \frac{1}{2}$

$\frac{1}{2}$ shows that there are 1 out of 2 chances that the coin will show a head.

Example 7:

Sadaf has a bag with ten marbles. 2 are red, 4 are green, 3 are yellow and 1 is blue. What is the probability that if she chooses one marble randomly without seeing, will be red?

Solution:

Number of possible outcomes = 10

Number of favorable (desired) outcomes = 2

Probability of an event = $P(E) = \frac{\text{Number of favorable (desired) outcomes}}{\text{Total number of possible outcomes}}$
 $= \frac{2}{10}$

Probability of an event = $P(E) = \frac{1}{5}$ The probability of choosing red is $\frac{1}{5}$.

Quick Check

What is the probability of choosing yellow marble?



Exercise 11.4

1 A spinner is spun.

Event A: The arrow landed on a purple

Event B: The arrow landed on yellow.

Find:

- If the two events mutually exclusive
- $P(A \cup B)$
- $P(A \cap B)$



2 A die is rolled. What is the probability that the dots on the top are less than 3?

3 A die is rolled. What is the probability of getting a 3 or 4? Are the events mutually exclusive?

4 A bag contains 5 blue, 7 red, 3 green and 2 yellow marbles. Ali picked a marble randomly and recorded the result. Then he put the marble back in the bag. He again picked a marble randomly. Tell if the events are independent or not. Explain your answer. Also calculate the probability that the first marble is green and the second marble is red.

NOT FOR SALE

297

5 Two dice are rolled.

Event A: the sum of the numbers shown on the two dice is 3
 Event B: both the numbers are odd.

Find:

- The sample space
- If the two events mutually exclusive
- $P(A \cup B)$
- $P(A \cap B)$

6 A coin is tossed 4 times. What is the probability of every time getting a tail?

7 Nida picks a card randomly.

Event A: randomly selected card will have a 7.

Event A: randomly selected card will be even.

Find:

- If the two events mutually exclusive
- $P(A \cup B)$
- $P(A \cap B)$



11.5 Theoretical and Experimental Probability

We have learnt that the probability of an event occurring, denoted by $P(E)$, is:

Probability of an event = $P(E) = \frac{\text{Number of favorable (desired) outcomes}}{\text{Total number of possible outcomes}}$

This is the **Theoretical probability** of an event. Theoretical probability of a single event is the ratio or comparison of favorable (desired) outcomes to the total number of possible outcomes.

Note it down

Probability of an event built on known facts is called **theoretical probability**.

Example 1: If a die is rolled, what is the probability of getting a 6?

Solution:

Theoretical probability = $P(6) = \frac{\text{Number of favorable (desired) outcomes}}{\text{Total number of possible outcomes}}$
 $= \frac{1}{6}$



Example 2: If the spinner is spun, what is the probability of getting a 3?

Solution:

$$\text{Theoretical probability} = P(3) = \frac{\text{Number of favorable (desired) outcomes}}{\text{Total number of possible outcomes}} = \frac{2}{8} = \frac{1}{4}$$



We can estimate the probability of a simple event by carrying out experiments. This probability is known as **experimental probability**.

Experimental Probability can be found using the formula:

$$\text{Experimental Probability of an Event} = \frac{\text{Number of times an event occurs}}{\text{Total number of trials}}$$

A single performance of an experiment is called a **trial**.

Note it down

Probability of an event that is built on the outcomes obtained by performing an experiment is known as **experimental probability**.

Example 3:

Look at the spinner. There are 5 portions in different colours. Dania performed an experiment to find out the probability of landing on a blue. She spinned the wheel 40 times and recorded the result in a table.



Colour of the portion as outcome	Frequency
Blue	8
Green	12
Red	6
Yellow	14
	$\Sigma = 40$

Compare the experimental and theoretical probabilities of the event.

Solution: Based on the experiment, the spinner landed 8 times on blue portion. So,

$$\text{Experimental probability of the event} = \frac{\text{Number of times an event occurs}}{\text{Total number of trials}} = \frac{8}{40} = \frac{1}{5}$$



Practically involve students in finding the experimental probability of various events and then ask them to compare the results with the theoretical probability.

NOT FOR SALE

Let's find and compare the theoretical probability.

$$\text{Theoretical probability of the event} = \frac{\text{Number of favorable (desired) outcomes}}{\text{Total number of possible outcomes}} = \frac{2}{8} = \frac{1}{4}$$

We can see that the experimental probability is close to the theoretical probability but not exactly the same.

Example 4:

Ibrahim performed an experiment. He had a pack of candies containing 1 mango flavoured candy, 4 orange flavoured candies, 2 chocolate flavoured candies and 3 strawberry flavoured candies. He picked one candy at a time without looking into the bag and recorded the outcome.



He put the candy back and in the same way he repeated this experiment and recorded the results in a table.

Flavour of the candy as outcome	Frequency
Mango	4
Orange	3
Chocolate	5
Strawberry	8
	$\Sigma = 20$

Find the experimental Probability of the following events:

- Picking a chocolate candy
- Picking a strawberry candy

Also compare the results with the theoretical probabilities of these events.

Solution:

a) To find the experimental probability of Picking a chocolate candy, use the formula:

$$\text{Experimental probability} = \frac{\text{Number of times an event occurs}}{\text{Total number of trials}} = \frac{5}{20} = \frac{1}{4}$$

NOT FOR SALE

Let's find the theoretical probability of picking a chocolate candy.

$$\text{Theoretical probability of the event} = \frac{\text{Number of favorable (desired) outcomes}}{\text{Total number of possible outcomes}} = \frac{2}{10} = \frac{1}{5}$$

b) To find the experimental probability of Picking a strawberry candy, use the formula:

$$\text{Experimental probability} = \frac{\text{Number of times an event occurs}}{\text{Total number of trials}} = \frac{8}{20} = \frac{2}{5}$$

Let's find the theoretical probability of picking a strawberry candy.

$$\text{Theoretical probability of the event} = \frac{\text{Number of favorable (desired) outcomes}}{\text{Total number of possible outcomes}} = \frac{3}{10} =$$

By comparison, we can see that there is more difference between the experimental and theoretical probabilities of "Picking a strawberry Candy" than the experimental and theoretical probabilities of "Picking a chocolate Candy"

Exercise 11.5

1 What is the difference between theoretical and experimental probability?

2 Zara performed an experiment. She tossed a coin 50 times. The results are represented in the table below.

Outcome	Frequency
Head	28
Tail	22

a) Calculate the experimental probability of getting a head and compare it with the theoretical probability of getting a head.

b) Calculate the experimental probability of getting a tail and compare it with the theoretical probability of getting a tail.

NOT FOR SALE

3 Nida performed an experiment. She rolled a die 40 times. The results are represented in the table below.

Outcome	Frequency
1	8
2	6
3	7
4	9
5	4
6	6

- Calculate the experimental probability of getting a 5 and compare it with the theoretical probability of getting a five.
- Calculate the experimental probability of getting an odd number and compare it with the theoretical probability of getting an odd number.
- Calculate the experimental probability of getting a number less than 2 and compare it with the theoretical probability of getting a number less than 2.
- Calculate the experimental probability of getting a factor of 12 and compare it with the theoretical probability of getting a factor of 12.
- Calculate the experimental probability of getting a multiple of 3 and compare it with the theoretical probability of getting a multiple of 3.

4 Sehrish performed an experiment. She had a bag of buttons containing 3 red, 4 yellow, 6 blue and 5 green buttons. She picked one button at a time without looking into the bag and recorded the outcome. She put the button back and in the same way he repeated this experiment and recorded the results in a table.

Flavour of the candy as outcome	Frequency
Red	10
Yellow	8
Blue	10
Green	12
	$\Sigma = 40$

NOT FOR SALE

Find the experimental Probability of the following events:

- Picking a red button.
- Picking a blue button.
- Not picking a green button.
- Picking a yellow button.

Also compare the results with the theoretical probabilities of these events.

5 Ahmed performed an experiment. He spun the spinner 45 times and recorded the outcomes.



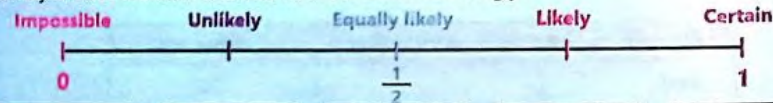
Flavour of the candy as outcome	Frequency
Red	18
Yellow	12
Blue	15
	$\Sigma = 45$

Calculate the probabilities and complete the table.

Events	Theoretical probability	Experimental probability
Arrow landed on red		
Arrow landed on blue		
Arrow landed on yellow		

Think Higher

Think of any desired outcome and then fill in the boxes accordingly.



--	--	--	--	--

Probability line for desired outcome of: _____

NOT FOR SALE

Summary

- The difference between the upper and lower class boundaries of the same class is called the size or width of the class.
- The closer the probability to 0, the more is the chances that the event will not occur and the closer the probability to 1, the more area the chances that the event will occur.
- Two or more events that cannot occur simultaneously are called mutually exclusive events.
- A desired or favorable outcome is an outcome which we want to get from an experiment or the one we are looking for.
- For mutually exclusive events:
 - $P(A \text{ or } B) = P(A \cup B) = P(A) + P(B)$
 - $P(A \text{ and } B) = P(A \cap B) = 0$
- To describe the probability of a single event A occurring, we use the notation $P(A)$.
- The probability of two independent events can be found by multiplying the probability of the first event by the probability of the second event.
- $P(A \text{ and } B) = P(A) \cdot P(B)$
- Probability of an event built on known facts is called **theoretical probability**
- Probability of an event that is built on the outcomes obtained by performing an experiment is known as **experimental probability**.

Vocabulary

- Variance
- Range
- Standard deviation
- Frequency polygon
- Histogram
- Mutually exclusive events
- Independent events
- Theoretical probability
- Experimental probability

Review Exercise

1 Choose the correct option.

- Discrete data is in the form of:
 - Fraction
 - decimal number
 - Rational number
 - whole numbers
- The length of different iron rods is the example of the data:
 - Grouped data
 - ungrouped data
 - Continuous data
 - discrete data
- The presentation of frequency distribution in vertical rectangle bars with no gap is called:
 - bar graph
 - multiple bar graph
 - histogram
 - block graph
- The _____ of each bar in a histogram represent the size of the class.
 - length
 - width
 - area
 - height

NOT FOR SALE

- e) Two events that not occur at the same time is called
 i) Independent event ii) combined event
 iii) Mutually exclusive even iv) equally likely event
- f) _____ event is not affected by the previous event:
 i) Independent event ii) combined event
 iii) Mutually exclusive even iv) equally likely event
- g) The formula to find the standard deviation is:

i) $\sqrt{\frac{\Sigma(x - \bar{x})^2}{n}}$ ii) $\sqrt{\frac{\Sigma(y - \bar{y})^2}{n}}$ iii) $\sqrt{\frac{\Sigma(x + \bar{x})^2}{n}}$ iv) $\sqrt{\frac{\Sigma(x - \bar{x})^2}{2n}}$

- h) When we plot class marks along x-axis and frequency along y-axis and representation of data is in form of closed polygon then it is called:
 i) Frequency distribution ii) histogram
 iii) line graph iv) frequency polygon
- i) The difference between the highest and the lowest value of the data is called:
 i) Variance ii) class boundary
 iii) Range iv) standard deviation
- j) When a coin is tossed 6 times. The theoretical probability of the coin head is:
 i) $\frac{1}{6}$ ii) $\frac{2}{6}$ iii) $\frac{4}{6}$ iv) $\frac{1}{2}$

2 Define the following terms.

- a) Variance b) histogram
 c) ungrouped data d) Standard deviation
 e) polygon frequency f) Mutually exclusive Events
 g) independent events

3 Calculate the standard deviation for the data: 4, 9, 5, 1, 2 and 8.

4 Calculate the variance and standard deviation for the given data: 3, 5, 7, 11, 13, 17, 19 and 23.

5 Calculate the range for the following observations.
 45.2, 67.9, 88.5, 33.2, 11.6, 99.2, 101.5

6 Sara collected data of marks out of 50 of students of grade 8. The data is: 32, 47, 49, 43, 39, 28, 35, 45, 41, 50, 47, 43, 49

NOT FOR SALE

7 Construct the histogram for the given frequency distribution.

Class intervals	10 - 15	15 - 20	20 - 25	25 - 30	30 - 35
Frequency	5	9	12	10	7

8 The following data shows the ages of different people who came for the issuance of smart id card on a certain day. Draw a histogram for this data.

Class intervals	20 - 30	30 - 40	40 - 50	50 - 60	60 - 70
Frequency	8	30	20	25	22

9 The given data is the temperature of the Lahore in 30 days of May is:

31°, 33°, 35, 37, 32, 31, 31, 34, 38, 40, 30, 32, 32, 33, 34, 34, 33, 30, 30, 35, 34, 29, 42, 39, 42, 40, 39, 42, 40, 41

Draw frequency distribution table for the given data.

10 Draw a histogram and a frequency polygon by using this data.

Class intervals	Frequency (f)
25 - 29	2
30 - 34	4
35 - 39	7
40 - 44	10
45 - 49	8
50 - 54	6
55 - 59	3

11 A die is rolled. What is the probability of rolling an even or odd number?

12 Two dice are rolled. What is the probability of getting a factor of 6s on both?

13 Ahamd is rolling dice what is probability that a dice shows 3 or 6.

NOT FOR SALE

- 14 In a box there are 4 blue beads, 6 red beads and 2 yellow beads. Sara picked a bead randomly and recorded the result. Then she put the bead back in the bag. She again picked a bead randomly. Tell if the events are independent or not. Explain your answer. Also calculate the probability that the first bead is blue and the second bead is yellow.

- 15 Zaman performed an experiment. He rolled a die 30 times. The results are represented in the table below.

Outcome	Frequency
1	5
2	4
3	3
4	6
5	5
6	7

- Calculate the experimental probability of getting a 6 and compare it with the theoretical probability of getting a head.
- Calculate the experimental probability of getting an odd number and compare it with the theoretical probability of getting a tail.
- Calculate the experimental probability of getting a number less than 4 and compare it with the theoretical probability of getting a head.
- Calculate the experimental probability of getting a factor of 6 and compare it with the theoretical probability of getting a tail.

Math Project

Material Required:

- Dice
- Coins
- Spinners
- Marbles
- Buttons
- Beads
- Recording sheet



Procedure:

- Work in groups.
- Each group will be assigned a different experiment (like choosing marbles, spinning the spinner, tossing a coin etc.).
- The group will perform their respective experiments for about 40-50 times and record the results in a table.
- Calculate the experimental probabilities of each event.
- Then calculate the theoretical probability of these events and compare them with the experimental probabilities.

NOT FOR SALE

Answer Key

Unit 1: Approximation and Estimation

Exercise 1.1

- 67200
 - 8230
- 83570
 - 93650
- 8723500
 - 8348300
- 87.3
 - 2.66
- 72.56
 - 6.099
- 4.1099
 - 0.23487
- 150,480
- 6723.9 litres
- 99200
- 53800
- 6380
- 239000
- 178900
- 83480
- 873600
- 5362900
- 4528400
- 9024700
- 5245900
- 58.3
- 4.82
- 9.25
- 89.04
- 23.23
- 56.89
- 65.237
- 65.720
- 872.12
- Rs. 2,237,800
- Rs. 723.66
- 23.77 gram

Review Exercise

- iv
 - iii, iv
 - i
 - ii
 - iii
- 6700000
 - 7600000
 - 24
 - 0.67
 - 46
 - 6730000
 - 7610000
 - 23.9
 - 0.672
 - 45.8
 - 6729000
 - 7609000
 - 23.87
 - 0.6725
 - 45.76
 - 6729000
 - 7609200
 - 23.871
 - 0.67245
 - 45.762
- 130000
 - 127000
 - 127500
 - 127460
- 1496800
- 453 km
 - 453.0 km
 - 452.99 km

Unit 2: Real Numbers

Exercise 2.1

- Terminating: c, f, g, j, k
- Recurring: c, e, h
- 0.666666..... (recurring)
 - 0.142857..... (Non-Terminating)
 - 0.952380..... (Non-Terminating)
 - 0.571428..... (Non-Terminating)
- Non-terminating: a, b, d, e, h, i, l
- Non-recurring: a, b, d, f, g
- 3.75 (Terminating)
- 1.375 (Terminating)
- 1.625 (Terminating)
- 1.533333..... (Recurring)

NOT FOR SALE

4. a) $0.\overline{35}$ b) $0.\overline{127}$ c) $7.\overline{4}$ d) $8.\overline{123}$
 e) $0.\overline{6}$ f) $3.\overline{46}$ g) $12.\overline{3}$ h) $9.\overline{3}$
5. In standard form: b, c, d, f Not in standard form: a, e, g, h
- a) $-\frac{12}{16}$ or $-\frac{3}{4}$ (In Standard Form) b) $-\frac{41}{61}$ (Not in Standard Form)
 c) $-\frac{21}{2}$ (In Standard Form) d) $\frac{3}{4}$ (In Standard Form)
 e) $\frac{5}{15}$ or $\frac{1}{3}$ (In Standard Form) f) $\frac{17}{4}$ (In Standard Form)
 g) $\frac{44}{88}$ or $\frac{1}{2}$ (In Standard Form) h) $-\frac{75}{135}$ or $-\frac{5}{9}$ (In Standard Form)
6. a) $\frac{7}{9} = 0.777 \dots$ (Non-Terminating) b) $\frac{8}{13} = 0.615 \dots$ (Non-Terminating)
 c) $\frac{7}{-6} = -1.1666 \dots$ (Non-Terminating) d) $\frac{10}{11} = 0.9090 \dots$ (Non-Terminating)
 e) $-\frac{3}{12} = -0.25$ (Terminating) f) $\frac{7}{5} = 1.4$ (Terminating)
 g) $\frac{13}{10} = 1.3$ (Terminating) h) $\frac{25}{15} = 1.666 \dots$ (Non-Terminating)

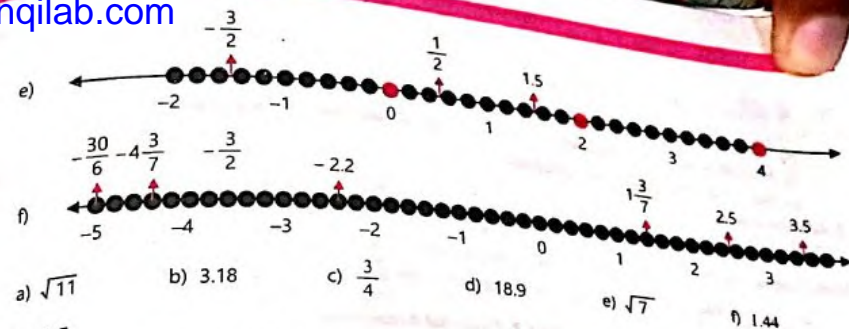
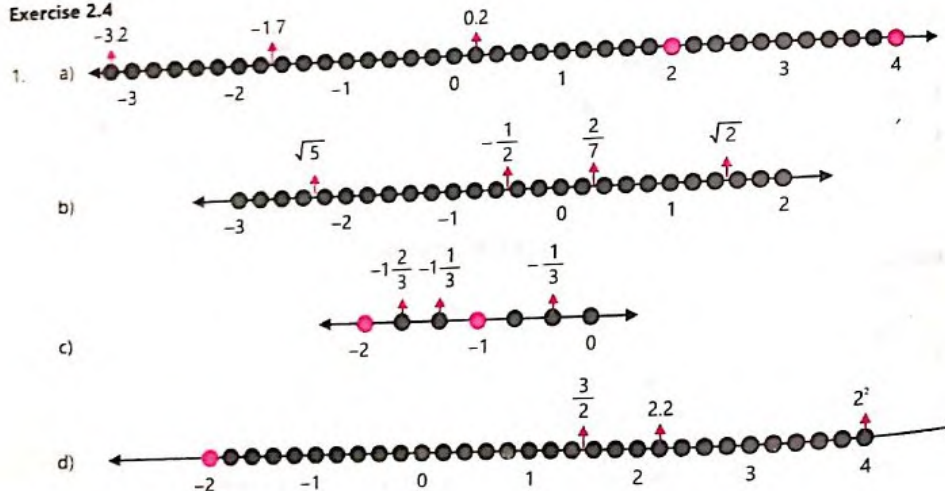
Exercise 2.2

1. $\frac{5}{6}$ liters 2. 21 kg 3. $1\frac{5}{9}$ km 4. a) 47.97 km b) 108.24 km 5. 22 km 6. 1.25 kg

Exercise 2.3

1. Rational Numbers: (a), (b), (c), (e), (g), (h), (k), (l), (n), (o)
 Irrational Numbers: (f), (i), (j), (m)
 (d) Not possible

Exercise 2.4



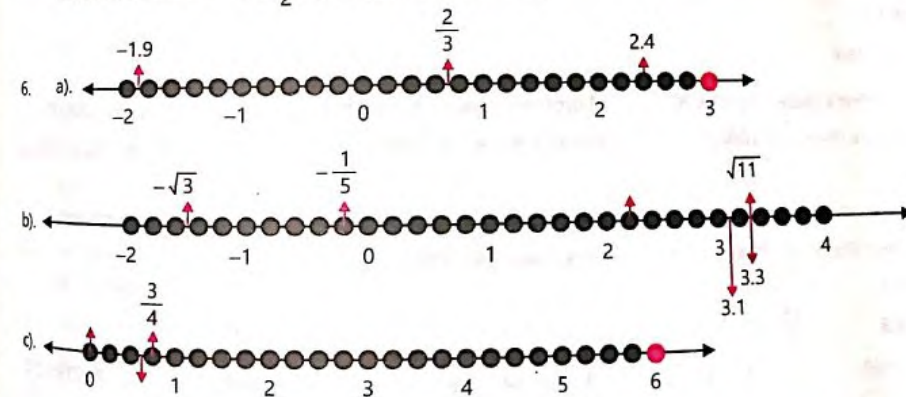
Exercise 2.5

1. a) Commutative property of addition b) Commutative property of multiplication
 c) Multiplicative inverse d) Additive identity
 e) Multiplicative inverse f) Commutative property of addition
 g) Multiplicative identity h) Multiplicative inverse
 i) Multiplicative inverse j) Distributive property of addition
 k) Distributive property of multiplication

2. a) -9 b) 3.4 c) $-\frac{3}{9}$ d) $-\frac{1}{\sqrt{5}}$ e) $-\frac{\sqrt{2}}{\sqrt{5}}$ f) 6 g) $\frac{4}{9}$ h) 0 i) -1
3. a) $\frac{1}{-10}$ b) 2 c) $\frac{1}{8.1}$ d) $\frac{4}{\sqrt{5}}$ e) $\frac{1}{\sqrt{11}}$ f) $\frac{1}{14}$ g) $\frac{10}{-9}$ h) Does not exist i) 1

Review Exercise

1. a) ii b) i c) iii d) iv e) ii f) iii g) iii h) ii
3. Terminating decimal numbers: a, d, e, g Non-terminating decimal numbers: b, c, f, h
4. Recurring decimal numbers: a, b, c, e, f, g Non-recurring decimal numbers: d, h
5. Rational numbers: 5.7 , $\frac{12}{5}$, $\frac{4}{7}$, $-3\frac{1}{2}$, $\frac{15}{38}$, 0 , 9 , 5.2
 Irrational numbers: -8π , $\frac{\pi}{2}$, $\sqrt{12}$, $\sqrt{25}$, $\sqrt{9}$, $\sqrt{17}$, $-\sqrt{16}$



7. a) $\sqrt{17}$ b) 0.06 c) $\frac{4}{5}$
8. a) Commutative property of addition b) Commutative property of multiplication
c) Multiplicative inverse Property d) No such property is addressed
e) Associative Property of Multiplication

9. Additive inverse
a) -10 b) -2.5 c) $-\frac{1}{9}$ d) $-\frac{1}{\sqrt{2}}$ e) $-\frac{\sqrt{2}}{3}$ f) 10 g) 0 h) -1

- Multiplicative inverse
a) $\frac{1}{10}$ b) $\frac{1}{2.5}$ c) 9 d) $\sqrt{2}$ e) $\frac{3}{\sqrt{2}}$ f) $-\frac{1}{10}$ g) 0 h) 1

Unit 3: Financial Arithmetic

Exercise 3.1

1. a) $y = 12$ b) $y = 3$ c) $y = 10$ d) $y = 16$ e) $y = 12$
f) $y = 49$ g) $y = 500$ h) $y = 8$ i) $y = 16$ j) $y = 50$
2. $\frac{280}{3}$ 3. 22 4. 10 5. $\frac{480}{11}$ 6. $1555\frac{5}{9}$ 7. 212.37 kg
8. 22500 cm³ 9. 16 10. 290, 98 $\frac{7}{16}$ 11. $77\frac{1}{7}$ minutes 12. 63 minutes

Exercise 3.2

1. a) 2.56 % loss b) 3.45 % loss c) 1.622 % profit d) 70 % profit
2. Rs 9910000 3. Rs 22693 4. Rs 60000 5. Rs 20727 6. Rs 39501 7. Rs 9153
8. 16.3 % profit 9. Rs 1951440 10. Rs 8936.17 11. Rs 144 per kg 12. Rs 10130, 3.65 %
13. 3.1 % 14. Rs 54012

Exercise 3.3

1. a) Rs 64 b) Rs 15833.34 c) 2.6 years d) Rs 294
2. Rs 3000 3. Rs 19500 4. 4.48 % 5. Rs 63960 6. Rs 2042.4 7. Rs 12.5 %

Exercise 3.4

1. $16\frac{1}{3}$ days
2. 1st brother's share: Rs 200000 2nd brother's share: Rs 200000 Sister's share: Rs 100000
3. Salman's share: Rs 10000 Ahmad's share: Rs 20000 Akram's share: Rs 15000
4. 243
5. Widow's share: Rs 18750 Daughter's share: Rs 43750 Son's share: Rs 87500
6. Aleem's share: Rs 7000 Riaz's share: Rs 14000
7. 2 days

Exercise 3.5

1. Rs 32500 2. 5% 3. Rs 154761.9 4. Rs 69886.25 5. Rs 719512.5

Exercise 3.6

1. a) Rs 60921.76 b) Rs 17334
e) Rs 5196.27 f) Rs 6500.25
2. a) 3.3486 USD b) 0.2415 CAD c) Rs 19564.72
e) 2.3719 USD f) 0.6762 CAD g) Rs 88640.16 h) Rs 59915.875
i) Rs 156739.929 d) Rs 2.52 GBP
j) 1.63 EUR d) 2.3604 GBP
- Review Exercise
1. (a) ii (b) iii (c) iv (d) ii (e) iii
5. profit Rs 1,660,000, profit percentage = 22.01%
6. Ahmad share = Rs 20,000 Haroon share = 15000 Ahsan share = Rs 8000
7. wife share = Rs 120,250 daughter share = Rs 168,350 each son's share = Rs 336,700

Unit 4: Square Roots and Cube Roots

Exercise 4.1

1. (i) 6 (ii) 10 (iii) 49 (iv) 640 (v) $\frac{5}{7}$
- (vi) $\frac{31}{101}$ (vii) $\frac{19}{21}$ (viii) 5.442 (ix) 0.09 (x) 3.4 (xi) 89.6457
2. (a) 67 (b) 98 (c) 87 (d) 168 (e) 35 (f) 270
3. (a) $\frac{17}{5}$ (b) $\frac{40}{13}$ (c) $\frac{27}{19}$ (d) $\frac{14}{22}$ (e) $\frac{65}{74}$
4. (a) 0.33 (b) 0.05 (c) 9.6 (d) 10.6
5. (a) 2.236 (b) 1.414 (c) 2.645 (d) 3.316 (e) 1.760
6. (a) 1.79 (b) 7.52 (c) 6.81 (d) 3.36 (e) 2.40

Exercise 4.2

1. 320 2. 315m 3. 850m 4. Rs 37500 5. 110cm 6. Rs 44370 7. 396m

Exercise 4.3

1. (a) 8 (b) 12 (c) 19 (d) 25 (e) 28 (f) 32
- (g) 36 (h) 39 (i) 44 (j) $\frac{5}{16}$ (k) $\frac{13}{19}$ (l) $\frac{11}{20}$
- (m) $\frac{4}{9}$ (n) $\frac{3}{8}$ (o) 48 (p) 23 (q) 35 (r) 11
2. 19m 3. 42m 4. 21m
5. (a) 300763 (b) 1728 (c) 941192 (d) 125000 (e) 85184 (f) 389017
- Review Exercise
1. (a) ii (b) iii (c) ii (d) i (e) iv
- (f) i (g) iii (h) i (i) iv
2. (a) 29 (b) 35 (c) 42 (d) $\frac{50}{31}$ (e) $\frac{6}{19}$
- (f) 1.1840 (g) 1.11 (h) 0.26 (i) 2.53
3. (a) 77 (b) 64 (c) 52 (d) $\frac{21}{13}$ (e) $\frac{38}{9}$
- (f) 6.9 (g) 12.1

4. (a) 274625 (b) 456533 (c) 27000 (d) 5832 (e) 10648 (f) 970299
5. (a) 4 (b) 5 (c) 8 (d) 11 (e) 42 6. (a) 3 (b) 6 (c) 9 (d) 14 (e) 24
7. 41 8. 900m 9. 115
10. 312 by 312 11. 20 by 20 by 20 12. Rs 37,400
- Unit 5: Sets**

Unit 5: Sets

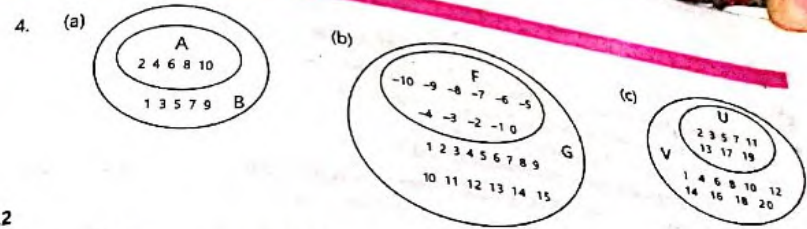
Exercise 5.1

- Exercise 3.1
1. (i) W=The set of whole numbers
Z=The set of +ve integers
E=The set of non-negative even numbers
O=The set of positive odd numbers
P=The set of prime numbers

- (ii) $W = \{0, 1, 2, 3, \dots\}$
 $Z = \{1, 2, 3, 4, \dots\}$
 $E = \{0, 2, 4, 6, \dots\}$
 $O = \{1, 3, 5, 7, \dots\}$
 $P = \{2, 3, 5, 7, 11, \dots\}$

2. i) $A = \{1, 2, 3, \dots, 9\}$
A is a set of first nine Natural numbers
- ii) $B = \{2, 3, 5, 7, 11, 13, 17, 19\}$
B is a set of Prime numbers less than 20
- iii) $C = \{4, 6, 8, 10, 12, 14\}$
C is a set of Even numbers between 2 and 15
- iv) $D = \{2, 4, 6, 8, 9, 10, 12, 14\}$
D is a set of Composite numbers between 1 and 15
- v) $E = \{0, 1, 2, \dots, 10\}$
E is a set of whole numbers less or equal to 10
- vi) $F = \{-10, -9, -8, \dots, -1, 0, 1, 2, \dots, 9\}$
F is a set of Integers between -11 and 10

3. (a) $E = \{x | x \in E\}$
 (c) $A = \{x | x \in W\}$
 (e) $D = \{x | x = n^2, n = 1, 2, 3, 4, 5\}$
 (g) $G = \{x | x \in O \wedge 1 \leq x \leq 9\}$
 (i) $E = \{x | x \text{ is a 30 days month}\}$
 (k) $\{x | x \text{ is a common divisor of } 3, 4\}$
 (m) $\{x | x \in P \wedge 11 < x < 13\}$
 (o) $\{x | x \text{ is a consonant}\}$
 (q) $\{x | x \text{ is a factor of } 24\}$
 (b) $T = \{x | x \in N \wedge 101 \leq x \leq 109\}$
 (d) $V = \{x | x \text{ is a set of vowels}\}$
 (f) $F = \{x | x \in N \wedge x \geq 11\}$
 (h) $B = \{x | x = 11n, n = 2, 3, 4, 5\}$
 (j) $\{x | x \text{ is a common factor of } 18, 30, 42\}$
 (l) $\{x | x \text{ is a factor of } 81 \text{ between } 5 \text{ and } 30\}$
 (n) $\{x | x \in E \wedge x \leq 100\}$
 (p) $\{x | x \in W \wedge 20 < x < 50\}$



Exercise 5.2

- Exercice 2

1. (i) T $\Phi, \{2\}$ (iv) T $\Phi, \{2\}$ (v) F

2. (i) Number of subsets = 2 $\Phi, \{2\}$

(ii) Number of subsets = 4 $\Phi, \{4\}, \{8\}, \{4, 8\}$

(iii) Number of subsets = 16 $\Phi, \{-1\}, \{0\}, \{1\}, \{2\}, \{-1, 0\}, \{-1, 1\}, \{-1, 2\}, \{0, 1\}, \{0, 2\}, \{1, 2\}, \{-1, 0, 1\}, \{-1, 0, 2\}, \{-1, 1, 2\}, \{0, 1, 2\}, \{-1, 0, 1, 2\}$

(iv) Number of subsets = 1 Φ

(v) Number of subsets = 8 $\Phi, \{5\}, \{10\}, \{15\}, \{5, 10\}, \{5, 15\}, \{10, 15\}, \{5, 10, 15\}$

(vi) Number of subsets = 32 $\Phi, \{2\}, \{3\}, \{5\}, \{7\}, \{11\}, \{2, 3\}, \{2, 5\}, \{2, 7\}, \{2, 11\}, \{3, 5\}, \{3, 7\}, \{3, 11\}, \{5, 7\}, \{5, 11\}, \{7, 11\}, \{2, 3, 5\}, \{2, 3, 7\}, \{2, 3, 11\}, \{2, 5, 7\}, \{2, 5, 11\}, \{2, 7, 11\}, \{3, 5, 7\}, \{3, 5, 11\}, \{3, 7, 11\}, \{5, 7, 11\}, \{2, 3, 5, 7\}, \{2, 3, 5, 11\}, \{2, 3, 7, 11\}, \{2, 5, 7, 11\}, \{3, 5, 7, 11\}, \{2, 3, 5, 7, 11\}$

(vii) Number of subsets = 16 $\Phi, \{5\}, \{7\}, \{11\}, \{13\}, \{5, 7\}, \{5, 11\}, \{5, 13\}, \{7, 11\}, \{7, 13\}, \{11, 13\}, \{5, 7, 11\}, \{5, 7, 13\}, \{5, 11, 13\}, \{7, 11, 13\}, \{5, 7, 11, 13\}$

(viii) Number of subsets = 8 $\Phi, \{-1\}, \{0\}, \{1\}, \{-1, 0\}, \{-1, 1\}, \{0, 1\}, \{-1, 0, 1\}$

3. (i) $P(X) = \{\Phi, \{x\}, \{y\}, \{z\}, \{x, y\}, \{x, z\}, \{y, z\}, \{x, y, z\}\}$

(ii) $P(A) = \{\Phi, \{+\}, \{-\}, \{\times\}, \{\div\}, \{+, -\}, \{+, \times\}, \{+, \div\}, \{-, \times\}, \{-, \div\}, \{\times, \div\}, \{+, -, \times\}, \{+, -, \div\}, \{+, \times, \div\}, \{-, \times, \div\}, \{+, -, \times, \div\}\}$

(iii) $P(I) = \{\Phi, \{i\}, \{s\}, \{f\}, \{a\}, \{m\}, \{i, s\}, \{i, a\}, \{i, f\}, \{i, m\}, \{s, a\}, \{s, f\}, \{s, m\}, \{a, f\}, \{a, m\}, \{f, m\}, \{i, s, a\}, \{i, s, f\}, \{i, s, m\}, \{i, a, f\}, \{i, a, m\}, \{i, f, m\}, \{s, a, f\}, \{s, f, m\}, \{s, a, m\}, \{a, f, m\}, \{i, s, f, a\}, \{i, s, f, m\}, \{i, s, a, m\}, \{i, f, a, m\}, \{s, f, a, m\}, \{i, s, a, f, m\}\}$

(iv) $P(B) = \{\Phi, \{0\}\}$

(v) $P(T) = \{\Phi, \{-3\}, \{-2\}, \{-3, -2\}\}$

Exercise 5.3

- $$\begin{aligned} A' &= \{-10, -9, -8, \dots, -1, +6, +7, +8, +9, +10\} \\ B' &= \{-9, -7, -5, -3, -1, 0, +1, +3, +5, +7, +9\} \\ X' &= \{-10, -9, -8, \dots, 0\} \\ Y' &= \{-10, -9, -8, -7, -6, +6, +7, +8, +9, +10\} \\ M' &= \{-10, -9, -8, \dots, -2, +2, +3, \dots, +10\} \\ T' &= \phi \end{aligned}$$

Review Exercise

- (a) (i) (b) (i) (c) (ii) (d) (i) (e) (iv)
(f) (i) (g) (iv) (h) (i) (i) (ii)

3. (a) $\{x \in E \wedge 0 < x < 26\}$
 4. (a) Set of Whole numbers less than 20
 (b) Set of Prime numbers greater than 5
 (c) Set of multiples of 4 between 0 and 24
 (d) Set of Composite numbers between 12 and 25
 5. (a) $\{18, 27, \dots, 99\}$ (b) $\{-5, -4, -3, \dots, 20\}$ (c) $\{1, 3, 5, \dots, 29\}$ (d) $\{0, 2, 4, 6, \dots\}$
 6. (a) $\{\phi, \{1\}, \{2\}, \{3\}, \{1, 2\}, \{1, 3\}, \{2, 3\}, \{1, 2, 3\}\}$ (c) $\{\phi, \{0\}\}$ (d) $\{\phi, \{12\}, \{13\}, \{12, 13\}\}$
 (b) $\{\phi, \{5\}, \{8\}, \{9\}, \{11\}, \{5, 8\}, \{5, 9\}, \{5, 11\}, \{8, 9\}, \{8, 11\}, \{9, 11\}, \{5, 8, 9\}, \{5, 8, 11\}, \{5, 9, 11\}, \{8, 9, 11\}, \{5, 8, 9, 11\}\}$

Unit 6: Sequences and Algebra

Exercise 6.1

- Add 3 to get the next term
14, 17, 20
 - Add 3 to get the next term
13, 16, 19
 - Add 4 to get the next term
20, 24, 28
 - Multiply by 2 to get the next term
32, 64, 128
 - Multiply by 3 to get the next term
243, 729, 2187
 - Multiply by 3 to get the next term
162, 486, 1458
 - Subtract 4 to get the next term
48, 44, 40
 - Divide by 3 to get the next term
27, 9, 3
 - Multiply by 6 to get the next term
42, 48, 54
 - Multiply by 2 to get the next term
400, 800, 1600
- 2, 5, 8, 11, ... (Arithmetic Sequence)
 - 3, 9, 27, 81, ... (Geometric Sequence)
 - 80, 72, 64, 56, ... (Arithmetic Sequence)
- $a_n = 59$ (Arithmetic Sequence)
- $a_n = 72$ (Add 11 to get the next term)
- 3, 9, 16, 25, 36, 49, 64, ... $a_n = 64$
 - 6, 12, 20, 30, 42, 56, 72, ... $a_n = 72$

- Add 3 to get the next term (Arithmetic Sequence)
 - Add 4 to get the next term (Arithmetic Sequence)
 - Add 3, 5, 7, 9, ... to get next consecutive terms
 - Add 3, 5, 7, 9, ... to get next consecutive terms
 - Add 3, 5, 7, 9, ... to get next consecutive terms
 - Not a sequence
 - Add 2, 4, 6, 8, ... to get next consecutive terms
 - Divide by 2 to get the next term (Geometric Sequence)
 - Not a sequence
 - Double the number to get the next term
- 11, 14
 - 28
 - 9, 81
 - 36
 - 32
- $a_n = 26$
- $a_{11} = 110$
- $a_n = 37, a_{11} = 53, a_{20} = 81$

Exercise 6.2

- $d = 4$
- $a_n = 4n - 2, a_1 = 2$
 - $a_n = 6n - 1, a_1 = 47, a_{10} = 59$
 - $a_n = 12n + 6, a_{11} = 138, a_{14} = 174$
 - $a_n = 0.5n + 10.5, a_{24} = 22.5$
 - $a_n = 16 - 9n, a_{14} = -200$
 - $a_n = 10n - 9, a_1 = 81, a_{17} = 161$
- $d = 5$
- $a_{17} = 114$
- $a_{18} = 28$
- $a_{11} = 65,000$
- 120 m
- $a_n = 19,200$

Exercise 6.3

- Variable: x ; Constant: 3
 - Variable: x, y, z ; Constant: 0
 - Variable: p ; Constant: -3
- ii, iii, vi
 - i, iv
 - v
 - vii, viii
- Linear Polynomials: i, viii, xii
 - Quadratic Polynomials: ii, iii, iv, ix, xiii
 - Cubic polynomials: v, xiv, xv
 - Bi Quadratic Polynomials: vi, vii, x, xvi
- ii
- iii, iv, ix
- xiv
- xii

Exercise 6.4

- $2x^2 + y^2 + 2xy$
 - $5p^3 + 2p^2q$
 - $5x^5 - 7x^4 + 3x^3 - 6x^2 + 5$
 - $21x^3 + 18x^2y - 10xy^2$
 - $20x^5 + 5x^4 - 8x^3 + 6x - 10$
 - $9x^3 + 8x^2y - 4y^3$
 - $15p^4 + 2p^3q - 5p^2q^2 + q^3$
 - 25u
- $8x^2 + 18y^2 + 7xy$
 - $7pq + 11qr - 11rs - 28$
 - $61x^3 - 5x^2 + 38x - 94$

Exercise 6.5

1. (i) $20a-22b+10c$
(iv) $25x^3-9x^4-20x^2+12x+8x+23$

- (ii) $3a^3-2b^3$
(v) $27x^3-20x^4y^3-13x^3y^2+31y^5$

- (iii) p^3+10p^3+30

Exercise 6.6

1. (i) $15x^3+4xy+6y$
(iii) $49f^2-25g^2$
(v) $p^4+p^2q^2-p^2r^2+2p^3+2pq^2-2pr^2$

- (ii) $10x^2-15x^2y+21xy^2+6xy^2-10y^2$
(iv) $10p^2-13pr+15pq-12qr+4r^2$
(vi) $x^4-6x^3+16x^2-24x+16$

- (ii) $3x^5-3xy^4$

- (iii) $24abc-40bc^2-2a^2b$

2. (i) x^3+2x^2-35x
(iv) $p^3-2p^2q+q^4$

- (ii) $11x^3+8x^2z-4x^2+28xy-15xz$
(v) $-2x^2+104xy+92y^2$

- (iii) $-9p^2+120p-46$
(vi) $-3x^2-24xy+4y^2$

3. (i) $3a^2+7a-18$
(iv) $5x^3-37x^2+55x-175$

Exercise 6.7

1. (i) $2x^2-4x+5$
(vi) x^3+xy+y^2

- (ii) $3a+5b$
(vii) $x-y$

- (iii) $2p-3q$
(viii) $2x+5$

- (iv) $x-y$
(ix) y^2+1

- (v) $x+y$

2. (i) $x-3$ (ii) $3x+5y$

- (iii) a^2-ab+b^2

- (iv) $4x^2-x$ (v) $p+q$

- (iii) $x^4+x^3y+x^2y^2+xy^3+y^4$

3. (i) $x+3$
(iv) $p^3+pq^2-p^2q+q^3$

- (ii) $2r+5s$
(v) $x^2-xy+4y^2$

Exercise 6.8

1. (i) $2b^2+12ab$
(iii) $4c+8cd^2+8cd$
(v) $10xy-6x^2y-8xy^2-6x^4$
(vii) $24a^2-4ab^4-4ab^3+4b^2c$

- (ii) $-x+2y-2xy-4x^2y^2-10x^2y$
(iv) $5m-10m^2-9mn-2n^2-2$
(vi) $5e^2+5f-4ef$
(viii) $4y-4y^2z-yz^3+40y^2z^2+16y^3z$

Exercise 6.9

1. (i) $x^2+9x+20$
(vi) $5m^2+6m+5$

- (ii) y^2-3y+2
(vii) $16z^2-5z+4$

- (iii) $a^2+13a+42$
(viii) $9n^2+9n+18$

- (iv) $b^2-2b-35$
(ix) $49t^2+8t-9$

- (v) $4a^2-a-20$

2. (a) 1156
(e) 41616

- (b) 10609
(f) 1010025

- (c) 14641
(g) 26896

- (d) 6084
(h) 91204

3. 70

4. 6

5. (a) 2

- (b) B

- (c) $25p^2$

- (d) $16b^2$

- (e) $9x$

6. (a) $4a^2+\frac{20}{7}a+\frac{25}{49}$

- (b) $9a^2+24ab+16b^2$

- (c) $t^2-\frac{2}{x^2}t^2+\frac{1}{x^4}$

- (d) $\frac{9}{25}x^2-\frac{4}{5}xy+\frac{4}{9}y^2$

- (e) $p^4-8p^3q+16q^2$

- (f) $x^2+\frac{1}{x^2}+2$

- (g) $x^2+\frac{1}{x^2}-2$

- (h) $x^4-\frac{2}{t^2}x^2+\frac{1}{t^4}$

- (i) $\frac{x^2}{4y^2}+\frac{4y^2}{5x^2}-2y$

- (j) $(a-7)(a+7)$

- (k) $(5x-3y)(5x+3y)$

- (l) $(1-5t)(1+5t)$

- (m) $4x+34$

- (n) $(9p-7q)(9p+7q)$

7. 7

8. 2207

9. 102

10. 322

12. 806402

13. (a) 9604
(f) 1452025
(k) 89375

- (b) 0.9604
(g) 10,240,000
(l) 999900

- (c) 225
(h) 15,210,000

- (d) 2.25
(i) 1.21

- (e) 0.0025
(j) 9975

Exercise 6.10

1. (a) $2(a+3)$
(f) $3(b^2+2b-5)$
(k) $(a+3)(a+b)$
(p) $(a-5b)(4y-z)$

- (b) $5(x-2)$
(g) $5(ax+2ay-5z)$
(l) $(a^2+b^2)(4+c^2)$
(q) $(r+s)(p+q)$

- (c) $x(x-1)$
(h) $7p(p+1)(p-3)$
(m) $(x+y)(3a-14)$
(r) $(u-v)(s-t)$

- (d) $3x^2(x+4)$
(i) $5pq(2p+5q-7r)$
(n) $5(4x-2xy-3yz)$
(s) $(3x-2y)(2x-3y)$
(t) $(3a-4b)(5x^2+7y)$

Exercise 6.11

1. (a) $(a+b)^2$
(e) $(3a-5b)^2$
(i) $\left(\frac{5t+2v}{5}\right)^2$
(m) $(3t-7q)(3t+7q)$
(p) $(a+5+c)(a+5-c)$

- (b) $(2a+5b)^2$
(f) $(5p+7q)^2$
(j) $(4x-7y)^2$
(n) $\left(\frac{3}{4}u-\frac{5}{7}v\right)\left(\frac{3}{4}u+\frac{5}{7}v\right)$
(q) $(3b+7-d)(3b+7+d)$

- (c) $(3x+2y)^2$
(g) $(7p-9q)^2$
(k) $(a-5)(a+5)$
(o) $(15-16y)(15x+16y)$
(r) $(5p-7q-5r)(5p-7q+5r)$

- (d) $(2a-3)^2$
(h) $(7t+11v)^2$
(l) $(2b-9)(2b+9)$

Exercise 6.12

1. (a) $(2a+3b)(4a^2-6ab+9b^2)$ (b) $(x+5y)(x^2-5xy+25y^2)$ (c) $(a-5b)(a^2+5ab+25b^2)$
(d) $(3p+4q)(9p^2-12pq+16q^2)$ (e) $(5p-7q)(25p^2+35pq+49q^2)$ (f) $(2t-3u)(4t^2+6tu+9u^2)$
(g) $\left(x+\frac{1}{x}\right)\left(x^2-1+\frac{1}{x^2}\right)$ (h) $\left(x-\frac{1}{x}\right)\left(x^2+1+\frac{1}{x^2}\right)$ (i) $\left(4p-\frac{1}{x}\right)\left(16p^2+p+\frac{1}{16}\right)$ (j) $\left(5x+\frac{1}{5x}\right)\left(25x^2-1\right)$
(k) 52 (l) 322 (m) 36 (n) 140 (o) 488 (p) 970

Review Exercise

1. (a) iv (b) ii (c) ii (d) iii (e) i (f) ii (g) iii (h) i (i) i
2. 229, 299 3. Rs 8500, Rs 11,300 4. 180 minutes, 285 minutes.
5. Rs 7500, Rs 10,500 6. 211 7. $7x+11y+17z$ 8. $8x^4-3y^3+8x^2+10x-3$
9. $x^5+2x^3y+x^2yz+2y^2z+2x^2y^2+4y^3$
10 a) $5v-4vu^2-20v^2u^2w-40v^3w-10vu^2w^2-20v^2w^2$ b) $6y-21xy^3+15x^4y+18xy^2+9x^3y^2$
11. $2x+1+\frac{y}{x}$ 12. a) y^2-25 b) $16t^2+25u^2+40tu$ c) 10404 13. 14
14. a) $(3x+4y)^2$ b) $3(s+2t)$ c) $(3a+b)^2$ 15. a) 198 b) 1692
16. $a_n = 7 + (n-1)9$ $a_{11} = 97$ $a_{12} = 108$

Unit 7: Exponents and Laws of Exponents

Exercise 7.1

1. (a) Exponent: 2 Base: 7 Value = 49
(b) Exponent: 3 Base: $\frac{1}{2}$ Value = $\frac{1}{8}$
(c) Exponent: 2 Base: 5 Value = 25
(d) Exponent: 4 Base: 3 Value = 81
(e) Exponent: 9 Base: 1 Value = 1

- (f) Exponent: 2
(g) Exponent: 8
(h) Exponent: 2
(i) Exponent: 3
(j) Exponent: 2
(k) Exponent: 2
(l) Exponent: 5

Base: 8
Base: $\frac{7}{13}$
Base: 12
Base: $\frac{1}{3}$
Base: $\frac{22}{133}$
Base: 144
Base: $\frac{3}{4}$

Value = 64
Value = $\frac{5764801}{815730721}$
Value = 144
Value = $\frac{1}{27}$
Value = $\frac{484}{17689}$
Value = 20736
Value = $\frac{243}{1024}$

2. (a) 4^5 (b) n^5 (c) $(ab)^5$ (d) e^5 (e) $(-\frac{1}{2})^5$ (f) $(\frac{1}{9})^5$ (g) $(\frac{a}{b})^5$ (h) $(-9)^5$ (i) $(-\frac{1}{10})^5$
4. (a) $(14)^5$ (b) $(2)^5$ (c) $(3)^5$ (d) $(\frac{1}{10})^5$ (e) $(\frac{2}{7})^5$ (f) $(\frac{1}{2})^5$ (g) $(3)^5$ (h) $(\frac{2}{3})^5$

Exercise 7.2

1. (a) $(-8)^5$ (b) $(x)^5$ (c) $(20)^5$ (d) $(18)^5$ (e) $(15)^5$ (f) x^5, y^5, z^5
(g) $(4)^5$ (h) $(\frac{5}{8})^5$ (i) $(\frac{5}{6})^5$ (j) $(3)^5$ (k) $2^5 \times 4^5 \times 5^5$ (l) $(\frac{2}{5})^5$
2. (a) $(3)^5$ (b) $(-7)^5$ (c) $(5)^5$ (d) $(-\frac{4}{10})^5$ (e) $(\frac{3}{7})^{50}$ (f) $(-\frac{1}{12})^5$
(g) $(3)^5$ (h) $(\frac{21}{37})^5 = 1$ (i) $(x)^{50}$ (j) $(-\frac{4}{5})^5 = 1$
3. (a) $(-n)^{18}$ (b) $(-3)^5$ (c) $(\frac{3}{7})^{18}$ (d) $(6)^{12}$ (e) $(-4)^{14}$ (f) $(\frac{4}{6})^5$
(g) $(-a)^{18}$ (h) $(a)^5$ (i) $(\frac{a}{b})^{18}$
4. (a) $(\frac{a}{b})^5$ (b) 6561 (c) 64 (d) $\frac{320}{3}$ (e) $\frac{1}{6561}$ (f) 4096
(g) 1679616 (h) 30 (i) $x^5 y^{16}$ (j) 2 (k) -1024 (l) $\frac{6561}{65536}$
(m) 5764801 (n) 131.8359 (o) $\frac{98}{15}$ (p) 268435456 (q) 282475249

Exercise 7.3

1. (a) ab (b) $\frac{11a^5b}{c}$ (c) $\frac{1}{6a^5}$ (d) $\frac{5a}{3}$ (e) $\frac{x^5bc^2}{2}$ (f) $\frac{432}{x^5y^3}$ (g) $\frac{b^5}{ax^2}$
3. (a) 6 (b) $\frac{1}{1536}$ (c) $\frac{4}{9}$ (d) $\frac{1}{729}$ (e) 135 (f) -165888 (g) 192
(h) $\frac{1^5}{y^5z^5}$ (i) $\frac{1}{896}$ (j) 0 (k) 1 5. (a) $\frac{1}{(-9)^5 \times (c-4)^5}$ (b) $(\frac{3}{2})^5$ (c) $x^5 \times y^5$
(d) $\frac{y^5}{z^5}$ (e) $\frac{1}{4^5 \times 5^5}$ (f) $\frac{1}{(-6rs)^5}$

Review Exercise

1. (a) i (b) iii (c) iv (d) ii (e) i (f) ii (g) iii 3. (a) 64 (b) 78125
(c) 49 (d) 1 (e) $\frac{16}{81}$ (f) $\frac{1}{243}$ 4. (a) 7^5 (b) $\frac{x^5}{y^5}$ (c) $16b^7$ (d) $\frac{16x^2}{9y^5}$
5. (a) 16 (b) $\frac{4}{7}$ (c) $(\frac{2}{5})^5$ (d) $(\frac{1}{100})^3$ (e) $(\frac{1}{9})^{39}$ (f) $(\frac{4}{6})^4$

NOT FOR SALE

Unit 8: Simultaneous Linear Equations

Exercise 8.1

1. $x-3y = 0$; $2x+y = 3960$
2. $x-y = -3$; $x+y = 11$
3. $x+y = 82$; $x-y = 23$

Exercise 8.2

1. (a) Not a pair of simultaneous equations
(c) A pair of simultaneous equations
(e) Not a pair of simultaneous equations
2. (a) $a = 5, b = 2$ (b) $m = 7, n = 3$ (c) $p = 9, q = 5$ (d) $x = 15, y = 10$ (e) $x = 4, y = 7$
(f) $x = 10, y = 15$ (g) $p = 2, q = -3$ (h) $a = 5, b = -1$ (i) $x = 12, y = 5$

Exercise 8.3

1. (a) $a = 7, b = 2$ (b) $a = 5, b = -4$ (c) $x = 5, y = 2$ (d) $x = 10, y = 6$ (e) $x = -3, y = 7$
(f) $p = -5, q = -7$ (g) $u = 8, v = 12$ (h) $x = 1, y = -1$
2. (a) $p = \frac{372}{275}, q = -\frac{148}{275}$ (b) $l = 8, m = 7$ (c) $u = -3, v = -5$ (d) $y = 11, z = 8$ (e) $x = 10, y = 15$
(f) $x = 6, y = 14$ (g) $x = 5, y = 9$ (h) $x = -10, y = 3$

Exercise 8.5

1. (a) $x = 3, y = 0$ (b) Solution does not exist (c) $x = 2, y = 0$ (d) $x = 7, y = -\frac{3}{2}$
(e) Solution does not exist (f) $x = 0, y = 2$
2. (a) $x = 6, y = -2$ (b) $x = \frac{31}{29}, y = -\frac{7}{29}$ (c) $x = \frac{27}{19}, y = -\frac{2}{19}$ (d) $x = \frac{9}{4}, y = -\frac{1}{2}$

Exercise 8.6

1. First number: 9, Second number: 6
2. First number: 20, Second number: 7
3. Price of a pencil: Rs 10, Price of a sharpener: Rs 6
4. Price of a chair: Rs 10, Price of a table: Rs 6
5. Price of Potatoes/kg: Rs 150, Price of Carrots/kg: Rs 50
6. Present age of Father: 40 years Present age of son: 10 years

Exercise 8.7

1. (a) $x < -\frac{10}{3}$ (b) $x < -2$ (c) $x > \frac{7}{3}$ (d) $x < -\frac{1}{7}$ (e) $x > 2$ (f) $x > 3$ (g) $x < 2$ (h) $x < -3$

Exercise 8.9

4. (a) $m = 1, c = -5$ (b) $m = 2, c = 4$ (c) $m = -4, c = 2$ (d) $m = 3, c = 3$ (e) $m = -1, c = 2$ (f) $m = -2, c = 3$
5. (a) $y = 2x - 1$ (b) $y = -2x + 2$ (c) $y = 2x - 1$ (d) $y = -\frac{5}{2}x$ (e) $y = \frac{1}{2}x$ (f) $y = 3x$ (g) $y = 3$

Review Exercise

1. (a) ii (b) iii (c) iv (d) iii (e) ii (f) i (g) ii (h) i (i) ii
(j) incomplete (k) iii (l) ii (m) i (n) iv

NOT FOR SALE

2. (a) $x = -\frac{17}{2}, y = \frac{11}{2}$ (b) $x = -\frac{25}{22}, y = -\frac{59}{22}$ (c) $x = \frac{27}{23}, y = \frac{87}{46}$ (d) $x = 2, y = -1$
 3. (a) $x = \frac{43}{15}, y = -\frac{1}{5}$ (b) $x = \frac{3}{11}, y = -\frac{92}{11}$ (c) $x = -\frac{7}{3}, y = \frac{17}{15}$ (d) $x = 3, y = -1$
 4. (a) $x = \frac{49}{22}, y = \frac{37}{22}$ (b) $x = -\frac{1}{3}, y = \frac{7}{3}$ (c) $x = -\frac{26}{31}, y = -\frac{135}{31}$
 5. Hina's weight: 45 kg
 Ariba's weight: 23 kg
 6. cost of potatoes = 31.44 per kg
 cost of tomatoes = 14.29 per kg
 7. Rabia's age: 31.5 years
 Daughter's age: 6.5 years
 8. (a) $x > \frac{2}{5}$ (b) $x < \frac{1}{2}$ (c) $x > -\frac{5}{4}$ (d) $x = 16$ 9. $x > \frac{7}{3}$ 10. $y = 8$

Unit 9: Surface, Area and Volume

Exercise 9.1

1. (i) $x = 6$ cm (ii) $x = 15$ cm (iii) $x = 13$ cm 2. (i) $x = 20$ cm (ii) $x = 10\sqrt{3}$ cm
 (iii) $x = 10\sqrt{7}$ cm 3. (i) $xy = 5\sqrt{5}$ cm (ii) $xz = 8\sqrt{5}$ cm (iii) $yz = \frac{5\sqrt{3}}{2}$ cm
 4. (i) $b = 15$ cm (ii) $c = \sqrt{527}$ cm (iii) $b = 2.5$ cm (iv) $a = \frac{9}{25}$ cm 5. $\sqrt{161}$ m
 6. 2.408 m 7. 28 m 8. 6 m 9. 4 cm 10. $6\sqrt{2}$ m 11. 264.575 m
 12. $5\sqrt{13}$ km 13. $6\sqrt{5}$ cm

Exercise 9.2

1. (i) 60.5 m ; 332.75 m² (ii) 10.48 cm ; 52.38 cm² (iii) 7.857 cm ; 11.79 cm²
 (iv) 44 m ; 264 m² (v) 22.35 cm ; 89.4 cm² (vi) 13.1 cm ; 32.74 cm²
 (vii) 23.05 cm ; 126.76 cm² (viii) 4.19 m ; 6.286 m² (ix) 8.73 m ; 17.46 m²
 2. (i) 4.19 cm ; 6.286 cm² (ii) 25.14 m ; 150.86 m² (iii) 9.603 cm ; 48.02 cm²
 (iv) 4.19 cm ; 6.286 cm² (v) 25.14 m ; 150.86 m² (vi) 9.603 cm ; 48.02 cm²
 (vii) 38.41 cm ; 384.13 cm² (viii) 31.43 m ; 235.71 m² (ix) 17.33 cm ; 2737.78 cm²
 (x) 55 cm ; 577.5 cm² (xi) 7.857 m ; 23.57 m² (xii) 67.57 cm ; 1452.79 cm²
 3. 5657.14 m² 4. 201.14 m² 5. 104.76 m²
 6. (i) 17957.93 m² (ii) 7922.62 m² (iii) 12148.02 m²

Exercise 9.3

1. (i) 221.76 cm² ; 310.464 cm³ (ii) 49896.62 cm² 1047816.464 cm³
 (iii) 887.04 m² ; 2483.712 m³
 2. 78.5 m² ; 65.42 m³ 3. (i) Rs. 2587.2 (ii) 38.808 m³
 4. (i) Rs. 78571.43 (ii) 2095240 liters 5. Rs. 8825.14

Exercise 9.4

1. (i) 2093 cm² (ii) 2619 cm² (iii) 3904 m² (iv) 2400 m² (v) 1656 m² (vi) 4700 cm²
 2. (i) 119 cm² (ii) 174.25 cm² (iii) 72 m² (iv) 261 cm² (v) 72 cm² (vi) 825 cm²
 3. (i) 6174 m³ (ii) 4224 m³ (iii) 795 cm² (iv) 1621.33 cm² (v) 552 m² (vi) 1275 m²
 4. 4840 cm² 5. (i) Rs. 3334500 (ii) 12000 m² 6. (i) 48 m² (ii) 26.67 m²
 7. (i) 1320 cm² (ii) 3066.67 cm² (iii) 501600

Exercise 9.5

1. (i) 125.6 cm² ; 204.1 cm² (ii) 514.96 cm² ; 726.09 cm²
 (iii) 37.68 cm² ; 65.94 cm² (iv) 150.72 cm² ; 263.76 cm²
 2. (i) 113.825 cm² ; 78.5 cm² (ii) 117.56 cm² ; 91.10 cm²
 (iii) 16.68 cm² ; 4.90 cm² (iv) 282.6 cm² ; 340.17 cm²
 3. Rs. 4910.714 4. 141.74 cm² 5. (i) 301.714 cm² (ii) Rs. 27.15
 6. (i) 188.57 cm² (ii) 267.14 cm² (iii) 285.70 cm²

Review Exercise

1. (a) iv (b) i (c) iv (d) ii (e) ii
 (f) i (g) iv (h) iv (i) iv (j) i
 2. (i) $2\sqrt{65}$ cm (ii) 9.54 cm (iii) 21 cm
 3. $15\sqrt{5}$ cm 4. $4\sqrt{41}$ cm 5. 75.43 cm² 6. 176 cm² 7. 4.365 cm
 8. (i) 495 m² ; 648.79 m³ (ii) 175.05 cm² ; 130.67 m³
 9. (i) 314 m² ; 523.33 m³ (ii) 4069.44 m² ; 24416064 m³ (iii) 1256 m² ; 4186.67 m³
 10. (i) 113.04 cm² ; 50.24 cm² (ii) 113.04 cm² ; 50.24 cm² (iii) 2813.44 m² ; 9847.04 m³
 11. (i) 226.08 cm² ; 452.57 cm³ (ii) 904.32 m² ; 3620.57 cm³ 12. (i) Rs. 20790 (ii) 180 m³
 13. (i) 175.53 cm² (ii) 254.03 cm² (iii) 261.9 cm³

Unit 10: Geometry

Exercise 10.1

3. (a) (-9, -2) (b) (-2, 9) (c) (9, 2) 4. (a) (3, -5) (b) (5, 3) (c) (-3, 5)

Exercise 10.3

1. (i) D, E (ii) F, G (iii) A, B, C 2. Secants: v, y, z Tangents: u, x

Exercise 10.9

1. Congruent Figures: (b), (d), (f), (j) Similar Figures: (a), (c), (e), (g), (h)

2. (a) $\triangle ABC \sim \triangle DFE$ (b) $\triangle ABC \sim \triangle XZY$

Exercise 10.10

1. (a) $\triangle ABC \cong \triangle ADC$ (b) $\triangle ABC \cong \triangle DEF$ (c) $\triangle MJK$ not congruent to $\triangle LKJ$ (d) $\triangle GHI \cong \triangle$
 2. (a) $x = 20^\circ, y = 12$ cm (b) $m\angle TU = 8$ cm, $m\angle VW = 7$ cm
 3. (a) $OP \cong OT, OR \cong OS, RP \cong TS$ (b) $\triangle POR \cong \triangle TOS$ 5. $x = -50, y = 28$

Review Exercise

1. (a). iii (b). ii (c). i (d). ii (e). i
(f). iii (g). iii (h). i (i). i (j). iv
- 16 $\triangle ABC \cong \triangle DEF$

Unit 11: Data Management and Probability

Exercise 11.1

3. a) Discrete b) Discrete c) Discrete d) Continuous e) Continuous
f) Discrete g) Continuous h) Discrete i) Continuous

Exercise 11.2

1. a) $R=72$, $S^2=628.81$, $S=25.076$ b) $R=195$, $S^2=4018.49$, $S=63.39$
c) $R=19$, $S^2=30.67$, $S=5.38$ d) $R=60$, $S^2=448.57$, $S=21.179$
e) $R=59$, $S^2=35.53$, $S=18.31$ f) $R=12$, $S^2=17.14$, $S=4.14$
2. a) $R=62$ b) $S^2=300.066$ c) $S=17.32$ 3. a) $R=26$ b) $S^2=67.41$ c) $S=8.21$
4. $R=46$, $S=14.44$ 5. $R=125$, $S^2=1737.98$ 6. $R=64200$, $S^2=680145468.04$

Exercise 11.4

1. a) $P(A) = \frac{3}{8}$, $P(B) = \frac{2}{8}$ b) $P(A \cup B) = \frac{5}{8}$ c) $P(A \cap B) = 0$ 2. $1/3$ 3. $1/3$

4. Independent Events, $P(A \cap B) = \frac{21}{17}$

$$5. a) S = \left\{ \begin{array}{l} (1,1), (1,2), (1,3), (1,4), (1,5), (1,6), \\ (2,1), (2,2), (2,3), (2,4), (2,5), (2,6), \\ (3,1), (3,2), (3,3), (3,4), (3,5), (3,6), \\ (4,1), (4,2), (4,3), (4,4), (4,5), (4,6), \\ (5,1), (5,2), (5,3), (5,4), (5,5), (5,6), \\ (6,1), (6,2), (6,3), (6,4), (6,5), (6,6) \end{array} \right\}$$

$$b) A = \{(1,2), (2,1)\} B = \{(1,1), (3,3), (5,5)\}$$

$$P(A) = \frac{2}{36}, P(B) = \frac{3}{36}$$

$$c) P(A \cup B) = \frac{5}{36} \quad d) P(A \cap B) = 0 \quad 6. \frac{15}{16}$$

$$7. a) P(A) = \frac{2}{6}, P(B) = \frac{3}{6} \quad (0 \text{ is considered as an even number})$$

$$b) P(A \cup B) = \frac{5}{6} \quad c) P(A \cap B) = 1$$

Exercise 11.5

$$2. a) \text{ Theoretical Probability} = \frac{1}{2} \\ \text{Experimental Probability} = \frac{14}{25}$$

$$b) \text{ Theoretical Probability} = \frac{1}{2} \\ \text{Experimental Probability} = \frac{11}{25}$$

$$3. a) \text{ Theoretical Probability} = \frac{1}{6} \\ \text{Experimental Probability} = \frac{1}{10}$$

$$b) \text{ Theoretical Probability} = \frac{1}{2} \\ \text{Experimental Probability} = \frac{19}{40}$$

$$c) \text{ Theoretical Probability} = \frac{1}{6} \\ \text{Experimental Probability} = \frac{1}{5}$$

$$d) \text{ Theoretical Probability} = \frac{5}{6} \\ \text{Experimental Probability} = \frac{9}{10}$$

$$4. a) \text{ Experimental Probability} = \frac{1}{4} \\ \text{Theoretical Probability} = \frac{1}{6}$$

$$c) \text{ Experimental Probability} = \frac{7}{10} \\ \text{Theoretical Probability} = \frac{1}{18}$$

Review Exercise

1. a) iv b) iv c) iii d) ii e) iii f) i g) i h) iv i) iii j) iv
3. 2.91 4. 43.93 5. 89.9 6. $R=22$, $S=6.67$, $S^2=44.44$ 11. 1 12. $\frac{22}{3}$ 13. $\frac{4}{9}$ 14. $\frac{1}{3}$

$$15. \text{Independent Events, } \frac{1}{18}$$

$$16. a) \text{ Experimental Probability} = \frac{7}{30} \\ \text{Theoretical Probability} = \frac{1}{6}$$

$$b) \text{ Experimental Probability} = \frac{13}{30} \\ \text{Theoretical Probability} = \frac{1}{2}$$

$$c) \text{ Experimental Probability} = \frac{13}{30} \\ \text{Theoretical Probability} = \frac{1}{2}$$

$$d) \text{ Experimental Probability} = \frac{5}{6} \\ \text{Theoretical Probability} = \frac{2}{3}$$



Glossary

A

Absolute Value	The distance from origin and to that real number
Algebraic Expression	A combination of variables, constants, coefficients, exponents and symbols of operations
Altitude	The perpendicular distance from the base to the vertex
Angle bisector	A line segment from the vertex to the opposite side such that it bisects the angle at the vertex
Apex	common vertex of the triangular faces of pyramid
Approximation Error	The difference in the value between an exact value and the approximated value of something
Arc of circle	A part or portion of the circumference of a circle
Arithmetic sequence	A sequence of numbers in which each new term is obtained by adding or subtracting a fixed number in the previous term

B

Base	The number that is multiplied by itself
Binomial	Polynomial which contains two terms
Biquadratic polynomial	A polynomial with degree four

C

Cash price	The full payment that a customer has to pay to purchase an article
Chord	A line segment with end points on the circle
Compound proportion	The relationship between two or more proportions
Concentric circles	Two or more circles with same centre but different radii
Concyclic points	Points lying on the circumference of the circle
Congruent figures	The objects or figures that have same shape and size
Continuous data	A data that can be measured
Cost price	The price of goods or items at which it is purchased by a shopkeeper or wholesaler from the manufacturer
Creditor	A person who lends money
Cube root	A number whose cube is that number
Cube	A number is that number raised to the power 3.
Cubic polynomials	A polynomial with degree three

D

Data collection	The process of collecting information
Debtor	A person who borrows the money
Depreciation	The decreasing rate of vehicle or property
Discount	Reduction in the marked price of an item

Discrete data
Down payment

A quantitative data that is countable
The amount a customer has to pay at the beginning of an installment plan

E

Enlargement	A type of transformation where the shape of object remains the same but size increases
Equation	a mathematical statement that shows the equality of two expressions
Experimental Probability	To estimate the probability of a simple event by carrying out experiment
Exponent	The number above to the right of base
Exterior of a circle	The points which lie outside the circle

F

Factorization	The process of finding factors of an algebraic expression
Fibonacci sequence	A sequence in which each new term is obtained by adding the two previous terms together
frequency distribution	Representation the values and their respective frequencies in a table.
Frequency polygon	A multisided closed figure that is constructed by plotting class frequencies against their corresponding class marks
Frequency	A measure of the number of occurrences of a repeating event.

G

Geometric Sequence	A sequence of numbers in which each term is obtained by multiplying or dividing the preceding term by a fixed number
Gradient	measure of steepness
Hemisphere	one parts out of two equal parts of the sphere
Histogram	Representation of data where the class intervals (or boundaries) are taken along x-axis and the corresponding frequencies are on the y-axis

I

Inheritance	The act of giving any property or share upon the death of an individual
Insurance	To protect the financial well-being of an individual or a company in case of unexpected loss
Intercept	The point where a graph of an equation intersects the coordinate axes
Interior of a circle	The points lying inside the circumference of a circle
Irrational numbers	The numbers that are not in the form of p/q
Kite	A quadrilateral with two pairs of equal adjacent sides with unequal diagonals

L

Lateral faces	The faces other than base
Life insurance	a form of insurance that pays for medical expenses
Linear polynomial	A polynomial whose degree is one
Loss	When cost price of good are more than the selling price

M

Major arc
Markup
Minor arc
Monomials

An arc which is more than half of the circle
An amount that is added to the cost price to calculate the sale price
An arc which is smaller than half of the circle
Polynomial which contains only one term

N

Non-recurring
Decimals
Non-terminating
Decimals

A non-terminating decimal number that continues endlessly, with no digit or a block of digits repeating
Decimal numbers that have an infinite number of digits after the decimal point

O

Open sentence

statement (or sentence) that does not include enough information required to decide whether it is true or false
The extra expenses adding a surplus value to the cost of items or services

Overheads

P

Partnership

A business in which two or more than two individual or partners establish or manage a business

Polynomials

Polynomials are the algebraic expressions in which the powers of variables are whole numbers.

Prime factors

A number that are divisible by 1 and the number itself when selling price of goods are more than cost price.

Profit

the equality of two ratios is called proportion.

Proportion

3D solid with one polygon base and other lateral faces are triangular in shape

Pyramid

Pythagoras theorem

In a right angle triangle the square of the length of the hypotenuse is equal to the sum of the squares of the lengths of the other two sides.

Q

Quadratic polynomials A polynomial whose degree is two

R

Range

difference between the greatest and the smallest values given in the data

Rational number

The numbers in the form of p/q where $q \neq 0$

Raw data

The information that is collected initially without any organization or arrangement

arrangement

Real Numbers

The sum of rational and irrational numbers

Recurring Decimals

A decimal number that has a single digit or a block of digits that repeats itself an infinite number of times after the decimal point repeating event

Rhombus

A parallelogram in which all four sides are equal in measurement and none of the angle is 90° .

Rise

The vertical change between two points

Rotation

turning a shape around a fixed point

Run

The horizontal change between two points

S

Secant of a circle

A line intersect a circle at two distinct points

Sector of circle

The part of a circle enclosed by two radii and an arc

Segment of circle

A region of a circle which is separated by a chord

Selling price

The price of goods or items at which it is sold by a wholesaler or a shopkeeper to the customer

Set Builder Form

The shortest possible form that describes the elements of the set

Significant figure

The number of figures or digits that indicate the accuracy of any calculation or measurement

Similar figures

The objects or figures that have same shape but not same size

Sphere

A solid round figure which is generated by the rotation of circle on fixed point.

Square root

A factor of a number that when multiplied by itself, gives the number

Square

When a number is multiplied by itself

Symmetric property

interchanging the sides of an equation does not affect the result

T

Tangent to a circle

A line that touches a circle at only one point

Terminating Decimal

Decimal numbers that have finite number of digits after the decimal point

Numbers

Away of changing size or position of a shape or object

Transformation

A geometrical shape that has 3 sides and three angles

Triangle

polynomial which contain three terms

Trinomials

U

Unitary method

calculate the value of number of things by finding the value of one thing

V

Value

The result of any calculation

Vehicle insurance

insurance that covers the risk of fire, accidents, etc. against vehicles

Volume

the space occupied by any object

X

X-intercept

the x-coordinate of a point at which the graph intersects x-axis

Y-intercept

the y-coordinate of a point at which the graph intersects y-axis

Important Formulas

$$\text{Profit} = \text{selling price} - \text{cost price}$$

$$\text{Profit} = \text{S.P} - \text{C.P}$$

$$\text{Loss} = \text{cost price} - \text{selling price}$$

$$\text{Loss} = \text{C.P} - \text{S.P}$$

$$\text{Profit \%} = \frac{\text{Profit}}{\text{cost price}} \times 100\%$$

$$\text{Loss \%} = \frac{\text{Loss}}{\text{cost price}} \times 100\%$$

$$\text{Discount} = \text{M.P} - \text{S.P}$$

$$\text{Percentage Discount} = \frac{\text{Discount}}{\text{M.P}} \times 100\%$$

$$(a + b)^2 = a^2 + b^2 + 2ab$$

$$(a - b)^2 = a^2 + b^2 - 2ab$$

$$(a + b)(a - b) = a^2 - b^2$$

$$\pi = \frac{\text{circumference}}{\text{diameter}} = \frac{C}{d}$$

$$\text{Circumference} = \pi \times \text{diameter}$$

$$C = \pi d$$

$$\text{Diameter} = 2 \times \text{radius}$$

$$\text{Area of a circle} = \pi r^2$$

$$\text{Circumference} = 2\pi r$$

$$\text{Total surface area of a cylinder} = 2\pi r(h + r)$$

$$\text{Area of each flat face of a cylinder} = \pi r^2$$

$$\text{Area of the curved face of a cylinder} = 2\pi rh$$

$$\text{Volume of a cylinder} = \pi r^2 h$$

$$\text{Total surface area of a rectangular prism} = 2 \times [(l \times w) + (l \times h) + (w \times h)]$$

$$\text{Area of cube} = 6 \times l^2$$

$$\text{Area of Square prism} = 6 \times l^2$$

$$\text{Total surface area of a right prism} = bh + (s_1 + s_2 + s_3) \times L$$

$$\text{Volume of right rectangular prism} = l \times w \times h$$

$$\text{Volume of right square prism} = l \times l \times l$$

$$\text{Volume of the triangular prism} = \text{base area} \times \text{length of the prism} = \frac{1}{2} bh \times L$$

$$1 \text{ kilometre (km)} = 1000 \text{ metres (m)}$$

$$1 \text{ metre (m)} = 100 \text{ centimetres (cm)}$$

$$1 \text{ centimetre (cm)} = 10 \text{ millimetres (mm)}$$

$$1 \text{ hour} = 60 \text{ minutes}$$

$$1 \text{ minute} = 60 \text{ seconds}$$

$$1 \text{ hour} = 60 \times 60 \text{ seconds} = 3600 \text{ seconds}$$

$$\text{Speed} = \frac{\text{Distance}}{\text{Time}}$$

$$\text{Time} = \frac{\text{Distance}}{\text{Speed}}$$

$$\text{Distance} = \text{Speed} \times \text{Time}$$

$$\text{Average speed} = \frac{\text{total distance covered}}{\text{total time taken}}$$

$$\text{Sum of interior angles} = 180^\circ \times (n-2),$$

where n is number of sides of the polygon.

$$\text{Each interior angle of a regular polygon}$$

$$= \frac{\text{Sum of interior angles of the regular polygon}}{\text{Number of interior angles of the regular polygon}}$$

$$= \frac{180^\circ \times (n-2)}{n} \quad (\text{Where } n \text{ is the number of sides of regular polygon})$$

$$\text{The sum of exterior angles of an n-sided polygon}$$

$$= \frac{\text{the sum of interior and exterior angles of an n-sided polygon} - \text{the sum of interior angles of an n-sided polygon}}{\text{angles of an n-sided polygon}}$$

$$\text{Measure of an exterior angle of any polygon} = \frac{360^\circ}{\text{Number of sides of the polygon}}$$

$$\text{Angle of each sector} = \frac{\text{Frequency of each observation}}{\text{Total number of observations}} \times 360^\circ$$

$$\bar{X} = \frac{\sum x}{n}$$

$$\text{Mean of grouped data} = \bar{X} = \frac{\sum fx}{\sum f}$$

$$\text{Probability of an event} = P(E)$$

$$= \frac{\text{Number of favourable (desired) outcomes}}{\text{Total number of possible outcomes}}$$

Web Links

www.mathopenref.com

<https://www.math-only-math.com/significant-figures.html>

<https://www.onlinemath4all.com/estimation-word-problems.html>

<https://www.vedantu.com/maths/difference-between-rational-and-irrational-numbers>

<https://www.britannica.com/science/real-number>

https://math.answers.com/questions/What_do_you_mean_by_absolute_value_of_a_real_number

https://proofwiki.org/wiki/Ordering_Properties_of_Real_Numbers

<https://mathbitsnotebook.com/Algebra1/RealNumbers/RNProp.html>

<https://schoolbag.info/mathematics/basic/14.html>

<https://www.quora.com/What-is-the-difference-between-terminating-and-non-recurring-number>

<https://www.onlinemathlearning.com/proportions.html>

<https://www.edudose.com/maths/profit-loss-discount-formulas-tricks/>

<https://www.math-only-math.com/square-root-of-a-perfect-square-by-using-the-long-division-method.html>

<https://byjus.com/maths/how-to-find-cube-root/>

<https://www.cuemath.com/algebra/set-builder-notation/>

<https://www.mytutor.co.uk/answers/1044/GCSE/Maths/What-are-the-differences-between-arithmetic-and-geometric-sequences/>

<https://www.mathplanet.com/education/pre-algebra/right-triangles-and-algebra/the-pythagorean-theorem>

<https://www.khanacademy.org/math/cc-eighth-grade-math/cc-8th-geometry/cc-8th-volume/e/volume-of-cylinders--spheres--and-cones-word-problems>

https://www.youtube.com/watch?v=PUNHdOI-E_w

<https://www.statstutor.ac.uk/resources/uploaded/varstddev.pdf>

https://saylordotorg.github.io/text_introductory-statistics/s07-01-sample-spaces-events-and-their.html

<https://study.com/academy/lesson/comparing-theoretical-experimental-probability.html>

Index

A	22	Enlargement	243
Absolute value	113	Exponent	149
Algebraic expression	127	F	134
Algebraic Identities	2	Factorization	65
Approximation error	106	Factorization Method	284
Arithmetic sequence		Frequency distribution	288
B	149	Frequency polygon	
Base	116	G	107
Biquadratic polynomial		Geometric sequence	190
C	241	Gradient	171
Centre of rotation	246	Graphical method	
Chord	215	H	286
Circle	113	Histogram	252
Closed sentence	292	Hypotenous	
Combined event	48	I	293
Compound Proportion	248	Independent event	48
Concentric circle	248	Inheritance	45
Concyclic point	230	Installment	51
Cone	262	Insurance	34
Congruent figures	277	Inverse proportion	18
Continuous data	77	Irrational numbers	
Cube root	76	K	259
Cube	116	Kite	
Cubic polynomial		L	260
D	276	Line bisector	186
Data	97	Linear inequalities	114
De-Morgan's Law	87	Linear polynomial	40
Descriptive form	34	Loss	
Direct proportion	42	M	44
Discount	277	Markup	278
Discrete data		Measure of dispersion	
E	167	N	10
Elimination method		Non-terminating Decimal	

Number sequence	105	Set builder form	87
O	113	Significant figure	2
Open sentence	40	Similar figures	263
Overheads	257	Simplification	125
Parallelogram	48	Simultaneously linear equation	164
Partnership	76	Sphere	219
Perfect cube	62	Square root	64
Perfect square	114	Square	61
Polynomial	154	Standard Deviation	281
Power law	92	Subset	90
Power set	291	Substitution method	169
Probability	151	Symmetric property	24
Product law	40	T	
Profit	34	Tabular form	87
Proportion	224	Terminating Decimal	9
Pyramid	209	Transitive Property	24
Pythagoras theorem		Trapezium	258
Q	115	Triangle	249
Quadratic polynomial	277	V	
Quantitate Data	152	Variance	280
Quotient law		Venn diagram	88
R	278	X	
Range	18	X-intercept	192
Rational Numbers	20	Y	
Real number	10	Y-intercept	192
Recurring Decimals	24		
Reflexive Property	256		
Rhombus	239		
Rotation			
S	215		
Sector of circle	247		
Semicircle			

Authors' Profiles

Madeeha Nuzhat

Madeeha Nuzhat has done her Masters in Mathematics from the University of Peshawar and did her B. Ed from AIOU Islamabad. She also holds a 2-year certificate in Professional Manager in Educational Planning and Management. She has more than 15 years of experience in the field of education. She has taught mathematics in leading schools of Lahore. She has been associated with AFAQ Research and Development department since 2011. Currently she is working as a Senior Subject Head (Mathematics) at AFAQ. Her publications include various textbooks for local and international students of mathematics including the Primary Mathematics series (2018) and the textbooks based on the Single National Curriculum (SNC 2020) approved by the Ministry of Federal Education and Professional Training. Additionally, she has a vast experience of developing teacher training manuals, assessment papers, lesson plans, etc.

Sadia Manzoor

Sadia Manzoor is working for AFAQ (Association for Academic Quality) R&D department as a Subject Specialist in Mathematics. She has done Masters of Science in Mathematics and MEd. She has been in the field of education for more than 7 years. She has also worked with Dar-e-Arqam School System as a senior mathematics teacher. She has developed 29 books on mathematics.

قومی ترانہ

پاک سر زمین شاد باد کشورِ حسین شاد باد
تُو نشانِ عزمِ عالی شان ارضِ پاکستان!

مرکزِ یقین شاد باد پاک سر زمین کا نظام
قوتِ اُخوتِ عوام قوم، ملک، سلطنت
پابندہ باد پابندہ باد

شاد باد منزلِ مراد پرچم ستارہ و ہلال
رہبرِ ترقی و کمال ترجمانِ ماضی، شانِ حال
جانِ استقبال! سایہ خدائے ذوالجلال

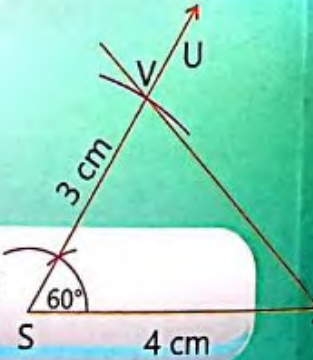


**Linear
Equations**

Gradient of a
straight line

Horizontal
Change

Geometry



Printed by

MALIK HOUSE, LHR

Quantity

339,844

Code No.

PSP/MHP-56-204-2223(O)